ITECH

ITECH TUTORING SERVICES

COMM 308

INTRODUCTION TO CORPORATE FINANCE

CHAPTER 5- Time Value of Money

Why TVM

Mark is selling his cell phone. Jen offers to pay Mark $100 today and Maria offers to pay $100 one year from now. Who would Mark sell his phone to?

- Obviously Mark would sell his phone to Jen since she is paying earlier.

This example clearly proves that we should not only focus on how much we are getting paid but also when we would get paid. Thus, time value of money is an important issue and we will analyze this issue in depth within this chapter.

Present Value & Future Value Relationships

Example 1

Mark deposits $300 in to a savings account that offers 5% annual interest. How much would he accumulate in 6 years?

Solution:

First we have to clear our financial calculator:

2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,

300 PV; 5 I/Y; 6 N
CPT FV: $402.03 would be accumulated in 6 years

Example 2

Mark is willing to purchase a car in 2 years. He estimates the car price to be $5,000 by that time. He is planning to invest in a savings account that pays 5.5% interest annually. How much should he invest today?

Solution:

First we have to clear our financial calculator:
2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,
5,000 FV; 5.5 I/Y; 2 N CPT PV: $4,492.26 should be invested today

Example 3

Mark can only invest $2,300 today in to the 5.5%-savings account. In that case how long would it take him to purchase the $5,000 worth car?

Solution:

First we have to clear our financial calculator:
2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,
-2,300 PV; 5.5 I/Y; 5,000 FV CPT N: 14.5 years

ORDINARY ANNUITIES

Equal amount of cash flows paid/received regularly at the end of every period for a finite period of time.

Rule 1: PV of an ordinary annuity always belongs to one period before the first payment.

Rule 2: FV of an ordinary annuity belongs to the last payment period.

Example

An investment opportunity will pay Mark $500 annually for 7 years. First payment is due 4 years from now. Annual interest rate is 6%. How much Mark would be willing to pay for this investment opportunity?

Solution:

First we have to clear our financial calculator:
2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,
500 PMT; 6 I/Y; 7 N CPT PV: $2,791.19 @ the end of year 3 (one year before the first payment)

(please note that for the calculations involving annuities, N value would always equal to the number of payments)

Now, we have to find the value of $2,791.19 (@ t=3) today:

2^{nd} FV
2^{nd} CE/C
2,791.19 FV; 6 I/Y; 3 N CPT PV: $2,343.54 today = Ans

Example 5

Mark will deposit the proceeds from this investment to a savings account that pays 6% interest per year. What would his account balance be at the end of year 15?

Solution:
First we have to clear our financial calculator:
2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,
500 PMT; 6 I/Y; 7 N CPT FV: $4,196.92 @ the end of year 10

The account balance would keep on growing @ 6% rate between year 10 and 15. There would be no additional deposit during this time period.

2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,
4,196.92 PV; 6 I/Y; 5 N CPT FV: $5,616.42 @ the end of year 15

ANNUITY DUE

Equal amount of cash flows paid/received regularly at the beginning of every period for a finite period of time

Rule 1: PV of an annuity due always belongs to the same period as the first payment.

Rule 2: FV of an annuity due belongs to one period after the last payment.

Example 6

An investment opportunity will pay Mark $500 annually at the beginning of every year for 7 years. First payment is at the beginning of the 4^{th} year. Annual interest rate is 6%. How much Mark would be willing to pay for this investment opportunity?

Solution:
First we have to change the format of the calculator to "BGN"
2^{nd} PMT
2^{nd} Enter
CE/C
(Now you should see the BGN abbreviation at the top right corner of your financial calculator)

Then we have to clear our financial calculator:
2^{nd} FV
2^{nd} CE/C

Now we can plug in the relevant data,
500 PMT; 6 I/Y; 7 N CPT PV: \$2,958.66 @ the end of year 3 (or at the beginning of year 4)

Now, we have to find the value of \$2,958.66 (@ t=3) today:

2^{nd} FV
2^{nd} CE/C
2,958.66 FV; 6 I/Y; 3 N CPT PV: \$2,484.15 today = Ans

CAUTION: Once you are done with the annuity due question, DO NOT FORGET to switch the calculator format back to "END".
2^{nd} PMT; 2^{nd} Enter ; CE/C

