ADM 2304 Midterm Solutions
Winter 2011

Question 1 [10 marks]

a) Without reference to the data distributions but taking into account the manner in which the data were collected, identify two appropriate tests for determining whether there has been an overall increase in prices.

[1] *Since the cost price and the current price correspond to a single random sample of stocks, the two samples are **paired**. The two possible tests are the paired t or the paired Wilcoxon.*

b) Now look at the boxplots of the data. Which of the two tests above is the most appropriate? Explain briefly with reference to specific boxplots.

[2] *-Only the boxplot of the differences is relevant. The outliers suggest that we cannot assume that the sample of differences come from a normal distribution.*
-Therefore the best test is the paired sample Wilcoxon test.

c) Ignoring your answer in (b), perform the most appropriate parametric test to determine if there has been an overall increase in prices. Use the .10 significance level.

[4] *Ho: mean change = 0, Ha: mean change > 0 ,*
where change is market minus cost price

t = .1307 / 2.2574 = 0.058

Do not reject Ho since t is not > 1.345 (t(.10) based on 13 d.f.))

Conclude that there has not been an overall increase in prices.

d) Ignoring your answer in (b), perform the most appropriate non-parametric test.

[3] *Ho: median change = 0; Ha: median change > 0*

p-value is 0.19 is not < .10

Do not reject Ho. Conclude no overall increase in prices.

Question 2 [10 marks]

a. [4]

$$\hat{p} = \frac{x}{n} = \frac{57}{125} = 0.456$$

S1:

H_0: $p \geq (p_0 = 0.54)$ Ha: $p < (p_0 = 0.54)$ (1 mark)

S2:

$$SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}} = \sqrt{\frac{0.54(0.46)}{125}} = 0.0446$$ (0.5 mark)

Since np_0, $nq_0 > 10$

$$Z_{Calc} = \frac{\hat{p} - p_0}{SD(\hat{p})} = \frac{0.456 - 0.54}{0.0446} = -1.8843$$ (1 mark)

S3:

LS = $\alpha = 0.05$, $Z_{Crit} = Z_\alpha = Z_{0.05} = 1.645$ (0.5 marks)

S4:

Since $\{|Z_{Calc}| = 1.8843\} > \{Z_{Crit} = 1.645\}$ and $Z < 0$ ➔ Reject H0. (1.0 marks)

Based on the statistical evidence, it is appropriate to say that the proportion of people who agree with Friedman's contention is lower than 54%.

Note that $|Z_{Calc}| > 1.645$ is not enough to reject the null hypothesis without making sure that $Z_{Calc} < 0$. For example, suppose p-hat = .624 and Z = (.624 - .54)/.0446 = +1.8843. Since p-hat = .628 is not < .54, we should not reject the null H. However |1.8843| > 1.645 and based on this rule alone, we might have rejected the null H.

b. [2]

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.456(0.544)}{125}} = 0.0445$$ (0.5 marks)

$$UB = \hat{p} + Z_\alpha SE(\hat{p}) = 0.456 + 1.645 * 0.0445 = 0.5293$$ (1 mark)

Since $\{p0 = 0.54\} > UB = 0.5293$, H_0 must be rejected. (0.5 marks)

Many students calculated the wrong 1-sided CI, using a lower bound of p-hat – 1.645*.0445. This is appropriate for an alternative hypothesis that p > .54. Obviously, this interval covers the hypothesized value of p=.54 and results in accepting the null hypothesis that $p \leq .54$. This is completely reasonable since p-hat = .456 < .54 is consistent with the null hypothesis and inconsistent with the alternative hypothesis. This solution is worth only 0.5 marks.

Some students calculated a 2-sided interval and this covers the hypothesized value of .54, resulting in a non-rejection of the null hypothesis. This solution is worth 1 mark maximum since a 1-sided interval was requested.

Test and CI for One Proportion

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Test of p = 0.54 vs p < 0.54

                                    95% Upper
Sample    X    N  Sample p      Bound  Z-Value  P-Value
1        57  125  0.456000   0.529275    -1.88    0.030

Using the normal approximation.
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c. [2]

$n = \frac{Z_{\alpha/2}^2 pq}{ME^2}$ Since H_0 was rejected, the true value of 'p' is unknown. We could use p= 0.5

$n = \frac{Z_{\alpha/2}^2 pq}{ME^2} = \frac{1.96^2 * 0.5 * 0.5}{(0.02)^2} = 2401$ (2 marks)

Using the most recent estimate p-hat = .456, we get:

$n = \frac{Z_{\alpha/2}^2 \hat{p}\hat{q}}{ME^2} = \frac{1.96^2 * 0.456 * 0.544}{(0.02)^2} = 2382.4067 \approx 2383$ (2 marks)

Or,

$n = \frac{Z_{\alpha/2}^2 pq}{ME^2} = \frac{1.96^2 * 0.54 * 0.46}{(0.02)^2} = 2385.63 \approx 2386$ (2 mark)

