Solution to Test 1(B)

MAY1320C, Fall 2014

1. [1 point] Solve for x: $2^{5x-6} = 16$.

Solution. $2^{5x-6} = 2^4$. $5x - 6 = 4$, $5x = 10$, $x = 2$.

2. [2 points] Find the limit $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$.

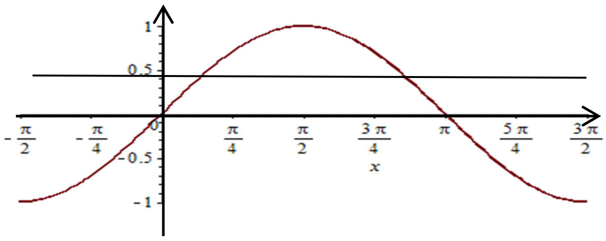
Solution. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3}\frac{\left(\sqrt{x+1}-2\right)\left(\sqrt{x+1}+2\right)}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to 3}\frac{(x+1)-4}{(x-3)\left(\sqrt{x+1}+2\right)}$

$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$.

3. [1 point] If $\frac{\pi}{2} < x < \frac{3\pi}{2}$, what is arcsin(sin($x$))?

Understand the question: If $u = \sin x$, then $x^* = \arcsin u$ is an angle in the range $-\frac{\pi}{2} \le x^* \le \frac{\pi}{2}$, such that $\sin x^* = u = \sin x$. Thus this question asks: what is an angle x^*, $-\frac{\pi}{2} \le x^* \le \frac{\pi}{2}$, such that $\sin x^* = \sin x$.

Solution. From the graph of the sine function, we see that $x^* = \pi - x$.



4. [3 points] Use the definition of the derivative to find $f'(x)$, if $f(x) = \frac{2x+3}{x-1}$.

Solution. $f'(x) = \lim_{h\to 0}\frac{1}{h}\left(\frac{2(x+h)+3}{x+h-1} - \frac{2x+3}{x-1}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{(2x+2h+3)(x-1)-(2x+3)(x+h-1)}{(x+h-1)(x-1)}\right)$

$= \lim_{h\to 0}\frac{1}{h}\left(\frac{h(2x-2-2x-3)}{(x+h-1)(x-1)}\right) = \lim_{h\to 0}\frac{-5}{(x+h-1)(x-1)} = -\frac{5}{(x-1)^2}$.

5. [2 points] Use known formulas to find the derivative of the function $y = \dfrac{\sin x}{x^3}$.

Solution. $y' = \dfrac{x^3 \cos x - 3x^2 \sin x}{x^6} = \dfrac{x \cos x - 3 \sin x}{x^4}$.

6. [3 points] For which value of x, does function $y = \sqrt{e^x - 3x}$ have a horizontal tangent line?

Solution. Let $y' = \dfrac{e^x - 3}{2\sqrt{e^x - 3x}} = 0$. $e^x = 3$, $x = \ln 3$.

7. [3 points] Consider function $y = f(x)$, where $f(1) = 3$, $f(2) = 1$, $f(3) = 2$, $f'(1) = 2$, $f'(2) = 3$, $f'(3) = -1$.

(a) Find $(f \circ f)(1)$.

(b) Find the derivative of the composite function $(f \circ f)$ at $x = 1$.

Solution. (a) $(f \circ f)(1)= f(f(1)) = f(3) = 2$.

(b) Let $u = f(x)$. Then $y = f(u)$. When $x = 1$, $f'(1) = 2$. When $x = 1$, $u = 3$, $f'(3) = -1$. $(f \circ f)'(1) = -2$.