GROWING ANNUITY

The cash flows that are paid regularly and that also grow at a constant rate for a limited time period

$$PV = \frac{PMT_1}{k-g} * (1 - \frac{(1+g)^n}{(1+k)^n})$$

PERPETUITY

Equal amount of cash flows paid/received regularly at the end of every period forever

PV = PMT/k; Where k = discount rate per period

- PV belongs to one period before the first payment

Example 7

You want to create a scholarship fund that is capable of awarding $25,000 annually in perpetuity. The first award is to be paid exactly 4 years from today. The annual interest rate is 6%. How much money should you donate today?

Solution:

25,000/0.06 = $416,666.67 is the PV of the perpetuity but which time period does this $ amount belong to?

The first pmt is at the end of year 4 therefore PV belongs to end of year 3.

To find the PV today (at time 0):

416,666.67 FV; 3 N; 6 I/Y CPT PV = $349,831.37 should be donated today.

GROWING PERPETUITY

The cash flows that are paid regularly and that also grow at a constant rate forever

PV = PMT / (k-g)

Where g = annual growth rate

* PV belongs to one period before the first payment

Example 8

You want to donate $100,000 to a scholarship fund will be award $X exactly four years from now. The award will grow at 5% rate annually and award will be paid forever. The annual interest rate is 11%. X = ?

Solution:

PV (at t=3) = $\frac{X}{0.11-0.05} = \frac{X}{0.06}$ is also the FV of the $100,000 3 years from now

100,000 PV; 3 N; 11 I/Y CPT FV = $136,763.10

X = 0.06 * (136,763.10) = $8,205.79

EAR – APR RELATIONSHIP

APR = Annual percentage rate = Quoted rate = Stated rate = Nominal rate

"10% interest compounded semi-annually" statement:

- refers to APR of 10%
- interest is paid on a semi annual basis. Semi-annual rate = 10%/2 = 5%

Example 9

You will invest $100 today. Interest rate is 10% compounded semi-annually.

a) How much is your investment worth at the end of 6 months?
b) How much is your investment worth at the end of a year?
c) How much % return would you earn in one year?

Solution:

a)
Semi-annual rate: 5%
In 6 months; 100 * (1.05) = $105 would be accumulated

b)
During the second 6-month period $105 would earn 5% as well.
At the end of the year; 105 * (1.05) = $110.25 would be accumulated

c)
$110.25 - $100 = $10.25 earned in interest over $100 investment
% return = 10.25/100 = 10.25%

How come the actual annual return is > 10% APR?

10.25% is called the EAR (Effective annual rate). This would be the actual return earned over 1-year period.

EAR formula: $EAR = (1+\frac{APR}{m})^{m} - 1$ where m = # of compounding per year

How to find EAR on the financial calculator?

2^{nd} 2

NOM	10	Enter	↓↓	(NOM = APR value)
C/Y	2	Enter	↑	(C/Y = number of compounding per year)
EFF	CPT = 10.25			(EFF = EAR)

Example 10

An investment opportunity offers you $100 per month for 3 years. Payments are to be made at the end of every month. First payment is due a month from now.

a) If interest rate is 8% compounded monthly. How much would you be willing to pay for this investment today?

b) If interest rate is 8% compounded semi-annually. How much would you be willing to pay for this investment today?

Solution:

a)

$100/month is the PMT; 36months is the N; 8/12 = 0.667 is the I/Y
CPT PV = $3,191.18 today

The effective monthly interest rate is the appropriate I/Y to use in the PV calculation since both the PMT and N has "month" as the time unit. If a question involves PMT; I/Y should always be the effective per pmt period rate.

b)

Unlike part (a); the compounding frequency doesn't match the pmt frequency.
Hence I/Y = Effective per pmt period = $(1+\frac{APR}{m})^{(m/f)} - 1 = (1+\frac{0.08}{2})^{(2/12)} - 1 = 0.65579\%$
where f = # of pmts per year and m = # of compounding per year

$100/month is the PMT; 36months is the N; 0.65579 is the I/Y
CPT PV = $3,197.32 today

LOANS & MORTGAGES

- Amount borrowed = Sum of the PV of all loan payments
- Loan balance at any given point in time = Sum of the PV of the remaining loan payments
- Canadian mortgages are always compounded semi-annually

Example 11

You have borrowed $10,000. Loan will be paid off in 3 years with monthly payments that are to start a month from now. Interest rate is 9% compounded monthly.

a) How much would be the monthly payments?
b) What is the loan balance after a year?

