Variables

P = Power [W]
V = Voltage [V]
I = Current [A]
W = Energy [J]
R = Resistance [Ω]
ρ = resistivity
G = Conductance [Ω^{-1}]
σ = conductivity
λ = flux linkage [Wb-turn]
L = Inductance [H]
C = Capacitance [F]
ε_i = permittivity of i [F m^{-1}]
E = Electro Motive Force [V]

P = Real Power [W]
Q = Reactive Power [var]
↳ only imaginary
S = Complex Power [VA]
$|S|$ = Apparent Power

H = Magnetic Field Intensity [A m^{-1}]
J = Electric Current Density [A m^{-2}]
B = Magnetic Flux Density [J = Wb m^{-2}]
$\mathcal{F}$ = MagnetoMotive Force [A-turn]
ϕ = Magnetic Flux [Wb]
μ_i = Permeability of i [H m^{-1}]
$\mathcal{P}$ = Permeance [Wb A^{-1}]
$\mathcal{R}$ = Reluctance [A^{-1} Wb]

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e = induced voltage in coil terminals [V]
Z = Impedance [Ω]
X = Reactance [Ω]

$$P = VI = RI^2$$
$$W = \int RI^2 dt = \int P dt$$
$$R = \frac{\rho l}{S} \rightarrow l = \text{length},\ S = \text{Cross Section}$$
$$G = \frac{1}{R} \rightarrow \sigma = \frac{1}{\rho}$$

Inductor

$$\lambda = \int V dt$$
$$V = L\frac{di}{dt} \rightarrow i = \frac{1}{L}\int_0^T v\,dt + I_0$$
$$W = \frac{L(i_0)^2}{2}$$
$$X_L = \omega L = 2\pi f L$$

Capacitor:

$$q = VC$$
$$i = C\frac{dV}{dt}$$
$$V = \frac{1}{C}\int_0^T i\,dt + V_0$$
$$W = \frac{CV^2}{2}$$
$$C = \frac{\varepsilon_0 \varepsilon_r S}{d} \rightarrow S = \text{area},\ d = \text{gap}$$
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

Kirchoff's laws

- Electric
  - Voltage = 0 in a loop
  - Current = 0 at a node
- Magnetic
  - MMF = 0 in a loop
  - Flux = 0 at a node

→ Summing $\mathcal{R}$
- Series: $R_{tot} = R_1 + R_2$
- Parallel: $1/R_{tot} = 1/R_1 + 1/R_2$

Phasors

$$I = I_{max}(\cos(\omega t) + j\sin(\omega t))$$

→ Resistor: $V = IR$
→ Inductor: $V = -\omega L I_{max}(\sin(\omega t) + j\cos(\omega t))$
→ Capacitor: $V = \frac{I_{max}}{\omega C}(\sin(\omega t) - j\cos(\omega t))$

$$Ae^{j\omega t} = A(\cos(\omega t) + j\sin(\omega t))$$
$$Ae^{j(\omega t + \theta)} = A(e^{j\theta})(e^{j\omega t})$$
↳ At $t=0$, angle $= \theta$

Resistor: In Phase, Real power
Inductor: V leads I, Reactive Power
Capacitor: V lags I, Reactive Power

$$S = P + Q = v \times i$$

Power Factor: $\frac{P}{S} = \cos\phi$
where ϕ is the phase angle

$$I_{RMS}\sqrt{2} = I_{max}$$
$$V_{RMS}\sqrt{2} = V_{max}$$

Fourier and Laplace

1. Take Periodic Input
2. Convert into Sinusoidal functions
3. Find the output signal for each
4. Combine all output signals

Fourier: $\int_{-\infty}^{\infty} f(x)e^{-j\omega x}dx$ | Laplace $\int_0^{\infty} f(t)e^{-st}dt$

Magnetic Circuits

$$\mathcal{F} = Ni = H_c l_c = \frac{l_c B_c}{\mu_c} + \frac{g B_g}{\mu_0}$$
→ l_c = core length
g = gap length

$$\phi = BA \rightarrow A = \text{area}$$
$$B = \mu H$$
$$\mu_0 = 4\pi \times 10^{-7}\ H\ m^{-1}$$
$$\mathcal{R} = \frac{l}{\mu A} \rightarrow \mathcal{R}_g = \frac{g}{\mu_0 A}$$
$$\phi = \frac{\mathcal{F}}{\mathcal{R}_c + \mathcal{R}_g}$$

On a Conductor In a Magnetic Field:
$F_m = i(\text{length} \times B)$
$E = l(\text{velocity} \times B)$

Flux linkage

$$\lambda = \int e\,dt = N\phi$$
$$L = \frac{\lambda}{i} = \frac{N^2}{\mathcal{R}_{tot}}$$

Multicoil

$$\mathcal{F} = N_1 i_1 + N_2 i_2$$
$$\phi = \frac{N_i + N_i}{\mathcal{R}_{tot}} \approx \frac{N_i + N_i}{\mathcal{R}_g}$$
$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$
$$\lambda_1 = \left(\frac{(N_1)^2}{\mathcal{R}_g}\right)i_1 + \left(\frac{N_1 N_2}{\mathcal{R}_g}\right)i_2$$
$$P = ie = i\frac{d\lambda}{dt}$$

Stored energy $W = \frac{\lambda^2}{2L} = \frac{Li^2}{2}$

Transformers

→ 2 types: Core and Shell

In No-load conditions, an EMF is induced in the primary coil.

$$V = R_1 i_\phi + e$$
↳ induced EMF

In no load $R \sim 0$
↳ $V_1 = e_1 = \omega N_1 \phi_{max}\cos(\omega t)$

$V_{RMS} = E_{RMS} = \sqrt{2}\,f N_1 \phi_{max}$ $\quad f$ = frequency

$$\rightarrow \phi_{max} = \frac{V_1}{\sqrt{2}\pi f N_1}$$

Exciting Current: i_ϕ → waveform is different
In phase Component supplies power for core losses
Out of phase current lags 90° behind

Core loss $= P_c = E_1 I_\phi \cos\theta_c \rightarrow I_c = \frac{P_c}{E_1}$

Ideal Transformer:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} \quad P = V_1 I_1 = V_2 I_2 \quad Z_1 = \frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Secondary leakage Flux
$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$