# DYNAMICS OF MECHANISMS
# MECH-314

**MIDTERM EXAM**
Wednesday, March 19, 2014, 13:05 – 14:25

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

- STUDENTS ARE ONLY ALLOWED A ONE-PAGE (8.5” X 11”) CRIB-SHEET, WITH WRITING ON BOTH SIDES.
- PLEASE ANSWER ALL QUESTIONS.
- QUESTIONS MUST BE ANSWERED DIRECTLY ON THE EXAM PAPER, IN THE SPACE PROVIDED BELOW OR BESIDE THE QUESTIONS.

Student Name:

Student Number:

<u>Question 1:</u> [ 5 ]
What are the first order kinematic coefficients?

*They are the first derivatives of the kinematic pair variables of a mechanism with respect to the input variables.*

<u>Question 2:</u> [ 5 ]
Enunciate the Aronhold-Kennedy theorem.

*Choose any three bodies in a mechanism. The ICV's for the three pairs that can be formed with these three bodies lie on the same straight line*

<u>Question 3:</u> [ 5 (bonus) ]
Kinematic pairs represent the geometric or kinematic constraints imposed on the motion of the links of a mechanical system. What is the main difference between higher and lower kinematic pairs?

*In a lower kinematic pair, the contact between the links takes place in a surface.*

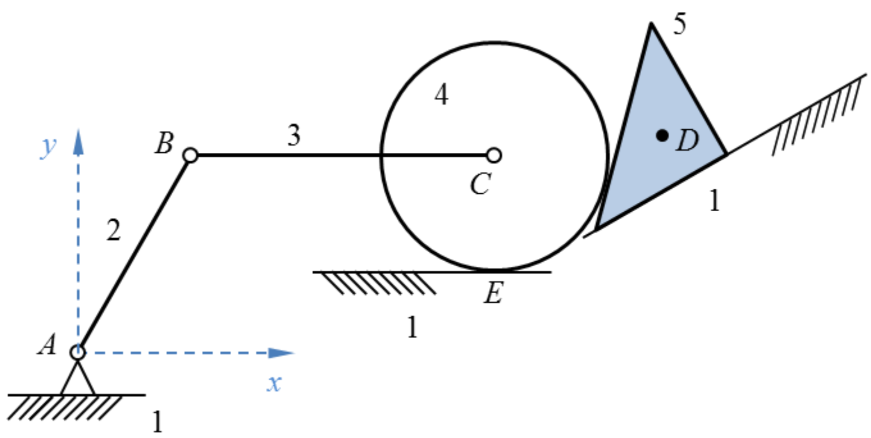
*In a higher kinematic pair, the contact between the links takes place in a point or a line.*

Problem 1: [25 + 15]

The figure shows the scaled kinematic diagram of a mechanism (scale 1 cm : 1 cm). The following dimensions are known: $R_{BA}$ = 3 cm, $R_{CB}$ = 4 cm, $R_{CE}$ = 1.5 cm. The angular velocity of link 2 is $\omega_2$ = 10 rad/s, ccw. The contacts between link 4 and links 1 and 5 are general direct contacts, where slip is possible. The contact between links 5 and 1 can be considered a prismatic pair.

a) Determine the apparent velocity of point $C_3$ with respect to link 5 ($V_{C3/5}$), and the velocity of point $D_5$. Use graphical velocity analysis and scale 1 cm : 5 cm/s. Give the detailed set of necessary velocity difference and/or apparent velocity equations and construct the velocity vector diagram. What is the angular velocity of link 5?

b) Assume rolling at point $E$ between links 1 and 4. For this case, determine the angular velocity of link 4, $\omega_4$.

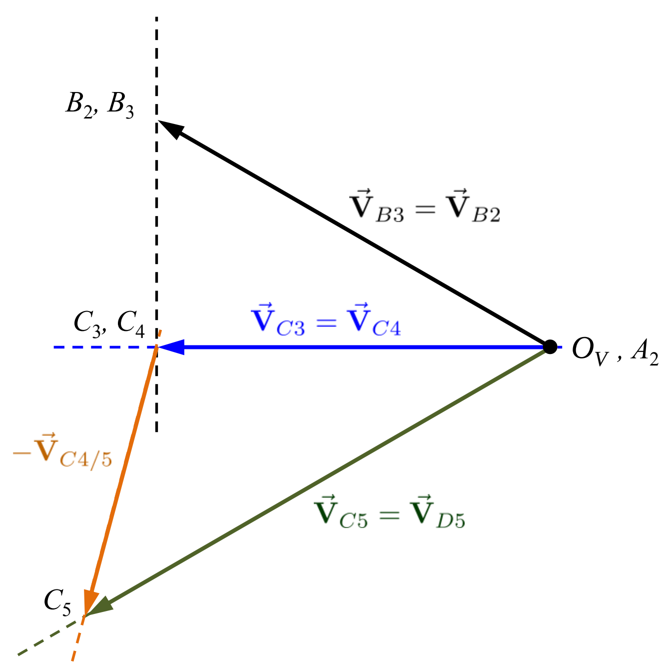



*Links 2 and 3*

$$\vec{\mathbf{V}}_{B2} = \cancel{\vec{\mathbf{V}}_{A2}} + \underbrace{\vec{\boldsymbol{\omega}}_2 \times \vec{\mathbf{R}}_{BA}}_{30\ \mathrm{cm/s}\ \perp\ \vec{\mathbf{R}}_{BA}}$$