- Pmts are monthly and compounding is also monthly therefore I/Y = 9%/12 = 0.75%

10,000 PV; 36 N; 0.75 I/Y CPT PMT = $318/month

- Please recall that "**Loan balance at any given point in time = Sum of the PV of the remaining loan payments"**

  After a year; the loan has two more years (or 24 more months) left in its life.

  318 PMT; 0.75 I/Y; 24 N CPT PV = $6,960.69

Example 12

You have borrowed $10,000. Loan will be paid off in X years with monthly payments of $200 that are to start a month from now. Interest rate is 9% compounded semi-annually. How long would it take to pay off the loan?

Solution:

PMT frequency is monthly but compounding frequency is semi-annually therefore

$I/Y = \text{Effective per pmt period} = (1+\frac{APR}{m})^{(m/f)} - 1 = (1+\frac{0.09}{2})^{(2/12)} - 1 = 0.7363\%$

10,000 PV; -200 PMT; 0.7363 I/Y CPT N = 62.58 months

LOAN AMORTIZATION TABLE

- Interest pmt (i) = Beginning loan balance (i) * I/Y
- Principal payment (i) = Loan pmt (i) – Interest pmt (i)
- End balance (i) = Beginning loan balance (i) – Principal pmt (i)

Principal payment is the amount that the loan balance would decline by.
Interest payment is the fee that is paid due to benefiting from the advantages of a loan.

Period	Beg. balance	PMT	Int. Pmt.	Princ. Pmt	End balance
1	10,000	318	75	243	9,757
2	9,757	318	73.18	244.82	9,512.18
3	.	.	.	.	.

Let's fill in the table above for the loan in example 11.

Interest payment (1) = $10,000 * 0.75% = $75
Principal payment (1) = $318 - $75 = $243
End balance (1) = $10,000 – $243 = $9,757 = Beginning balance (2)
Interest payment (2) = $9,757 * 0.75% = $73.18
Principal payment (2) = $318 - $73.18 = $244.82
End balance (2) = $9,757 - $244.82 = $9,512.18

What would be the principal payment portion of the 8th instalment?

Principal payment (8) = $318 – Interest payment (8)

Interest payment (8) = Beginning balance (8) * 0.75%

Beginning balance (8) = PV of the remaining 29 payments (36 total payments – 7 payments that have already been paid before the beginning of period (8))

318 PMT; 0.75 I/Y; 29 N CPT PV = $8,260.33 = Beginning balance (8)

Interest payment (8) = 0.75% * $8,260.33 = $61.95

Principal payment (8) = 318 – 61.95 = $256.05

CHAPTER 6- BOND VALUATION

How do the firms raise the funds they need for investments?

One possible scenario is to borrow money from the market.

You can actually think of bonds as written contracts. By selling the bond to a bondholder, the firm is actually borrowing money. In return the firm makes a payment promise. The nature of this promise is specified on the bond. Bondholders would receive two types of payments from the firm:

Payment type 1: Face value (Par Value)
This is a single payment that the bondholder receives at the end of the bond's life. Unless it is specified otherwise, we'd assume face value to be $1,000.

Payment type 2: The coupon payment
Coupon payments can be received annually, semi-annually, quarterly etc.
Coupon payment = (Coupon rate * Par value of the bond) / # of coupon payments per year

Yield to maturity: Annual % return that the bondholder would earn if he/she keeps the bond until the end of its life.

Yield to maturity = Current Yield + Capital gains yield
Where Current yield = $ value of coupon payments per year / P(0)
Please note that P(0) is the bond price today and P(1) is the price of the bond a year from now.
And Capital gains yield = (P(1) – P(0)) / P(0)
Capital gains yield is also known as the % change in bond price over a year.

How to find the bond value?

In order to calculate the bond value you can use your financial calculator. Here is what you'd punch in your calculator.

