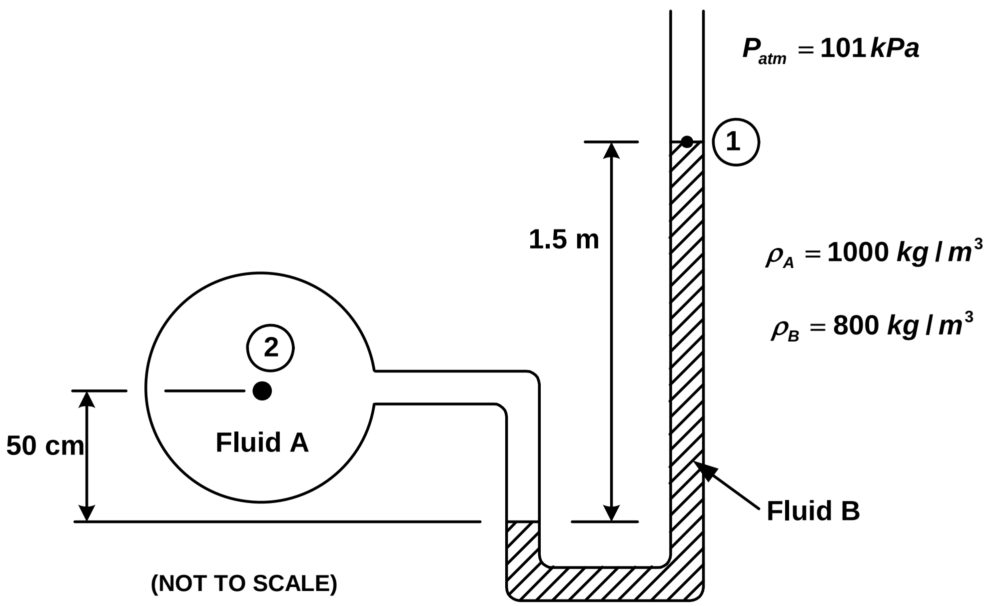
Department of Mechanical and Aerospace Engineering
CARLETON UNIVERSITY

MECH 3310 Biofluid Mechanics
PROBLEM SET #1

1. The blood pressure of an individual undergoing strenuous physical activity is measured at "190 over 95", meaning a systolic pressure of 190 mm Hg and a diastolic pressure of 95 mm Hg. Note that these values would be dangerously high under conditions of normal activity. If the atmospheric pressure at the time is 99.5 kPa(a), what is the pressure in the aorta at the end of systole, end of diastole, in: (a) Pa(a); (b) Pa(g)? The density of mercury is 13,600 kg/m^3. (Ans. (b) 12.48 x 10^5 Pa(a), 11.22 x 10^5 Pa(a))

2. The manometer arrangement is being used to measure the pressure at point 2. The tube is open to atmosphere at point 1.



What is the pressure at point 2 expressed in: (a) Pa(a); (b) Pa(g); (c) metres of water, m H_20(g); (d) mm Hg(a); (e) mm Hg(g)? (Ans. (a) 1.079 x 10^5 Pa(a); (d) 808.5 mm Hg(a))

3. In considering the blood pressures within the body, we generally neglect the effects of changes in elevation. Is this valid? Neglect any pressure losses in the arteries.
For a standing human, with a systolic pressure of 130 mmHg, what is the difference in mmHg between the systolic pressure and the maximum blood pressure seen in (i) the brain, which is about 0.5 m above the heart, and (ii) the bottom of the feet which are about 1.5 m below the heart? Take the density of blood to be 1060 kg/m^3. (Ans. (i) 39 mm Hg)

4. When the pressure in our surroundings changes (such as by swimming under water, climbing a mountain, or flying an aircraft at altitude), the pressure in the circulatory system adjusts so that the minimum pressure in the system is essentially at the pressure of the surroundings. This is accomplished by changing the total volume of the vessels and chambers making up the system. For example, as the external pressure increases, the vessels are compressed. This reduces the total volume of the system. But since the total mass of blood remains constant, the blood is also compressed, until the blood pressure just balances the external pressure again. We want to estimate change in circulatory system volume that occurs for some typical changes in pressure that we might experience. The compressibility of a liquid is expressed through is bulk modulus, K, where

$$K = -V\left(\frac{\partial P}{\partial V}\right)$$

and V is the volume.