$$\vec{\mathbf{V}}_{B2} = \vec{\mathbf{V}}_{B3}$$

$$\vec{\mathbf{V}}_{C3} = \vec{\mathbf{V}}_{B3} + \underbrace{\vec{\boldsymbol{\omega}}_3 \times \vec{\mathbf{R}}_{CB}}_{\vec{\mathbf{V}}_{C3B3}\ \perp\ \vec{\mathbf{R}}_{CB}}$$




*Higher pair between links 1 and 4*

$$\vec{\mathbf{V}}_{C3} = \vec{\mathbf{V}}_{C4}$$

$$\vec{\mathbf{V}}_{C4} = \cancel{\vec{\mathbf{V}}_{C1}} + \underbrace{\vec{\mathbf{V}}_{C4/1}}_{\text{horizontal}}$$

*Prismatic pair between links 1 and 5*

$$\vec{\boldsymbol{\omega}}_1 = \vec{\boldsymbol{\omega}}_5 = \vec{\mathbf{0}} \longrightarrow \vec{\mathbf{V}}_{D5} = \vec{\mathbf{V}}_{C5}$$

$$\vec{\mathbf{V}}_{C5} = \cancel{\vec{\mathbf{V}}_{C1}} + \underbrace{\vec{\mathbf{V}}_{C5/1}}_{\parallel\ \text{contact line 1-5}}$$

*Velocity of point $D_5$ (from diagram):*

$$\vec{\mathbf{V}}_{D5} = \vec{\mathbf{V}}_{C5}$$

$$V_{D5} = 35.5\ \mathrm{cm/s} \swarrow$$

*Higher pair between links 4 and 5*

$$\vec{\mathbf{V}}_{C4} = \vec{\mathbf{V}}_{C5} + \underbrace{\vec{\mathbf{V}}_{C4/5}}_{\parallel\ \text{contact line 4-5}}$$

$$\longrightarrow \quad \vec{\mathbf{V}}_{C5} = \vec{\mathbf{V}}_{C4} - \vec{\mathbf{V}}_{C4/5}$$

*If there is rolling between 1 and 4 at E:*

$$\vec{\mathbf{V}}_{E4} = \vec{\mathbf{V}}_{E1} = \vec{\mathbf{0}}$$

$$\vec{\mathbf{V}}_{C4} = \cancel{\vec{\mathbf{V}}_{E4}} + \vec{\boldsymbol{\omega}}_4 \times \vec{\mathbf{R}}_{CE}$$

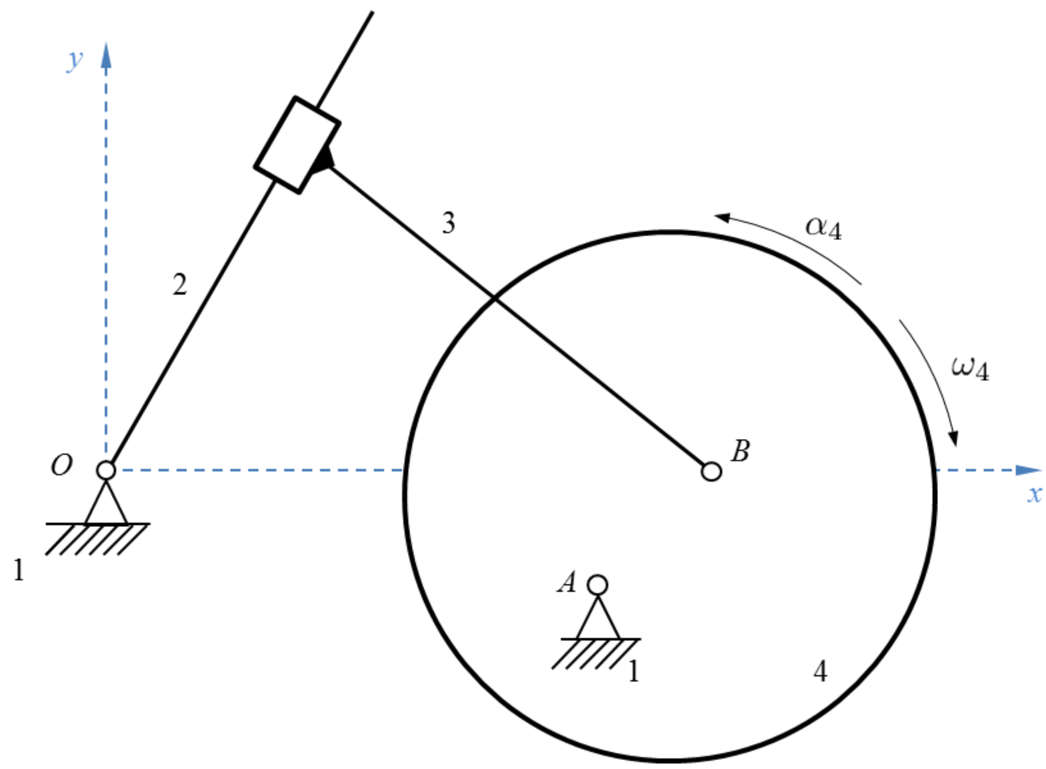
$$\longrightarrow \quad \omega_4 = \frac{V_{C4}}{R_{CE}} = \frac{26\ \mathrm{cm}}{1.5\ \mathrm{cm}} = 17.3\ \mathrm{rad/s\ CCW}$$