2^{nd} FV

2^{nd} CE/C

Face value would be the FV;

Coupon payment would be the PMT;

# of years left in bond's life * # of coupon pmts per year would be the N;

YTM / # of coupon payments per year would be the I/Y

CPT PV = Bond price at any given point in time

DISCOUNT & PREMIUM BONDS:

Bond's are classified in to three groups depending on their prices.

(i) Discount bonds: Trade at less than $1,000. Coupon rate < YTM

(ii) Premium bonds: Trade at more than $1,000. Coupon rate > YTM

(iii) Bonds that trade at par: Priced @ $1,000. Coupon rate = YTM

Some facts about bonds:

Assuming that the YTM stays constant throughout the bond's life:

- As maturity date gets closer, discount bond's price would keep on increasing and would be equal to $1,000 on the maturity date.
- As maturity date gets closer, premium bond's price would keep on decreasing and would be equal to $1,000 on the maturity date.

If the YTM changes during bond's life:

- Bond prices and YTM values are inversely related. As YTM goes up, bond price declines and as YTM declines, bond price goes up.
- Bonds with longer time to maturity would be more price-sensitive to changes in YTM.
- Bonds with lower coupon rate would be more price-sensitive to changes in YTM.
- If two bonds have the same coupon rate and time to maturity; the bond with the less frequent coupon payments would be more price-sensitive to changes in YTM.

Example 1

Bond A has 10-year life. It has been issued two years ago. Current YTM is 10%. The bond makes semi-annual coupon payments at a rate of 6%.

a) How much would you pay the purchase the bond today?
b) After 3 years, the YTM goes down by 2% and you decide to sell your bond. How much would you sell your bond for?
c) What's the HPR (Holding period return) of your 3-year investment horizon?

Solution:

a)

\$1,000 FV; $\frac{6\% * 1{,}000}{2}$ = \$30 PMT; 8 years remaining * 2 pmts/year = 16 N; 10/2 = 5 I/Y

CPT PV = \$783.24 is the bond price today

b)

1,000 FV; 30 PMT; 5 years remaining * 2 pmts/year = 10 N; (10-2)/2 = 4 I/Y

CPT PV = \$918.89

c)

918.89 FV; 30 PMT; -783.24 PV; 6 N CPT I/Y = 6.29% per 6-month

APR = 2 * 6.29% = 12.58% compounded semi-annually

HPR = EAR = $(1+\frac{0.1258}{2})^{(2)} - 1 = 12.98\%$

Example 2

Bond C is currently trading at 90% of the par value. If the YTM is 10% and the bond is making semi-annual payments and has 8-year life what would be the coupon rate?

Solution:

90% * 1,000 = $900 is the current bond price therefore

-$900 is the PV; 1,000 FV; 8*2 = 16 N; 10/2 = 5 I/Y CPT PMT = $40.77

$40.77 = $\frac{Coupon\ rate * 1{,}000}{2}$ therefore coupon rate = 8.154%

Example 3

	Bond A	Bond B
Face Value	$1,000	$1,000
Semi-annual coupon	$50	$70
Years to maturity	10	10
Price	$885.30	????

If these bonds are identical except for coupon and prices, what is the price of Bond B?

Solution: In order to find the price of Bond B; we would need to find out the YTM of Bond B. Since Bond A and B are identical, they should have the same YTM.

Let's first find the YTM of Bond A:

1,000 FV; 50 PMT; 10*2 = 20 N; -885.30 PV CPT I/Y = 6% per 6-months

YTM = 2*6% = 12% = YTM(A) = YTM(B)

Now we can calculate the price of Bond B:

1,000 FV; 70 PMT; 10*2 = 20 N; 12/2 = 6 I/Y CPT PV = $1,114.70 = Price (B)

Example 4

Four years ago XYZ Inc. sold $100,000,000 worth of bonds with eight years until maturity. The bonds were sold at par and carry an annual coupon of 10%. Itech Inc. has just sold $50,000,000 worth of bonds to investors also at par. Itech bonds have four years until maturity and carry a coupon rate of 8% with coupons paid semi-annually. Neither the XYZ nor the Itech bonds have any complex features (they are not callable, they are not convertible etc.). Their terms are the same and they recently received identical ratings from the major bond rating agencies. What is the EAR on XYZ bonds today?

