Multiple Choice Section Questions (1-5)

Question 1 *On what interval is the following function concave down?*

$$f(x) = x^4 + 2x^3 - 12x^2 + x + 5$$

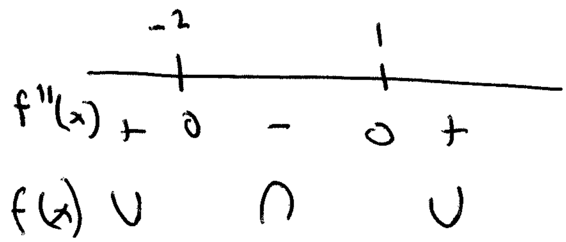
A) (0,4) **B)** (-2,1) **C)** (1,4) **D)** (-3,2) **E)** (-1,3)

$f'(x) = 4x^3 + \overset{6}{\cancel{8}}x^2 - 24x + 1$

$f''(x) = 12x^2 + \overset{12}{\cancel{16}}x - 24$

$= 12(x^2 + x - 2)$

$= 12(x+2)(x-1)$



Question 2 *Suppose that the demand function for a product is given by* $p = 216 - 30\sqrt{x}$. *What is the elasticity of demand when* $x = 9$? *Is demand elastic or inelastic?*

A) $\eta = -\frac{14}{5}$, inelastic **B)** $\eta = -\frac{14}{5}$, elastic **C)** $\eta = -\frac{1}{6}$, elastic
D) $\eta = -\frac{1}{6}$, inelastic **E)** $\eta = -1$, unit elastic

$p = 216 - 30\sqrt{9} = 126$

$x = 9$

$\frac{dp}{dx} = \frac{-15}{\sqrt{x}} = -5$

$\eta = \frac{\frac{126}{9}}{-5} = \frac{-126}{9 \cdot 5} = \frac{-14}{5}$ which is elastic

Question 3 *Consider the function $f(x) = xe^{3x}$. Which of the following statements is correct?*

A) There is a local min at $x = -\frac{1}{3}$.
B) There is a local max at $x = -\frac{1}{3}$.
C) There is a local max at $x = -3$.
D) There is a local min at $x = -3$.
E) There is a local max at $x = 1$.

$$f'(x) = e^{3x} + 3xe^{3x}$$
$$= (1+3x)e^{3x}$$

There is a CP at $x = -1/3$

		$x = -1/3$	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$		$\nearrow$

Question 4 *What is the absolute minimum value for the function $f(x) = x^3 + 3x^2 + 4$ on the interval [-3,-1]?*

A) -1 B) 0 C) 20 D) 4 E) There is no absolute minimum value.

$$f'(x) = 3x^2 + 6x = 3x(x+2)$$

CPs at $0, -2$

x	$f(x)$
-3	4
-2	8
-1	6

Question 5 *Consider the following function:*

$$f(x) = \frac{x^2 - 25}{x^2 - 2x - 15}$$

Which of the following statements is correct?

A) There is a horizontal asymptote at $y = 1$ and no vertical asymptotes.
B) There is a horizontal asymptote at $y = 1$ and only one vertical asymptote at $x = 5$.
C) There is a horizontal asymptote at $y = 1$ and only one vertical asymptote at $x = -3$
D) There is a horizontal asymptote at $y = 1$ and vertical asymptotes at $x = 5$ and $x = -3$.
E) There is no horizontal asymptote and only one vertical asymptote at $x = 5$

$$f(x) = \frac{(x+5)(x-5)}{(x+3)(x-5)}$$

VA at $x = -3$
NO VA at $x = 5$

$$\lim_{x \to \infty} f(x) = 1$$

Multiple Choice Section Questions (1-5)

Question 1 *On what interval is the following function concave down?*

$$f(x) = x^4 - 4x^3 - 18x^2 + x + 5$$

A) (0,4) **B)** (3,5) **C)** (1,4) **D)** (-3,2) **E)** (-1,3)

$f'(x) = 4x^3 - 12x^2 - 36x + 1$

$f''(x) = 12x^2 - 24x - 36$

$= 12 (x^2 - 2x - 3)$

$= 12(x-3)(x+1)$

		-1		3	
$f''(x)$	+		−		+
$f(x)$	∪		∩		∪

Question 2 *Suppose that the demand function for a product is given by* $p = 216 - 20\sqrt{x}$*. What is the elasticity of demand when* $x = 9$*? Is demand elastic or inelastic?*

A) $\eta = -\frac{26}{5}$, elastic **B)** ~~$\eta = -\frac{26}{5}$, inelastic~~ **C)** $\eta = -\frac{1}{6}$, elastic
D) $\eta = -\frac{1}{6}$, inelastic **E)** $\eta = -1$, unit elastic