The last calculation is questionable since we decided that p < .54 in parts a and b. However, the last two calculations are hard to tell apart. If $Z_{\alpha/2}$ is assigned the value 1.645, then deduct 1 mark for any of the above three since a 2-sided interval is being calculated.

d. [2]

S1:

H_0: $p \geq (p_0 = 0.25)$ Ha: $p < (p_0 = 0.25)$ (Optional)

S2:

Since np_0, $nq_0 < 10$, the normal approximation is not applicable.

p-Val = P[X ≤ 1]
= P[X = 0] + P[X = 1}
= 0.1001 + 0.2670, using the Binomial (n=8, p=.25)
= 0.3671 (1 mark)

Since {p-Val = 0.3671} > {LS = α = 0.05}➔ Do **not** Reject H_0 (1 mark)

If normal approximation is used then p-Val = 0.2071 and one would still not reject H_0, but it should be given only 1 mark out of 2 if all the steps are shown, other wise, only 0.5 marks.

Question 3. [10 marks]

a) Test whether there is sufficient evidence that the mean wait time is more than twice the target wait time. Use the .01 level of significance. Justify the use of the test you use.

[4]

-Ho: mu = 180, Ha: mu > 180

-t = (221-180)/(310/sqrt(120)) = 41/28.34 = 1.44

-The rejection region is t > 1.658 or z > 1.645

-Do not reject the null H and conclude that the mean time is not more than 180 days.

b) What is the p-value for the test in (a)?

[1]

Using normal approximation, the p-value is .075.

Using the t-table, the p-value is between .05 and .10.

c) Is the test in (a) valid given the distribution of the data? Explain briefly.

[1]

We can accept either "yes, it is valid since, despite the skewed distribution, the sample size is large enough and therefore the CLT justifies the test" or "no, it is not valid since despite the larger sample size, the distribution is so skewed, I would still be concerned about the application of the CLT". In other words, as long as they bring up the CLT and how it may or may not apply in this instance, they get the mark.

A very common answer was that the data need to be normally distributed. This is not required when the sample size exceeds 30. Then the CLT implies that the sample mean is normally distributed, but not the data, provided that the population data is not extremely skewed.

d) Use the Minitab output in Appendix A to determine whether in fact the median wait time is less than 180.

[3]

-Ho: median = 180, Ha: median < 180

-p-value of 0.061 not < .05

-Do not reject the null H; we cannot conclude that the true median is less than 180.

e) A larger sample determined that the mean wait time was in fact more than twice the target wait time and that the median wait time was in fact less than twice the target wait time. Does this coincide with your results in (a) and (d)? If not, why not?

[1]

To get the mark, I would expect the student to bring up the fact that in failing to reject the null we are not saying that the null is true and that since the p-values were almost less than 0.05, it is clear that the likelihood is still that the null is false. If they got both (a) and (d) wrong then they don't get this mark either. *Many solutions imply that the larger sample will result in normally distributed data. This is clearly not the case.*

Question 4. [10 marks]

a) Is there sufficient evidence, at the 5% significance level, to show that adult women spend more time watching TV than adult men? **What assumptions are required and how would you check them?**

[5]

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Sample    N   Mean  StDev  SE Mean
1        35  25.60   7.20      1.2
2        50  20.90   7.00     0.99

Difference = mu (1) - mu (2)
Estimate for difference:  4.70000
95% lower bound for difference:  2.08591
T-Test of difference = 0 (vs >): T-Value = 3.00  P-Value = 0.002  DF = 72
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-Ho: μ(women) = μ (men); Ha: μ (women) > μ (men) (μ is population mean)

-t=2.996, assuming unequal variance.
-df = 72, critical value = 1.671 (60 df) or 1.665 (75 df) or 1.645, p-value < 0.005
OR
-t=3.011, assuming equal variance.
-df = 83, critical value = 1.665 or 1.645, p-value < 0.005

-Reject null H, conclude women watch more TV than men.

-Assumptions: Since the sample sizes are both large, we only need to assume that the two populations of TV time are not extremely skewed. Check these assumptions by plotting the data.

b) Calculate the appropriate confidence interval to confirm the conclusion from (a). What added information does the confidence interval give you?

[3]

Without assuming equal population variances, the LB is 2.0879 (if using d.f. = 60) or LB = 2.0785 (if using d.f. = 75)
Assuming equal population variances, the LB is 2.101

1 mark for correctly identifying the need for a lower bound, 0.5 mark for using the appropriate critical value, 0.5 mark for the correct LB. Only 1 for correctly calculated 2-sided interval or for the wrong 1-sided interval.

The confidence interval allows you to confirm the conclusion from (a). That's standard. Added information: Moreover, it measures how far the interval is from not covering the hypothesized difference of zero in terms of the number of hours.

1 mark for answering the question about added information regardless of the interval calculated.

(c) - Here I expect them to point out that to make sure that they detect any real difference they need to make the potential of falsely attributing a difference where there isn't any a little higher and therefore the significance level of 0.1 is most appropriate.