

QUESTION 2. Find the derivatives of the following functions. Do not simplify.

a. $f(x) = e^{\arcsin^5 x + \arcsin x^5}$

$$f'(x) = (\arcsin^5 x + \arcsin x^5)' e^{\arcsin^5 x + \arcsin x^5}$$

$$= \left(5 \arcsin^4 x \frac{1}{\sqrt{1-x^2}} + \frac{5x^4}{\sqrt{1-x^{10}}} \right) e^{\arcsin^5 x + \arcsin x^5}$$

$$f'(x) = \left(5 \arcsin^4 x \frac{1}{\sqrt{1-x^2}} + \frac{5x^4}{\sqrt{1-x^{10}}} \right) e^{\arcsin^5 x + \arcsin x^5}$$

b. $f(x) = \arctan(\cos \sqrt{x}) + \arccos(\tan \sqrt{x})$

$$f'(x) = \frac{(\cos \sqrt{x})'}{1 + \cos^2 \sqrt{x}} - \frac{(\tan \sqrt{x})'}{\sqrt{1 - \tan^2 \sqrt{x}}} = \frac{-\frac{1}{2\sqrt{x}} \sin \sqrt{x}}{1 + \cos^2 \sqrt{x}} - \frac{\frac{1}{2\sqrt{x}} \sec^2 \sqrt{x}}{\sqrt{1 - \tan^2 \sqrt{x}}}$$

$$f'(x) = \frac{-\sin \sqrt{x}}{2\sqrt{x} (1 + \cos^2 \sqrt{x})} - \frac{\sec^2 \sqrt{x}}{2\sqrt{x} \sqrt{1 - \tan^2 \sqrt{x}}}$$

c. $f(x) = (\sqrt[3]{\sin^2 x})^{\arctan x}$

[Hint: Rewrite the function using the identity $a = e^{\ln a}$ and then calculate its derivative.]

$$\ln f(x) = \ln (\sqrt[3]{\sin^2 x})^{\arctan x}$$

$$\ln f(x) = \arctan x \ln (\sqrt[3]{\sin^2 x})$$

$$\ln f(x) = \frac{2}{3} \arctan x \ln \sin x$$

$$f(x) = e^{\ln f(x)} \rightarrow e^{\frac{2}{3} \arctan x \ln \sin x}$$

$$f'(x) = \left(\frac{2}{3} \arctan x \ln \sin x \right)' e^{\frac{2}{3} \arctan x \ln \sin x}$$

$$f'(x) = \frac{2}{3} \left(\frac{1}{1+x^2} \ln \sin x + \frac{\cos x}{\sin x} \arctan x \right) e^{\frac{2}{3} \arctan x \ln \sin x}$$

QUESTION 3. Find all local and global maxima and minima of the function $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{15}{8}x^2 + \frac{2}{3}$ on the interval $[-3, 3]$.

$$f'(x) = x^3 - x^2 - \frac{15}{4}x \quad \text{Domain of } f(x) : \mathbb{R}$$

To find critical points:

$$f'(x) = 0 \Rightarrow x^3 - x^2 - \frac{15}{4}x = 0 \Rightarrow x(x^2 - x - \frac{15}{4}) = 0$$

$$\begin{cases} x = 0 \\ \text{or} \\ x^2 - x - \frac{15}{4} = 0 \Rightarrow (x - \frac{5}{2})(x + \frac{3}{2}) = 0 \end{cases} \begin{matrix} \rightsquigarrow x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2} \\ \rightsquigarrow x + \frac{3}{2} = 0 \Rightarrow x = -\frac{3}{2} \end{matrix}$$

since all those points are in domain of $f(x)$, then there exist three critical points: $\{0, -\frac{3}{2}, \frac{5}{2}\}$. (Note: $f(x)$ exists on \mathbb{R})

To find local extrema;

x	$-\infty$	$-\frac{3}{2}$	0	$\frac{5}{2}$	∞	
x	-	0	-	0	+	
$x^2 - x - \frac{15}{4}$	+	0	-	0	+	
$f'(x)$	-	0	+	0	-	
$f(x)$	Decreasing		Increasing		Decreasing	
		↓	↑	↓	↑	
		local min	local max	local min		

Then, $f(-\frac{3}{2}) = -1.16 \Rightarrow$ local min

$f(0) = 0.67 \Rightarrow$ local max

$f(\frac{5}{2}) = -6.49 \Rightarrow$ local min and global min. 3

To find global extrema,

$f(3) = -4.96$

$f(-3) = 13.04 \Rightarrow$ global max.

QUESTION 4. Use the first and second order derivatives to sketch the graph of the function $f(x) = \frac{2}{x^2} + \frac{3}{x^3}$. You have to find the zeros, critical points, inflection points, intervals where the function is increasing and decreasing, and the vertical and horizontal asymptotes if any.

$$f(x) = \frac{2x+3}{x^3}$$

• Domain of $f(x)$ is $\mathbb{R} - \{0\}$.

• There isn't any y -intercept, since $x=0$ is not in domain.

There is one x -intercept, $f(x)=0 \Rightarrow 2x+3=0 \Rightarrow x = -\frac{3}{2}$

• $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{2x+3}{x^3} = \lim_{x \rightarrow \infty} \frac{2x}{x^3} = 0 \Rightarrow$ then $y=0$ is horizontal asymptote.

• $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{2x+3}{x^3} = +\infty$, $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{2x+3}{x^3} = -\infty$

then $x=0$ is vertical asymptote.

$$f'(x) = \frac{2(x^3) - 3x^2(2x+3)}{x^6}$$

$$= \frac{2x^3 - 6x^3 - 9x^2}{x^6}$$

$$= \frac{-4x^3 - 9x^2}{x^6}$$

$$= \frac{-4x-9}{x^4}$$

$f'(x)=0 \Rightarrow -4x-9=0 \Rightarrow x = -\frac{9}{4}$ is in domain $f(x)$

then $x = -\frac{9}{4}$ is critical point! (but $x=0$ isn't critical point!)

$$f''(x) = \frac{-4x^4 - 4x^3(-4x-9)}{x^8}$$

$$= \frac{12x + 36}{x^5}$$

$$\rightarrow f''(x) = 0 \rightarrow 12x + 36 = 0$$

$$\underline{x = -3}$$

Interval of Increase or Decrease

x	$-\infty$	$-\frac{9}{4}$	0	$+\infty$
$-4x-9$	+	0	-	-
x^4	+	+	+	+
$f'(x)$	+	0	-	-
$f(x)$	Increase	Decrease	Decrease	

\uparrow $f(-\frac{9}{4})$ \downarrow
 local max

Concavity

x	$-\infty$	-3	0	$+\infty$
$12x+36$	-	0	+	+
x^5	-	-	+	+
$f''(x)$	+	-	+	+
$f(x)$	Concave Up	Concave down	Concave Up	

\downarrow
 It's inflection point!

$$f(-3) = \frac{27}{243}$$

$$f(-\frac{9}{4}) = \frac{32}{243}$$

$$f(-\frac{3}{2}) = 0$$

