1.

a) $P = \begin{array}{c|ccc|} & C & S & G \\ C & 0.7 & 0.1 & 0.1 \\ S & 0.2 & 0.8 & 0.2 \\ G & 0.1 & 0.1 & 0.7 \end{array}$

b) $P^3 a_0 = \begin{vmatrix} 0.7 & 0.1 & 0.1 \\ 0.2 & 0.8 & 0.2 \\ 0.1 & 0.1 & 0.7 \end{vmatrix}^3 \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0.52 & 0.16 & 0.16 \\ 0.32 & 0.68 & 0.32 \\ 0.16 & 0.16 & 0.52 \end{vmatrix} \begin{vmatrix} 0.7 \\ 0.2 \\ 0.1 \end{vmatrix} = \begin{vmatrix} 0.412 \\ 0.392 \\ 0.196 \end{vmatrix}$

Hence. the probability of C is 0.412. the probability of S is 0.392, the probability of G is 0.196

c) The matrix is regular, so a steady-state vector exists, hence the process will converges to the fixed probabilities.

d) $P - I = \begin{vmatrix} -0.3 & 0.1 & 0.1 \\ 0.2 & -0.2 & 0.2 \\ 0.1 & 0.1 & -0.3 \end{vmatrix} \sim \begin{vmatrix} 1 & 1 & -3 \\ 0 & -4 & 8 \\ 0 & 4 & -8 \end{vmatrix} \sim \begin{vmatrix} 1 & 1 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{vmatrix}$ $x_1 - x_3 = 0$, $x_2 - 2x_3 = 0$

$\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} x_3 \\ 2x_3 \\ x_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 1 \end{vmatrix} x_3$. $P(C) = \frac{1}{4}$, $P(S) = \frac{2}{4} = \frac{1}{2}$, $P(G) = \frac{1}{4}$

∴ The percentage of C is 25%, of S is 50%. of G is 25%

...ges to the fixed probabilities

d) $P - I = \begin{vmatrix} -0.3 & 0.1 & 0.1 \\ 0.2 & -0.2 & 0.2 \\ 0.1 & 0.1 & -0.3 \end{vmatrix} \sim \begin{vmatrix} 1 & 1 & -3 \\ 0 & -4 & 8 \\ 0 & 4 & -8 \end{vmatrix} \sim \begin{vmatrix} 1 & 1 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{vmatrix}$ $\quad x_1 - x_3 = 0$, $x_2 - 2x_3 = 0$

$\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} x_3 \\ 2x_3 \\ x_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 1 \end{vmatrix} x_3$. $P(C) = \frac{1}{4}$, $P(S) = \frac{2}{4} = \frac{1}{2}$, $P(G) = \frac{1}{4}$

∴ The percentage of C is 25%, of S is 50%, of G is 25%

2.

a) $tI - A = \begin{vmatrix} t+3 & 0 & -1 \\ 0 & t+3 & -1 \\ 0 & 0 & t-2 \end{vmatrix} = (t+3)^2(t-2)$

b) $\begin{vmatrix} -3-\lambda & 0 & 1 \\ 0 & -3-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (-3-\lambda)^2(2-\lambda) = 0$. $\lambda_1 = -3$, $\lambda_2 = 2$

c) $\lambda = -3$, $\begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{vmatrix} \sim \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$, $x_3 = 0$. $\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} x_1 \\ x_2 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} x_1 + \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix} x_2$

$\lambda = 2$ $\begin{vmatrix} -5 & 0 & 1 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$ $\quad -5x_1 + x_3 = 0$, $-5x_2 + x_3 = 0$. $\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} \frac{1}{5}x_3 \\ \frac{1}{5}x_3 \\ x_3 \end{vmatrix} = \begin{vmatrix} \frac{1}{5} \\ \frac{1}{5} \\ 1 \end{vmatrix} x_3 = \begin{vmatrix} 1 \\ 1 \\ 5 \end{vmatrix} x_3$