$p = 216 - 20\sqrt{9} = 156$

$x = 9$

$dp/dx = \frac{-10}{\sqrt{x}} = \frac{-10}{3}$

$\eta = \frac{156/9}{\frac{-10}{3}} = -\frac{156}{9} \cdot \frac{3}{10} = \frac{-52}{10} = \frac{-26}{5}$

$\left|\frac{-26}{5}\right| > 1$ So demand is elastic

Question 3 *Consider the function $f(x) = xe^{-3x}$. Which of the following statements is correct?*

A) There is a local min at $x = \frac{1}{3}$.
B) There is a local max at $x = \frac{1}{3}$.
C) There is a local max at $x = 3$.
D) There is a local min at $x = 3$.
E) There is a local max at $x = -1$.

$$f'(x) = e^{-3x} - 3xe^{-3x}$$
$$= (1-3x)e^{-3x}$$

		1/3	
$f'(x)$	$+$	0	$-$
$f(x)$	$\nearrow$		$\searrow$

Question 4 *What is the absolute maximum value for the function $f(x) = x^3 - 3x^2 + 4$ on the interval [1,4]?*

A) -3 B) 0 C) 20 D)14 E) There is no absolute maximum value.

$$f'(x) = 3x^2 - 6x$$
$$= 3x(x-2)$$

CPs is 0,2

x	$f(x)$
1	2
2	0
4	20

Question 5 *Consider the following function:*

$$f(x) = \frac{x^2 - 9}{x^2 - 2x - 15}$$

Which of the following statements is correct?

A) There is a horizontal asymptote at $y = 1$ and no vertical asymptotes.
B) There is a horizontal asymptote at $y = 1$ and only one vertical asymptote at $x = 5$.
C) There is a horizontal asymptote at $y = 1$ and only one vertical asymptote at $x = -3$
D) There is a horizontal asymptote at $y = 1$ and vertical asymptotes at $x = 5$ and $x = -3$.
E) There is no horizontal asymptote and only one vertical asymptote at $x = 5$

$$\lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 2x - 15} = 1$$

So there is a HA at $y = 1$

Note $f(x) = \dfrac{(x+3)(x-3)}{(x+3)(x-5)}$

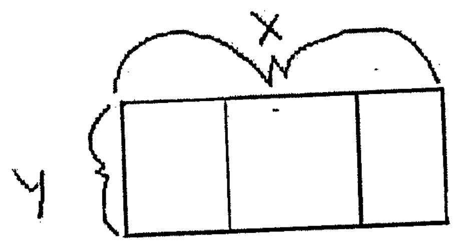
So there is a VA at $x = 5$

but <u>NOT</u> at $x = -3$

Long Answer Section Questions (6-7)

Question 6 (10 points)

A farmer must construct suitable fenced-in regions to contain his pigs. He must construct a rectangular area with two additional fences across its width, as shown in the following picture. Find the dimensions which maximize the area he can enclose with 2400 feet of fencing. You may assume all three rectangles have the same dimensions, but this will not affect the answer. **Be sure to explain why your answer is an absolute max, and not just local.**



We have $2x+4y=2400$ or $y=600-\frac{x}{2}$

We are trying to maximize

$$\text{Area} = xy = 600x - \frac{x^2}{2} = A(x)$$

$A'(x) = 600 - x$. So CP at $x=600$

It is an absolute max, since $A(x)$ is a CD parabola

If $x=600$, $y=?$

$$2(600)+4y=2400$$

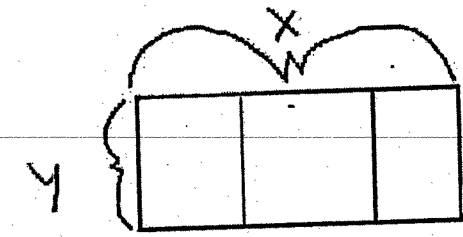
$$4y=1200$$

$$y=300$$

Long Answer Section Questions (6-7)

Question 6 (10 points)

A farmer must construct suitable fenced-in regions to contain his pigs. He must construct a rectangular area with two additional fences across its width, as shown in the following picture. Find the dimensions which maximize the area he can enclose with 2800 feet of fencing. You may assume all three rectangles have the same dimensions, but this will not affect the answer. **Be sure to explain why your answer is an absolute max, and not just local.**



$4y + 2x = 2800$. Maximize Area $= xy$

$y = 700 - \frac{1}{2}x$

So Area $= A(x) = 700x - \frac{x^2}{2}$

$A'(x) = 700 - x$

$x = 700$ is a CP.