Solution:

If two bonds have identical ratings (or the same risk level); these bonds' EAR would also be the same.

EAR (XYZ) = EAR (Itech)

Since Itech bonds have just sold at par; Itech's YTM = Itech's coupon rate = 8%

YTM = APR = 8% compounded semi-annually therefore:

EAR (Itech) = EAR (XYZ) = $(1+\frac{0.08}{2})^{(2)} - 1 = 8.16\%$

Important note: A common wrong answer in this problem is 10%.

XYZ bonds have sold at par four years ago therefore YTM (XYZ) four years ago = coupon rate of the XYZ bonds = 10%

However YTM fours year ago doesn't have to match the YTM today therefore 10% would be the wrong answer.

Example 5

Bond A ---- 5 years to maturity and zero coupon rate
Bond B ---- 5 years to maturity and 8% coupon rate (coupons paid semi-annually)
Bond C ---- 5 years to maturity and 8% coupon rate (coupons paid annually)
Bond D ---- 3 years to maturity and 10% coupon rate
Bond E ---- 3 years to maturity and 8% coupon rate

a) If you expect the interest rates to go up in the future which bond would you purchase today? Why?

If the interest rates go up; bond prices would go down. Would you want the price of your bond to go down a lot or just a little? Obviously you'd want your bond's price go down as little as possible. In other words you'd prefer to have a bond with low price sensitivity.
Lower time to maturity and higher coupon rate indicates lower price sensitivity.
So bond D would be the best option for this scenario.

b) If you expect the interest rates to go down in the future which bond would you purchase today? Why?

If the interest rates go down; bond prices would go up. Would you want the price of your bond to go up a lot or just a little? Obviously you'd want your bond's price go up as much as possible. In other words you'd prefer to have a bond with high price sensitivity.
Longer time to maturity and lower coupon rate indicates higher price sensitivity.
So bond A would be the best option for this scenario.

c) Rank these bonds from more interest rate price-sensitive to less interest rate price-sensitive.

A >C>B>E>D

CHAPTER 7- STOCK VALUATION

INTRODUCTION

Firms need to raise money to be able to invest in new projects. As discussed in chapter 6, issuing bonds would be a way to raise these required funds. Another way to raise funds would be issuing equity. Firm can either issue common equity (stock) or preferred equity (stock).

Preferred stocks would pay a fixed dividend and the stockholders of these shares can not vote at the shareholder meetings. However preferred stockholder have priority over common shareholders when it comes to the payment of the dividends.

Common stocks might or might not pay dividends. It would be the management that decides when and how much these dividends would be paid. Owners of common stocks can vote at the meetings.

STOCK PAYMENTS

Stockholders can receive payments when the firm pays out dividends and when the stockholder decides to sell his/her stock.

Value of the stock today would be calculated by finding the sum of the present value of all future payments the stock would offer to the stockholder.

There are four different cases for stock valuation.

STOCK VALUATION

CASE 1: ZERO-GROWTH FOREVER (CONSTANT DIVIDEND PMT FOREVER)

This case can also be referred to as the preferred stock case. Preferred stocks would pay constant annual dividends forever. Thus, the present value of these payments can be found by using the PV of perpetuity formula.

$$P_0 = stock \quad price \quad today = \frac{D}{k}$$

Where D = constant annual dividend
and k = required rate of return on the stock

D = Par value of the preferred stock * Dividend rate = Dividend yield (t) * Price (t)

CASE 2: CONSTANT DIVIDEND GROWTH RATE FOREVER

In some cases the dividend payments would grow at a constant rate forever. The constant annual growth rate would be referred to as "g". The price of the stock in this case would be calculated with the use of the PV of a growing perpetuity formula.

$$P_0 = \frac{D_1}{k - g}$$

If you want to find the price of the stock at the end of year (n);

$$P_{(n)} = \frac{D_{(n+1)}}{k - g}$$

* Most recent dividend payment = Current dividend payment = Dividend pmt today = D_0
* Expected dividend payment = Dividend payment a year from now = D_1

Dividend at the end of year n can be calculated as:

$$D_n = D_0 \times (1+g)^n = D_1 \times (1+g)^{n-1}$$

Example 1

A stock recently paid a dividend of $0.68 per share. Dividends are expected to grow at a constant rate of 4% per year. What is the expected value of the stock six years from now if investors require a return of 12%?