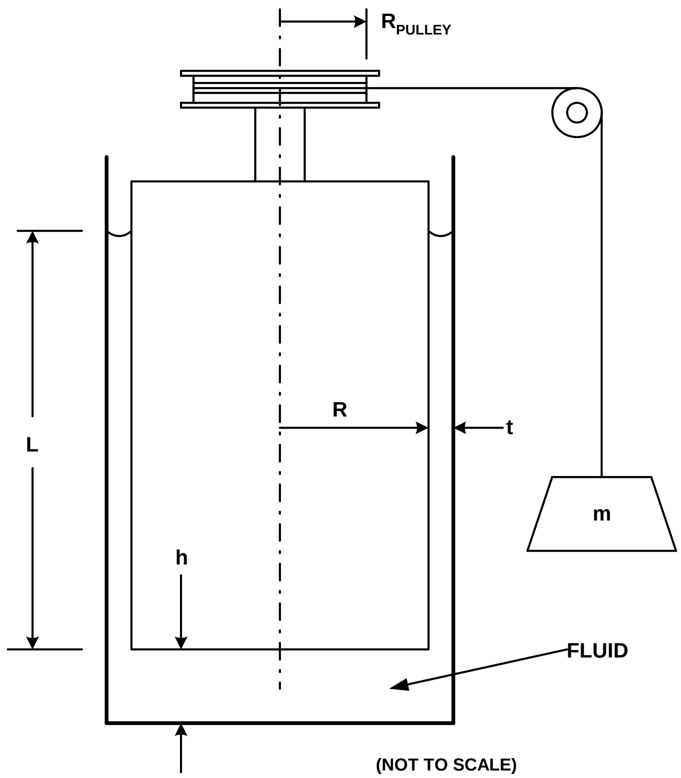
If the total volume of blood in the circulatory system is 6 litres at sea level atmospheric pressure (101.3 kPa) and we assume that blood has the same bulk modulus as water,
K = 2100 MPa, determine the change in volume of the circulatory system from sea level conditions to:
(a) The top of Mt. Everest, where the pressure is about 37 kPa.
(b) A depth of 50 m in the ocean. Recall that seawater has a density of about 1030 kg/m^3.
(Ans. (a) 1.837×10^{-4} litres)

5. The drawing shows a concentric-cylinder viscometer, a device for measuring the viscosity of a fluid. The fluid fills the gap between the two cylinders. The falling weight unreels a string from the pulley, thus rotating the inner cylinder. The viscosity of the fluid is then inferred from the rotational speed of the cylinder. Assume that the fluid velocity varies linearly both in the vertical gap between the cylinders and in the gap between the flat bottom of the inner cylinder and the bottom of the outer cylinder. The viscometer has the following dimensions: $R = 6$ cm, $R_{PULLEY} = 3$ cm, $L = 15$ cm, and $t = 0.1$ cm. When the falling weight has a mass of 30 gm, the cylinder is observed to rotate at 30 RPM.

(a) Estimate the viscosity of the fluid neglecting the frictional torque due to the fluid in the bottom gap.

(b) The fluid in the gap of height h at the bottom also contributes shear stress to the cylinder. Determine the gap h required so that the fluid at the bottom contributes less than 1% of the torque measured with the pulley.

(Ans. 1.38×10^{-2} Ns/m^2, 9.9 mm)



MECH 3310 - Biofluid Mechanics
Problem Set 1

1. Blood pressures

$\Delta h_{syst} := 190$ mmHg(g) $\Delta h_{dia} := 95$ mmHg(g)

Note that these are gauge pressures.

