(A)

MAT 1320 A   Fall 2014   October 1st, 8:30   Prof. Desjardins

TEST #1

Max = 15

Student Number: Solutions

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite. Graphs and sketches may be drawn in pencil.
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.
- During any test or exam, you must not have any type of electronic device. Having such a device is considered academic fraud.

Ⓐ

1. [1 point] Solve for x: $3^{2x+5} = 81$.

$$3^{2x+5} = 81 = 3^4$$

$$\text{so } 2x+5=4$$

$$2x=-1$$

$$\boxed{x = -1/2}$$

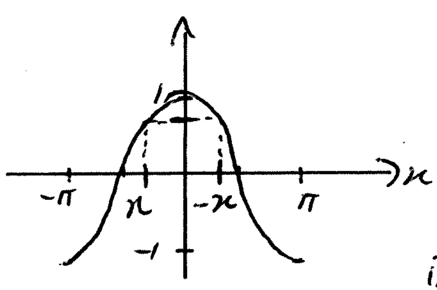
2. [2 points] Find the limit $\lim_{x \to 1} \dfrac{\sqrt{x+3}-2}{x-1}$. $\left(\frac{0}{0} \text{ indeterminate form}\right)$

$$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1} \left(\frac{\sqrt{x+3}-2}{x-1}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)$$

$$= \lim_{x\to 1} \frac{(x+3)-4}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \boxed{\frac{1}{4}}$$

3. [1 point] If $-\pi < x < 0$, what is $\arccos(\cos(x))$?



$$\arccos(\cos x) = \boxed{-x}$$

if $-\pi < x < 0$

then $0 < -x < \pi$

Ⓐ

4. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{x+h}{x+h+1} - \frac{x}{x+1}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+1)(x+h) - x(x+h+1)}{(x+h+1)(x+1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{x^2} + \cancel{x} + \cancel{xh} + h - \cancel{x^2} - \cancel{xh} - \cancel{x}}{(x+h+1)(x+1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{h}{(x+h+1)(x+1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{(x+h+1)(x+1)} = \boxed{\frac{1}{(x+1)^2}}$$

5. [2 points] Use known formulas to find the derivative of the function $y = \dfrac{\sin x}{e^x}$.

$$\frac{dy}{dx} = \frac{(\cos x)(e^x) - (\sin x)(e^x)}{(e^x)^2} = \boxed{\frac{\cos x - \sin x}{e^x}}$$

OR $y = e^{-x}\sin x$

so $$\frac{dy}{dx} = -e^{-x}\sin x + e^{-x}\cos x = \boxed{e^{-x}(\cos x - \sin x)}$$

A

6. [3 points] For which value(s) of x does the function $y = \sqrt{x^2+4x+16}$ have a horizontal tangent line?

$$\frac{dy}{dx} = \frac{1}{2}(x^2+4x+16)^{-1/2}(2x+4) = \frac{x+2}{\sqrt{x^2+4x+16}}$$

$$\frac{dy}{dx} = 0 \quad \text{if} \quad x = -2$$

so horizontal tangent when $\boxed{x = -2}$

7. [3 points] Consider the function $y = f(x)$, where $f(1) = 2, f(2) = 3, f'(1) = -1$, $f'(2) = 5, f'(3) = 7$.

(a) Find $(f \circ f)(1)$.
(b) Find the derivative of the composite function $(f \circ f)$ at $x = 1$.

a) $(f \circ f)(1) = f(f(1)) = f(2) = \boxed{3}$

b) $$\frac{d}{dx}(f \circ f)(x) = \frac{d}{dx}(f(f(x)))$$
$$= f'(f(x))f'(x)$$

so at $x = 1$, $f'(f(1))f'(1) = f'(2)(-1)$
$$= (5)(-1)$$
$$= \boxed{-5}$$

(B)

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(B)

1. [1 point] Solve for x: $5^{2x-1} = 125$.

$$5^{2x-1} = 125 = 5^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$\boxed{x = 2}$$

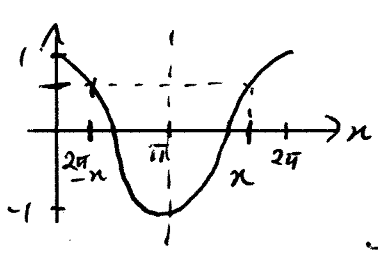
2. [2 points] Find the limit $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x+3}-2}$. $\left(\frac{0}{0} \text{ ind. form}\right)$

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} = \lim_{x\to 1} \left(\frac{x-1}{\sqrt{x+3}-2}\right)\left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right)$$

$$= \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{x-1}$$

$$= \lim_{x\to 1} (\sqrt{x+3}+2) = \boxed{4}$$

3. [1 point] If $\pi < x < 2\pi$, what is $\arccos(\cos(x))$?



$$\arccos(\cos x) = \boxed{2\pi - x}$$

if $\pi < x < 2\pi$

then $0 < 2\pi - x < \pi$

(B)

4. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x-2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{x+h}{x+h-2} - \frac{x}{x-2}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h)(x-2) - x(x+h-2)}{(x+h-2)(x-2)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{x^2} - \cancel{2x} + \cancel{xh} - 2h - \cancel{x^2} - \cancel{xh} + \cancel{2x}}{(x+h-2)(x-2)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{-2h}{(x+h-2)(x-2)}\right)$$

$$= \lim_{h\to 0} \frac{-2}{(x+h-2)(x-2)} = \boxed{\frac{-2}{(x-2)^2}}$$

5. [2 points] Use known formulas to find the derivative of the function $y = \dfrac{\cos x}{e^x}$.