∴ For $\lambda = -3$, the basis are $\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$, $\begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$. For $\lambda = 2$, the basis is $\begin{vmatrix} 1 \\ 1 \\ 5 \end{vmatrix}$.

d) We have 3 vectors, A is 3x3, so A is diagonalizable

$P = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{vmatrix}$. $D = \begin{vmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{vmatrix}$. $P^{-1} = \frac{1}{5} \begin{vmatrix} 5 & 0 & -1 \\ 0 & 5 & -1 \\ 0 & 0 & 1 \end{vmatrix}$

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b) $\begin{vmatrix} \frac{1}{2}-\lambda & \frac{1}{2} \\ \frac{3}{2} & -\lambda \end{vmatrix} = (\frac{1}{2}-\lambda)(-\lambda) - \frac{3}{2} = \lambda^2 - \frac{1}{2}\lambda - \frac{3}{2} = (\lambda+1)(\lambda-\frac{3}{2}) = 0.$ $\lambda_1 = -1,\ \lambda_2 = \frac{3}{2}$

Then $\lambda = \frac{3}{2}$ is the dominant eigenvalue

c) $\lambda = -1$ $\begin{vmatrix} \frac{3}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & 1 & 0 \end{vmatrix} \sim \begin{vmatrix} 3 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$, $3x_1 + x_2 = 0$, $\begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 1 \\ -3 \end{vmatrix} x_1$

$\lambda = \frac{3}{2}$, $\begin{vmatrix} -1 & \frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{3}{2} & 0 \end{vmatrix} \sim \begin{vmatrix} -2 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$, $-2x_1 + x_2 = 0$ $\begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \end{vmatrix} x_1$

$P = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix}$ $D = \begin{vmatrix} -1 & 0 \\ 0 & \frac{3}{2} \end{vmatrix}$ $P^{-1} = \frac{1}{5}\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$

$V_k = A_k V_0 = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} \begin{vmatrix} (-1)^k & 0 \\ 0 & (\frac{3}{2})^k \end{vmatrix} \frac{1}{5} \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} \begin{vmatrix} 60 \\ 40 \end{vmatrix}$

$= \begin{vmatrix} (-1)^k & (\frac{3}{2})^k \\ -3(-1)^k & 2(\frac{3}{2})^k \end{vmatrix} \begin{vmatrix} 16 \\ 44 \end{vmatrix}$

$\lambda = \frac{3}{2}$, $\begin{vmatrix} -1 & \frac{1}{2} & 0 \\ 3 & -\frac{3}{2} & 0 \end{vmatrix} \sim \begin{vmatrix} -2 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$, $-2x_1 + x_2 = 0$, $\begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \end{vmatrix} x_1$

$P = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix}$ $D = \begin{vmatrix} -1 & 0 \\ 0 & \frac{3}{2} \end{vmatrix}$ $P^{-1} = \frac{1}{5} \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$

$V_k = A_k V_0 = \begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} \begin{vmatrix} (-1)^k & 0 \\ 0 & (\frac{3}{2})^k \end{vmatrix} \frac{1}{5} \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} \begin{vmatrix} 60 \\ 40 \end{vmatrix}$

$= \begin{vmatrix} (-1)^k & (\frac{3}{2})^k \\ -3(-1)^k & 2(\frac{3}{2})^k \end{vmatrix} \begin{vmatrix} 16 \\ 44 \end{vmatrix}$

$= \begin{vmatrix} 16(-1)^k + 44(\frac{3}{2})^k \\ -48(-1)^k + 88(\frac{3}{2})^k \end{vmatrix}$

$\therefore a_k = 16(-1)^k + 44(\frac{3}{2})^k \quad j_k = -48(-1)^k + 88(\frac{3}{2})^k$

$a_k \to 0, \quad j_k \to 0$

d) $a_k = (\frac{3}{2})^k (16(-\frac{2}{3})^k + 44) \approx 44(\frac{3}{2})^k$

$j_k = (\frac{3}{2})^k (-48(-\frac{2}{3})^k + 88) \approx 88(\frac{3}{2})^k$