To convert to Pascals $\Delta P = \rho_{Hg} \cdot g \cdot \left(\frac{\Delta h_{mmHg}}{1000} \right)$

with $\rho_{Hg} := 13600$ kg/m3 $g := 9.81$ m/s2

Then $\Delta P_{syst} := \rho_{Hg} \cdot g \cdot \left(\frac{\Delta h_{syst}}{1000} \right)$ $\Delta P_{syst} = 25349$ Pa(g)

$\Delta P_{dia} := \rho_{Hg} \cdot g \cdot \left(\frac{\Delta h_{dia}}{1000} \right)$ $\Delta P_{dia} = 12674.5$ Pa(g)

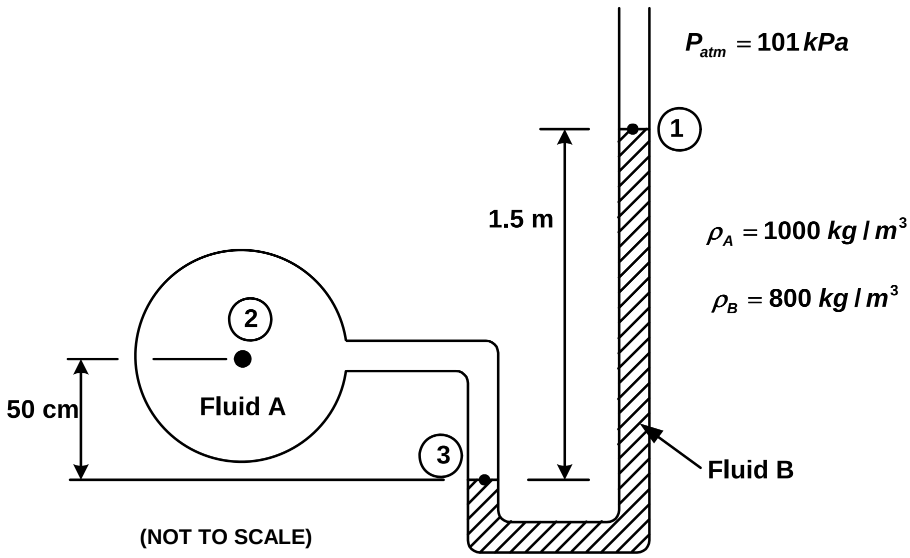
Corresponding absolute values, with $P_{atm} := 99.5 \cdot 1000$ Pa

$P_{syst} := \Delta P_{syst} + P_{atm}$ $P_{syst} = 1.248 \times 10^{5}$ Pa(a)

$P_{dia} := \Delta P_{dia} + P_{atm}$ $P_{dia} = 1.122 \times 10^{5}$ Pa(a)

MECH 3310 - Biofluid Mechanics
Problem Set 1

2. Manometer analysis:



Determining the pressure at 2.

$P_{atm} := 101000$ Pa(a) $g := 9.81$ m/s2

$P_1 := P_{atm}$ $P_1 = 1.01 \times 10^5$ Pa(a)

Using level at 3 as datum for elevation

$y_1 := 1.5$ m $\rho_A := 1000$ kg/m3

$y_2 := 0.5$ m $\rho_B := 800$ kg/m3

$y_3 := 0.0$ m

Always solve manometer problems one fluid at a time.

(a) $P_3 := P_1 + \rho_B \cdot g \cdot (y_1 - y_3)$ $P_3 = 1.128 \times 10^5$ Pa(a)

$P_2 := P_3 - \rho_A \cdot g \cdot (y_2 - y_3)$ $P_2 = 1.079 \times 10^5$ Pa(a)

(b) $P_{2g} := P_2 - P_{atm}$ $P_{2g} = 6.867 \times 10^3$ Pa(g)

(c) In m H_20(g) $\rho_{H2O} := 1000$ kg/m3

$P_g = \rho_{H2O} \cdot g \cdot h_{H2O}$ then $h_2 := \frac{P_{2g}}{\rho_{H2O} \cdot g}$ $h_2 = 0.7$ m H_2O(g)

(d) In mm Hg(a) $\rho_{Hg} := 13600$ kg/m3

$P_a = \rho_{Hg} \cdot g \cdot h_{Hg}$ then $h_2 := \frac{P_2}{\rho_{Hg} \cdot g}$ $h_2 = 0.809$ m Hg(a)

$h_2 \cdot 1000 = 808.501$ mm Hg(a)

(e) In mm Hg(g)