Solution:

$P_{(n)} = D_{(n+1)} / (k-g)$
$P_{(6)} = D_{(7)} / (k-g)$

$D_7 = D_0 * (1+g)^7$
$= 0.68 * (1.04)^7 = \$0.89483$

$P_{(6)} = 0.89483 / (.12 - .04) = \$11.19$

CASE 3: IRREGULAR DIVIDEND PAYMENTS

In some cases the dividend payments might not have any relationship at all. Then the price of the stock today is simply calculated by calculating the PV of all remaining dividend payments.

Short-cut would be to use the NPV function in your calculator.

$$\boldsymbol{P(0)} = \frac{\boldsymbol{D_1}}{\boldsymbol{(1+k)^1}} + \frac{\boldsymbol{D_2}}{\boldsymbol{(1+k)^2}} + \ldots\ldots\ldots\ldots + \frac{\boldsymbol{D_n}}{\boldsymbol{(1+k)^n}}$$

CASE 4: SUPERNORMAL GROWTH CASE

Sometimes the stock dividends grow at multiple growth rates throughout their life. Such cases are referred to as the "supernormal growth".

How to find the stock price for the supernormal growth case?

Step 1: Identify the moment in time where the last growth rate change takes place. Call this time period, period (t).

Step 2: Calculate all the dividend payments starting from D_1 until $D_{(t+1)}$

Step 3: Find $P_t = D_{(t+1)} / (k\text{-}g_{last})$

Step 4: Find the sum of the present values of D_1 , D_2 , D_3 ..... D_n , P_n
You can also use the NPV function in this step to get the stock price today

(Make sure to always enter the CF(0) as 0 since the current dividend payment is never considered as a part of the stock price)

Answer to step (4) = P_0

Example 2

A stock, which currently doesn't pay a dividend, is expected to pay its first dividend of $1 in five years. Thereafter the dividend is expected to grow at an annual rate of 25% for the next three years and then grow at a constant growth rate of 5% per year thereafter. The required rate of return is 10.3%. What is the expected price of the stock today?

Solution:

There are more than one growth rate in the question so it is supernormal growth case.

Step 1: The moment in time when the last growth change takes place?

t = 8

Step 2:

$D_1 = D_2 = D_3 = D_4 = 0$

$D_5 = \$1$

$D_6 = \$1 * (1 + 25\%) = \$1.25$

$D_7 = \$1.25 * (1 + 25\%) = \$1.5625$

$D_8 = \$1.5625 * (1 + 25\%) = \$1.95313$

$D_9 = \$1.95313 * (1 + 5\%) = \$2.05078$

Step 3:

$P_t = D_{(t+1)} / (k\text{-}g_{last})$

$P_8 = 2.05078 / (10.3\%\text{-}\ 5\%) = \$38.69$

Step 4: CPT NPV for the following inputs

CASH FLOW	AMOUNT
CF0	0
C01	0
F01	4
C02	1
F02	1
C03	1.25
F03	1
C04	1.5625
F04	1
C05	1.95313 + 38.69 = 40.64
F05	1
I	10.3

NPV = P_0 = $20.64

Or $P(0) = \frac{1}{(1+0.103)^5} + \frac{1.25}{(1+0.103)^6} + \frac{1.5625}{(1+0.103)^7} + \frac{40.64}{(1+0.103)^8} = \$20.64$

CASE 5: NPVGO METHOD

If the firm commits to a new project:

P(0) = EPS/r + NPVGO

Where NPVGO = NPV of growth opportunity (on a per share basis)

Example 3

Rite Bite Enterprises sells toothpicks. Gross revenues last year were $3M and total costs were $1.5M. Rite Bite has 1 million shares of common stock outstanding. Gross revenues and costs are expected to stay constant forever. Rite Bite pays no income taxes. All earnings are paid out as dividends.

a) If the appropriate discount rate is 15% and all cash flows are received at year's end, what is the price per share of Rite Bite stock?

b) Rite Bite has decided to produce toothbrushes. The project requires an immediate outlay of $15million. In one year, another outlay of $5million will be needed. The year after that, net cash inflows will be $6million. That profit level will be maintained in perpetuity. What effect will undertaking this project have on the price per share of the stock?

