

## Assignment 2

### Sketch of Solutions with Marking Scheme

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#### 1 QUESTION 1

*Algorithm Alternate alternates an electoral stage to the right and one to the left. The sequence of...*

##### 1.1 Answer

At each step, we add a node to separate a pair of nodes  $(n_1, n_2)$  where  $n_1 < n_2$  and where  $n_2$  lies in the current direction of the messages from  $n_1$ . This yields to:  $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 7, x_6 = 10$ .

To compute the number of nodes at stage  $i$ , we distinguish whether  $i$  is odd or even.

$$x_i = \begin{cases} x_{i-1} + x_{i-2}, & \text{if } i \text{ is odd} \\ x_{i-1} + x_{i-3}, & \text{if } i \text{ is even} \end{cases}$$

where  $x_1 = 1, x_2 = 2$ , and  $x_3 = 3$ .

This is worst than *Alternate*, for which the number of nodes in the worst case follows the *Fibonacci* sequence, which grows faster than previous formula (due to now  $x_i = x_{i-1} + x_{i-3}$  for  $i$  even). This results implies that *Alternate* has a lower number of steps (nodes are defeated faster), and thus less messages in total (there is always  $n$  messages per stage) in the worst case.

##### 1.2 Marking

- 25 points allocated as follows:

1. Filling the 5 empty rings in the figure - 10 pts.
2. Rule to generate number of nodes - 10 pts.
3. Better or Worst than *Alternate* (explain) - 5 pts.

#### 2 QUESTION 2

*Consider a tree network  $T$  where each entity has locally available a clock. Furthermore, the clocks...*

## 2.1 Answer

We can use saturation to find a node (or two nodes) that will send a timer to all nodes, after which all nodes will simultaneously start. Such timer is the maximum distance from the saturated nodes to a any other node.

We perform wake-up and saturation finding at the same time the *eccentricities* of the saturated nodes. In the resolution phase, the saturated nodes send  $v$  to every node except the saturated link, where  $v$  is the highest among the eccentricities of the saturated nodes, and waits  $v + 1$  units of times. (Note that the saturated nodes can know what is the maximum value they both received.) A node receiving a value  $v$ , will send  $v - 1$  to its children, and wait  $v$  units of times. After waiting its corresponding time, every node will start.

The message cost is the cost from *Activation* + *Saturation* + *Resolution*. This is  $M = (n + k_* - 2) + (n) + (n - 2) = 3n + k_* - 4$ , where  $k_*$  is the number of initiators.

## 2.2 Marking

- 25 points allocated as follows:

- Describe *Simultaneous Start-up* protocol (correct): 10 pts.
- Efficient: 5 pts. (If  $M > O(n)$ : 0 pts. If using Saturation twice: 2 pts.)
- Compute complexity: 10 pts.