$P_g = \rho_{Hg} \cdot g \cdot h_{Hg}$ then $h_2 := \frac{P_{2g}}{\rho_{Hg} \cdot g}$ $h_2 = 0.051$ m Hg(g)

$h_2 \cdot 1000 = 51.471$ mm Hg(g)

MECH 3310 - Biofluid Mechanics
Problem Set 1

3. Effect of elevation on blood pressures

Systolic pressure at heart $\Delta h_{syst} := 130$ mmHg(g)

$\rho_{blood} := 1060$ kg/m3 $g := 9.8$ m/s2 $\rho_{Hg} := 13600$ kg/m3

For standing human, relative to heart $\Delta y_{brain} := -0.5$ m

$\Delta y_{feet} := 1.5$ m

Change in pressure in blood due to elevation change

$$\Delta P = \rho_{blood} \cdot g \cdot \Delta y$$

Change from heart to brain $\Delta P_{brain} := \rho_{blood} \cdot g \cdot \Delta y_{brain}$ $\Delta P_{brain} = -5.194 \times 10^{3}$ Pa

or in mm Hg $\Delta P = \rho_{Hg} \cdot g \cdot \left(\frac{\Delta h_{mmHg}}{1000}\right)$ $\Delta h_{mmHg} = \frac{\Delta P \cdot 1000}{(\rho_{Hg} \cdot g)}$

$\Delta h_{mmHg} := \frac{\Delta P_{brain} \cdot 1000}{(\rho_{Hg} \cdot g)}$ $\Delta h_{mmHg} = -38.971$ mm Hg

Change from heart to feet $\Delta P_{feet} := \rho_{blood} \cdot g \cdot \Delta y_{feet}$ $\Delta P_{feet} = 1.558 \times 10^{4}$ Pa

or in mm Hg $\Delta h_{mmHg} := \frac{\Delta P_{feet} \cdot 1000}{(\rho_{Hg} \cdot g)}$ $\Delta h_{mmHg} = 116.912$ mm Hg

MECH 3310 - Biofluid Mechanics
Problem Set 1

4. Volume change of cardiovascular system due to changes in external pressure

Bulk modulus $$K = -V\cdot\left(\frac{dP}{dV}\right)$$

Then volume change due to a change in pressure

$$dP = -K\cdot\frac{dV}{V}$$

integrating $$\Delta P = -K\cdot\ln\left(\frac{V_2}{V_1}\right)$$

or $$\frac{V_2}{V_1} = e^{\frac{-\Delta P}{K}}$$

and $$\Delta V_A = V_1\cdot\left(e^{\frac{-\Delta P}{K}} - 1\right)$$

If we assume that the change is very small compared with V_1, then we could use the more approximate expression