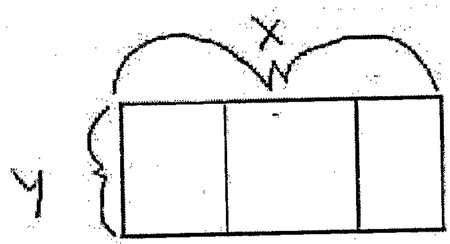
It is absolute since $A(x)$ is a CD parabola

If $x = 700$, $y = 350$

Long Answer Section Questions (6-7)

Question 6 (10 points)

A farmer must construct suitable fenced-in regions to contain his pigs. He must construct a rectangular area with two additional fences across its width, as shown in the following picture. Find the dimensions which maximize the area he can enclose with 3600 feet of fencing. You may assume all three rectangles have the same dimensions, but this will not affect the answer. **Be sure to explain why your answer is an absolute max, and not just local.**



$4y + 2x = 3600$

$y = 900 - \frac{x}{2}.$

Maximize $A(x) = 900x - \frac{x^2}{2}$

$A'(x) = 900 - x.$

So $x = 900$ is a CP.

If $x = 900$, $y = 450$

Again, $A(x)$ is a CD parabola.

Question 7 (10 points)

(a) (4 points) Suppose $f'(x) = \frac{4x+1}{\sqrt{x}}$*, and that* $f(4) = 2$*. Find* $f(x)$*.*

$f'(x) = 4\sqrt{x} + \frac{1}{\sqrt{x}} = 4x^{1/2} + x^{-1/2}$. $f(x) = \int (4x^{1/2} + x^{-1/2})\,dx$

$= \frac{4x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{8}{3}x^{3/2} + 2x^{1/2} + C$. $f(4) = \frac{64}{3} + 4 = \frac{76}{3} + C$

So $2 = \frac{76}{3} + C$ or $C = \frac{-70}{3}$

(b) (6 points) Calculate (Note this is a fourth root.)

$$\int \frac{x^2+1}{\sqrt[4]{x^3+3x+7}}dx$$

$$\boxed{\begin{array}{l} u = x^3+3x+7 \\ du = (3x^2+3)\,dx \end{array}}$$

$$= \frac{1}{3}\int \frac{3x^2+3\;dx}{\sqrt[4]{x^3+3x+7}}$$

$$= \frac{1}{3}\int u^{-1/4}\,du$$

$$= \frac{1}{3}\frac{u^{3/4}}{3/4} = \frac{4}{9}u^{3/4} = \frac{4}{9}(x^3+3x+7)^{3/4} + C$$

Question 7 (10 points)

(a) (4 points) Suppose $f'(x) = \frac{2x+1}{\sqrt{x}}$, and that $f(4) = 3$. Find $f(x)$.

$$f(x) = \int \frac{2x+1}{\sqrt{x}}\,dx = 2\int x^{+1/2}\,dx + \int x^{-1/2}\,dx$$

$$= 2\,\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{4}{3}x^{3/2} + 2x^{1/2} + C. \text{ We know } f(4)=3$$

So $f(4) = \frac{4}{3}\cdot 8 + 2\cdot 2 + C = \frac{32}{3} + \frac{12}{3} + C = \frac{44}{3} + C = 3$. So $C = \frac{-35}{3}$

(b) (6 points) Calculate (Note this is a cube root.)

$$\int \frac{x^2+1}{\sqrt[3]{x^3+3x+7}}dx$$

$$\boxed{\begin{aligned} u &= x^3+3x+7 \\ du &= (3x^2+3)\,dx = 3(x^2+1)\,dx \end{aligned}}$$

$$= \frac{1}{3}\int \frac{3(x^2+1)\,dx}{\sqrt[3]{x^3+3x+7}}$$

$$= \frac{1}{3}\int u^{-1/3}\,du = \frac{1}{3}\cdot\frac{u^{2/3}}{2/3} + C$$

$$= \frac{1}{3}\cdot\frac{3}{2}u^{2/3} + C = \frac{1}{2}(x^3+3x+7)^{2/3} + C$$

Question 7 (10 points)

(a) (4 points) Suppose $f'(x) = \frac{4x+1}{\sqrt{x}}$, and that $f(4) = 1$. Find $f(x)$.

$f'(x) = 4x^{1/2} + x^{-1/2}$ $\quad f(x) = \int (4x^{1/2} + x^{-1/2})\,dx$

$= \frac{4}{3}x^{3/2} + 2x^{1/2} + C$ $\quad f(4) = \frac{44}{3} + C = 1$. So $C = -\frac{41}{3}$

(b) (6 points) Calculate (Note this is a fifth root.)

$$\int \frac{x^2+1}{\sqrt[5]{x^3+3x+7}}dx$$

$$\boxed{\begin{aligned} &u = x^3+3x+7 \\ &du = (3x^2+3)\,dx = 3(x^2+1)\,dx \end{aligned}}$$

$$= \frac{1}{3}\int u^{-1/5}\,du$$

$$= \frac{1}{3}\frac{u^{4/5}}{4/5} = \frac{5}{12}u^{4/5} + C$$

$$= \frac{5}{12}(x^3+3x+7)^{4/5} + C$$