Problem 3: [35 + 15]

The scaled position diagram of a mechanism is shown in the figure: $R_{BO}$ = 8 cm, $R_{BA}$ = 2.1 cm. Disk 4 is attached to link 1 via a revolute joint at point $A$. The angular velocity of link 4 is $\omega_4$ = 1 rad/s cw. From the velocity analysis of the mechanism, it is known that $\omega_3 = \omega_4$ = 0.51 rad/s cw, and the apparent velocity of point $B_3$ with respect to link 2 ($V_{B3/2}$) is 3 cm/s, parallel to link 2 and pointing upwards. The input of the mechanism is $\alpha_4$ = 2 rad/s², ccw.

a) Perform the complete acceleration analysis of the mechanism using the graphical method. Obtain the acceleration images for points $B_2$, $B_3$, and $B_4$. Give the necessary sets of acceleration difference and/or apparent acceleration equations. Use scale 1 cm : 1 cm/s².

b) Determine the angular acceleration of link 2 ($\alpha_2$) and the acceleration of point $B_2$.




*Link 4*

$$\vec{\mathbf{A}}_{B4} = \vec{\mathbf{A}}_{A4} + \underbrace{\vec{\mathbf{A}}^t_{B4A4}}_{\substack{\vec{\boldsymbol{\alpha}}_4 \times \vec{\mathbf{R}}_{BA} \\ 4.2\ \text{cm/s}^2 \perp \vec{\mathbf{R}}_{BA}}} + \underbrace{\vec{\mathbf{A}}^n_{B4A4}}_{\substack{-\omega_4^2 \vec{\mathbf{R}}_{BA} \\ 2.1\ \text{cm/s}^2 \parallel \vec{\mathbf{R}}_{BA}}} = \vec{\mathbf{A}}_{B3}$$

*Link 2*

$$\vec{\mathbf{A}}_{B2} = \vec{\mathbf{A}}_{O2} + \underbrace{\vec{\mathbf{A}}^t_{B2O2}}_{\substack{\vec{\boldsymbol{\alpha}}_2 \times \vec{\mathbf{R}}_{BO} \\ \perp \vec{\mathbf{R}}_{BO}}} + \underbrace{\vec{\mathbf{A}}^n_{B2O2}}_{\substack{-\omega_2^2 \vec{\mathbf{R}}_{BO} \\ 2.08\ \text{cm/s}^2 \parallel \vec{\mathbf{R}}_{BO}}}$$

*Prismatic pair between links 2 and 3*

$$\vec{\mathbf{A}}_{B2} = \vec{\mathbf{A}}_{B3} + \vec{\mathbf{A}}^c_{B2B3} + \vec{\mathbf{A}}_{B2/3}$$

$$\vec{\mathbf{A}}^c_{B2B3} = 2\vec{\boldsymbol{\omega}}_3 \times \vec{\mathbf{V}}_{B2/3}$$

$3.06\ \text{cm/s}^2 \perp$ link 2

$\vec{\mathbf{A}}_{B2/3} \parallel$ link 2 and downwards

$\vec{\boldsymbol{\omega}}_3 = \vec{\boldsymbol{\omega}}_2$ CW

*From the diagram*

$$\alpha_2 = \frac{A^t_{B2O2}}{R_{BO}} = \frac{11.8\ \text{cm/s}^2}{8\ \text{cm}} = 1.475\ \text{rad/s}^2\ \text{CCW}$$

$A_{B2}$ : 11.95 cm/s²