$$\Delta P = -K\cdot\frac{\Delta V}{V_1}$$

and $$\Delta V_B = \frac{-\Delta P\cdot V_1}{K}$$

For blood assume $K := 2100\cdot 10^6$ Pa $V_1 := 6$ litres

$P_1 := 101300$ Pa

(a) Change in volume going to top of Mt. Everest where $P := 37000$ Pa

$\Delta P := P - P_1$ $\Delta P = -6.43\times 10^4$ Pa

Then change in volume

$$\Delta V_A := V_1 \cdot \left(e^{\frac{-\Delta P}{K}} - 1 \right)$$

$$\Delta V_A = 1.83717 \times 10^{-4}$$ litres

or from more approximate formula

$$\Delta V_B := \frac{-\Delta P \cdot V_1}{K}$$

$$\Delta V_B = 1.83714 \times 10^{-4}$$ litres

(b) Change in volume going to 50 m depth in ocean

$\rho_{H2O} := 1030$ kg/m3 $\Delta y := 50$ m $g := 9.8$ m/s2

$$\Delta P := \rho_{H2O} \cdot g \cdot \Delta y$$

$$\Delta P = 5.047 \times 10^{5}$$ Pa

Then change in volume

$$\Delta V_A := V_1 \cdot \left(e^{\frac{-\Delta P}{K}} - 1 \right)$$

$$\Delta V_A = -1.44183 \times 10^{-3}$$ litres

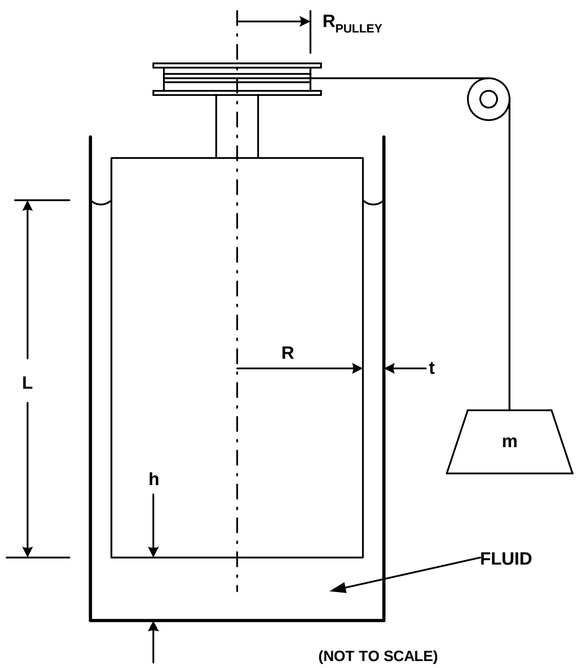
or from more approximate formula

$$\Delta V_B := \frac{-\Delta P \cdot V_1}{K}$$

$$\Delta V_B = -1.442 \times 10^{-3}$$ litres

MECH 3310 - Biofluid Mechanics
Problem Set 1

5. Cylindrical viscometer



Dimensions: $R := \frac{6}{100}$ $R = 0.06$ m $R_{pulley} := \frac{3}{100}$ $R_{pulley} = 0.03$ m

$L := \frac{15}{100}$ $L = 0.15$ m $h := \frac{0.1}{100}$ $h = 1 \times 10^{-3}$ m

$t := h$ $t = 1 \times 10^{-3}$ m

Falling weight: $m := \frac{30}{1000}$ $m = 0.03$ kg $g := 9.81$ m/s2

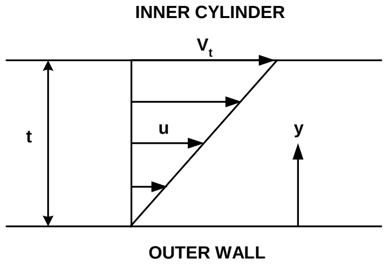
Thus $W := m \cdot g$ $W = 0.294$ N

For steady state conditions (weight falling at constant velocity), moment about puller axis

$M_{pulley} := W \cdot R_{pulley}$ $M_{pulley} = 8.829 \times 10^{-3}$ N-m

At steady state the inner cylinder is also rotating at a steady rotational speed. Since there is no angular acceleration, the moment applied by the pulley to the cylinder must be exactly balanced by the moment on the cylinder due to the fluid friction.

(a) Viscosity, taking into account the friction in the vertical gap only:



Velocity assumed to vary linearly from this value to zero on surface of the outer cylinder (no-slip condition)

Tangential velocity at edge of inner cylinder $V_t = R \cdot \omega$

Resulting shear stress on every plane in the gap (and thus at the inner cylinder wall) $\tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{\Delta u}{\Delta y} = \frac{\mu \cdot R \cdot \omega}{t}$

This acts on area $A = 2 \cdot \pi \cdot R \cdot L$ (wetted cylinder area in the gap)

Then total tangential force at surface of cylinder

$$F_t = \tau \cdot A = \frac{\mu \cdot R \cdot \omega}{t} \cdot 2 \cdot \pi \cdot R \cdot L = 2 \cdot \mu \cdot R^2 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

This acts at radius R so corresponding moment is

$$M_{gap} = F_t \cdot R = \frac{\mu \cdot R \cdot \omega}{t} \cdot 2 \cdot \pi \cdot R \cdot L \cdot R = 2 \cdot \mu \cdot R^3 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

Equating this to the pulley moment and solving for μ

$$M_{pulley} = 2 \cdot \mu \cdot R^3 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

