Solution to Test 1(A)

MAY1320C, Fall 2014

1. [1 point] Solve for x: $5^{3x-7} = 25$.

Solution. $5^{3x-7} = 5^2$. $3x - 7 = 2$, $3x = 9$, $x = 3$.

2. [2 points] Find the limit $\lim_{x\to 2}\frac{\sqrt{3x+3}-3}{x-2}$.

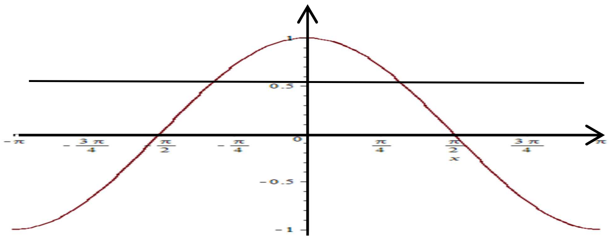
Solution. $\lim_{x\to 2}\frac{\sqrt{3x+3}-3}{x-2} = \lim_{x\to 2}\frac{\left(\sqrt{3x+3}-3\right)\left(\sqrt{3x+3}+3\right)}{(x-2)\left(\sqrt{3x+3}+3\right)} = \lim_{x\to 2}\frac{(3x+3)-9}{(x-2)\left(\sqrt{3x+3}+3\right)}$

$= \lim_{x\to 2}\frac{3}{\sqrt{3x+3}+3} = \frac{1}{2}$.

3. [1 point] If $-\pi < x < 0$, what is arccos(cos(x))?

Understand the question: If $u = \cos x$, then $x^* = \arccos u$ is an angle in the range $0 \le x^* \le \pi$, such that $\cos x^* = u = \cos x$. Thus this question asks: what is an angle x^*, $0 \le x^* \le \pi$, such that $\cos x^* = \cos x$.

Solution. From the graph of the cos function, we see that $x^* = -x$.



4. [3 points] Use the definition of the derivative to find $f'(x)$, if $f(x) = \frac{2x-1}{x+1}$.

Solution. $f'(x) = \lim_{h\to 0}\frac{1}{h}\left(\frac{2(x+h)-1}{x+h+1} - \frac{2x-1}{x+1}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{(2x+2h-1)(x+1)-(2x-1)(x+h+1)}{(x+h+1)(x+1)}\right)$

$= \lim_{h\to 0}\frac{1}{h}\left(\frac{3h}{(x+h+1)(x+1)}\right) == \lim_{h\to 0}\frac{3}{(x+h+1)(x+1)} = \frac{3}{(x+1)^2}$.

5. [2 points] Use known formulas to find the derivative of the function $y = \frac{\cos x}{x^2}$.

Solution. $y' = \dfrac{-x^2 \sin x - 2x\cos x}{x^4} = -\dfrac{x\sin x + 2\cos x}{x^3}$.

6. [3 points] For which value of *x*, does function $y = \sqrt{e^x - 2x}$ have a horizontal tangent line?

Solution. Let $y' = \dfrac{e^x - 2}{2\sqrt{e^x - 2x}} = 0$. $e^x = 2$, $x = \ln 2$.

7. [3 points] Consider function $y = f(x)$, where $f(0) = 2$, $f(1) = 3$, $f(2) = 1$, $f'(0) = 4$, $f'(1) = 5$, $f'(2) = 7$.

(a) Find $(f \circ f)(0)$.

(b) Find the derivative of the composite function $(f \circ f)$ at $x = 0$.

Solution. (a) $(f \circ f)(0)= f(f(0)) = f(2) = 1$.

(b) Let $u = f(x)$. Then $y = f(u)$. When $x = 0$, $f'(0) = 4$. When $x = 0$, $u = 2$, $f'(2) = 7$. $(f \circ f)'(0) = 28$.