$$\frac{dy}{dx} = \frac{(-\sin x)(e^x) - (\cos x)e^x}{(e^x)^2} = \boxed{\frac{-\sin x - \cos x}{e^x}}$$

OR $\quad y = e^{-x}\cos x$

so $\dfrac{dy}{dx} = -e^{-x}\cos x + e^{-x}(-\sin x) = \boxed{-e^{-x}(\cos x + \sin x)}$

(B)

6. [3 points] For which value(s) of x does the function $y = \sqrt{x^2 - 6x + 36}$ have a horizontal tangent line?

$$\frac{dy}{dx} = \frac{1}{2}(x^2-6x+36)^{-1/2}(2x-6) = \frac{x-3}{\sqrt{x^2-6x+36}}$$

so $\frac{dy}{dx} = 0$ if $x = 3$

thus horizontal tangent when $\boxed{x=3}$

7. [3 points] Consider the function $y = f(x)$, where $f(1) = 4, f(4) = 3, f'(1) = 2,$ $f'(4) = 5, f'(3) = 7$.

(a) Find $(f \circ f)(1)$.
(b) Find the derivative of the composite function $(f \circ f)$ at $x = 1$.

a) $(f \circ f)(1) = f(f(1)) = f(4) = \boxed{3}$

b) $\frac{d}{dx}((f \circ f)(x)) = \frac{d}{dx}(f(f(x)))$

$= f'(f(x))\, f'(x)$

so if $x = 1$ $\quad f'(f(1))\, f'(1) = f'(4)\,(2)$

$= (5)(2)$

$= \boxed{10}$

(C)

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C

1. [1 point] Solve for x: $3^{2x-1} = 81$.

$$3^{2x-1} = 81 = 3^4$$

$$2x - 1 = 4$$

$$2x = 5$$

$$\boxed{x = 5/2}$$

2. [2 points] Find the limit $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x+2}-2}$.

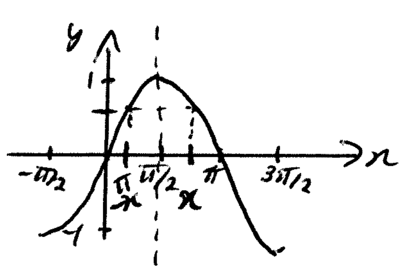
$\left(\frac{0}{0} \text{ ind form}\right)$

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x+2}-2} = \lim_{x\to 2} \left(\frac{x-2}{\sqrt{x+2}-2}\right)\left(\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right)$$

$$= \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{(x+2)-4} = \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{x-2}$$

$$= \lim_{x\to 2} (\sqrt{x+2}+2) = \boxed{4}$$

3. [1 point] If $\pi/2 < x < 3\pi/2$, what is $\arcsin(\sin(x))$?



if $\pi/2 < x < 3\pi/2$

then $-\pi/2 < \pi - x < \pi/2$

$$\arcsin(\sin x) = \boxed{\pi - x}$$

(C)

4. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+5}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{x+h}{x+h+5} - \frac{x}{x+5}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(x+h)(x+5) - x(x+h+5)}{(x+h+5)(x+5)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{x^2} + \cancel{5x} + \cancel{xh} + 5h - \cancel{x^2} - \cancel{xh} - \cancel{5x}}{(x+h+5)(x+5)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{5h}{(x+h+5)(x+5)}\right)$$

$$= \lim_{h\to 0} \frac{5}{(x+h+5)(x+5)} = \boxed{\frac{5}{(x+5)^2}}$$

5. [2 points] Use known formulas to find the derivative of the function $y = \dfrac{\sin x}{x^2}$.

$$\frac{dy}{dx} = \frac{(\cos x)(x^2) - (\sin x)(2x)}{(x^2)^2} = \boxed{\frac{x\cos x - 2\sin x}{x^3}}$$

or $y = x^{-2}\sin x$

so $\dfrac{dy}{dx} = -2x^{-3}\sin x + x^{-2}\cos x = \boxed{x^{-3}(x\cos x - 2\sin x)}$

C

6. [3 points] For which value(s) of x does the function $y = \sqrt{x^2+8x+48}$ have a horizontal tangent line?

$$\frac{dy}{dx} = \frac{1}{2}(x^2+8x+48)^{-1/2}(2x+8) = \frac{x+4}{\sqrt{x^2+8x+48}}$$

so $\frac{dy}{dx} = 0$ if $x = -4$

thus horizontal tangent when $\boxed{x=-4}$

7. [3 points] Consider the function $y = f(x)$, where $f(1) = 2, f(2) = 3, f'(1) = -2,$ $f'(2) = 6, f'(3) = 7$.

(a) Find $(f \circ f)(1)$.
(b) Find the derivative of the composite function $(f \circ f)$ at $x = 1$.

a) $(f \circ f)(1) = f(f(1)) = f(2) = \boxed{3}$

b) $\frac{d}{dx}((f \circ f)(x)) = \frac{d}{dx}(f(f(x)))$

$= f'(f(x))\, f'(x)$

so if $x=1$ $\quad f'(f(1))\, f'(1) = f'(2)(-2)$

$= (6)(-2)$

$= \boxed{-12}$