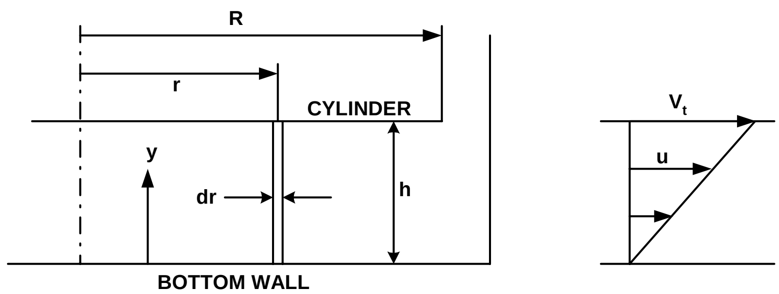
then $\mu = \frac{1}{2} \cdot \frac{M_{pulley}}{\left(R^3 \cdot \omega\right)} \cdot \frac{t}{(\pi \cdot L)}$

Since $N := 30$ RPM $\omega := \frac{2 \cdot \pi \cdot N}{60}$ $\omega = 3.142$ rads/s

and finally $\mu := \frac{1}{2} \cdot \frac{M_{pulley}}{\left(R^3 \cdot \omega\right)} \cdot \frac{t}{(\pi \cdot L)}$ $\mu = 1.3805 \times 10^{-2}$ N-s/m2

(b) Including friction on the bottom of the cylinder

For the gap at the bottom, again assume linear velocity variation across gap



Tangential velocity at radius r $\qquad V_t = r \cdot \omega$

then $\qquad \tau = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{\Delta u}{\Delta y} = \frac{\mu \cdot r \cdot \omega}{h}$

This acts on a ring of width dr, which has area $\qquad dA = 2 \cdot \pi \cdot r \cdot dr$

and resulting tangential force $\qquad dF_t = \tau \cdot dA$

Then the total frictional moment on the bottom of the cylinder

$$M_{bottom} = \int r \, dF_t$$

thus $\qquad M_{bottom} = \int_0^R \frac{r \cdot \mu \cdot r \cdot \omega}{h} \cdot 2 \cdot \pi \cdot r \, dr \qquad$ or $\qquad M_{bottom} = \int_0^R 2 \cdot r^3 \cdot \mu \cdot \frac{\omega}{h} \cdot \pi \, dr$

Since μ, ω and h are not a function of radius, integration gives

$$M_{bottom} = \frac{2 \cdot \pi \cdot \mu \cdot \omega}{h} \cdot \frac{R^4}{4}$$

From part (a) moment due friction in the gap

$$M_{gap} = 2 \cdot \mu \cdot R^3 \cdot \frac{\omega}{t} \cdot \pi \cdot L$$

and the total moment $$M_{total} = \frac{2\cdot\pi\cdot\mu\cdot\omega}{h}\cdot\frac{R^4}{4} + 2\cdot\mu\cdot R^3\cdot\frac{\omega}{t}\cdot\pi\cdot L$$

Thus if we want the friction on the bottom to add less than X% to the moment measured with the pulley, minimum required height at bottom:

$$\frac{M_{bottom}}{M_{total}} = 0.01$$

then $$\frac{\frac{2\cdot\pi\cdot\mu\cdot\omega}{h}\cdot\frac{R^4}{4}}{\frac{2\cdot\pi\cdot\mu\cdot\omega}{h}\cdot\frac{R^4}{4} + 2\cdot\mu\cdot R^3\cdot\frac{\omega}{t}\cdot\pi\cdot L} = \frac{X}{100}$$

or $$\frac{R}{(R\cdot t + 4\cdot L\cdot h)}\cdot t = \frac{X}{100}$$

Solving for h $$h = \frac{R\cdot t}{4\cdot L}\cdot\left(\frac{100}{X} - 1\right)$$ $R = 0.06$

For less than 1% contribution from the bottom $X := 1.0$

Then $$h := \frac{R\cdot t}{4\cdot L}\cdot\left(\frac{100}{X} - 1\right)$$

$h = 9.9\times 10^{-3}$ m

$h\cdot 1000 = 9.9$ mm