SOLUTION:

a. EPS = (3m-1.5m)/1m = $1.5
Dividend pay-out ratio = 100% → retention ratio = 1-100% = 0% → g = 0%

P(0) = $1.5/0.15 = $10/share

b. NPV $= C_0 + C_1 / (1+k) + [C_2 / k] / (1+k)$
$= -\$15{,}000{,}000 - \$5{,}000{,}000 / (1.15) + [\$6{,}000{,}000 / 0.15] / (1.15)$
$= \$15{,}434{,}782.61$

Divide the NPV by the number of shares to find the per-share effect of the new project.

NPVGO = $15,434,782.61 / 1,000,000
= $15.43

The share price will increase by the per-share NPV of the growth opportunity.

Share Price = PV(EPS) + NPVGO
= $10 + $15.43 = $25.43

Estimating the Growth Rate in Dividends

- The growth rate in dividends can be computed in one of the three possible ways as is shown below:

METHOD #1: Using the Gordon growth formula (only applicable if there a single constant dividend growth rate forever)

$$g = k - \frac{D_1}{P_0}$$

Example 3:

ITECH is expected to pay a 1$ dividend next year on its outstanding common stock which is currently priced at 10$/share. If the company's cost of equity is 20% what growth rate in dividends is foreseeable in the future?

$$D_1 = 1\$$$
$$P_0 = 10$$
$$k = 20\%$$
$$g = k - \frac{D_1}{P_0} = .20 - \frac{1}{10} = .10$$

METHOD #2: Using Historical Dividends

$$D_N = D_K(1+g)^{(N-K)}$$

$$\therefore g = \left(\frac{D_N}{D_K}\right)^{\frac{1}{(N-K)}} - 1$$

On the calculator we will simply press:

FV = Latest dividend payment
PV = - Earlier dividend payment
N = # of years in between
CPT I/Y = growth rate

Example 4:

ITECH just paid a 2$ dividend. The dividend paid 10 years ago was for 1$. If the historical growth rate in dividends is expected to continue for the foreseeable future, then what is the appropriate growth rate for ITECH's common stock?

$$g = \left(\frac{2}{1}\right)^{\frac{1}{10}} - 1 = .07177346$$

Or
2 FV; -1 PV; 10 N CPT I/Y = g = 7.177%

METHOD #3: Using the Retention Ratio and the Return on Equity:

g = growth rate = Retention ratio * ROE

Firm's net earnings would be allocated between dividend payments and retained earnings (the portion of the earnings the firm keeps for itself).

Dividend payout ratio = % of the annual net earnings that are paid as dividends
Dividend payout ratio = Total Dividend Payments/ Net Earnings = Dividend per share/EPS
Where EPS = Earnings per share = Net annual earnings/# of common shares outstanding (assuming no preferred shares exist)

Retention ratio = % of the annual net earnings that are kept within the firm as retained earnings
Retention ratio = 1 – dividend payout ratio

ROE = Return on Equity = Net Earnings / Book Value of Equity
ROE = Net Profit Margin (%) * Turnover Ratio * Leverage Ratio

EXAMPLE: The firm has just reported net earnings of $20M. The dividend pay-out ratio is 15%. The firm has 2 million shares of stock. The balance sheet information also indicates that firm's book value of equity is $200M. Required rate of return on the firm's stock is 12%. What is the current stock price?

Solution:
To find the stock price today, we need to know the value of k; D_1 and g
k = 12%
To find the growth rate:
Retention ratio = 1 – dividend pay-out ratio = 1-15% = 85%
ROE = Net Income (Earnings)/Book Value of Equity = 20M/200M = 10%

g = ROE * Retention Ratio = 85% * 10% = 8.5% = g

To find D(1):

Since the firm has just reported earnings of $20M, this would be the earnings at t=0 (today).

EPS(0) = NET EARNINGS (0)/# of common shares outstanding

EPS(0) = $20M/2M = $10 per share

EPS(0) * dividend pay-out ratio = D(0)

D(0) = $10 * 0.15 = $1.5 per share

D(1) = 1.5 * 1.085 = $1.6275

To find P(0) = stock price today:

P(0) = D(1)/(k-g) = 1.6275/(0.12 – 0.085) = $46.5/share

Estimating the Required Return

- The required rate of return is the return demanded by the investor for investing in a firm's equity. The required return is simply the sum of the dividend yield and the capital gains yield as is shown below

$$k_{Equity} = \frac{D_1}{P_0} + CGY$$

Dividend Yield = D_1/P_0

CGY = Capital Gains Yield = (P(1) – P(0)) / P(0)

Capital gains yield is also known as the % change in stock price over a year.

If there is a constant dividend growth rate forever; CGY would also equal to the growth rate.

Example 5:

ITECH just paid a $.9091 dividend this year on its outstanding common stock which is currently priced at 10$/share. If the capital gains yield on the firm's common stock is known is 10% then what is ITECH's cost of equity capital? (Please assume that there is a single constant dividend growth rate forever)

Solution:

$D_1 = 0.9091 * (1+10\%) = \$1$

$K_e = 1/10 + 10\% = 20\%$

CHAPTER 8 – RISK, RETURN & PORTFOLIO THEORY

In today's world there are many financial securities that an investor can choose to invest in. Out of so many choices which securities do the investors go for? How do they make their decision?

Investors focus on two important parameters when deciding which security to invest in:

- Return: $ profit earned for every $ invested in the security
- Risk: The variability of the returns from that financial security

MEASURING RETURNS

I) Calculating returns on an individual security based on historical information

Example (1):

The historical performance of stock A is provided:

Year	Return (A)
2003	10%
2004	12%
2005	15%
2006	3%
2007	8%

r_A = Avg. (arithmetic) return on stock (A) = $\frac{r(1)+r(2)+....+r(n)}{n}$

r_A = Avg. (geometric) return on stock (A) = $\{(1+r(1))*(1+r(2))*.....*(1+r(n))\}^{1/n} - 1$

r_A (arithmetic) = (10%+12%+15%+3%+8%)/5 = 9.6%

r_A (geometric) = $(1.1*1.12*1.15*1.03*1.08)^{(1/5)} - 1 = 9.53\%$

Note: Geometric return is always a better performance measure

Example (2):

IBM stock was trading at $20 per share exactly a year ago. During this year the stock paid $1.5 per share dividends. The current stock price is $25. What is the %return earned from the stock over the past year?

$$\%\ \text{Return} = \frac{D(1)}{P(0)} + \frac{P(1) - P(0)}{P(0)}$$

If you think of exactly a year ago as t=0

Now would refer to t=1

$$\%\ \text{return} = \frac{1.5}{20} + \frac{25 - 20}{20} = 32.5\%$$

The %return formula that has been provided above can calculate the annual return only if the investment horizon is 1 year. How can we find the annual % return if the investment horizon is longer?

Example (3):

IBM stock was trading at $20 per share on April, 1, 2006. On April 1, 2007, the stock paid $1.5 per share dividends. On April, 1, 2008 the stock also paid $1 per share dividends. The stock price on April 1, 2008 was $25. What is the % annual return earned from the stock between the April 1, 2006 and April 1, 2008 period?

Step 1: Identify the cash flows associated with the investment horizon

CF(0) = -$20

CF(1) = $1.5

CF(2) = 1+25 = $26

Step 2: Compute the IRR for these cash flows. The answer would the annual % return

IRR = 17.83% = annual % return over the past two years

II) Calculating returns on an individual security based on expectations

Sometimes investors make estimations about how the markets and the securities will perform in the future

Example (4):

	Probability	Return (A)
Boom	30%	12%
Normal	50%	9%
Recession	20%	5%

Expected return on the stock = E(r)

$= \sum Prob(i) * r(i)$ = Prob(boom)*return(boom) + …. + Prob (recession)*return (recession)

Where Prob(i) = Probability of economical state i taking place

i = boom, normal, recession

r(i) = return on the stock during economical state i

$E(r_A) = 30\% * 12\% + 50\% * 9\% + 20\% * 5\% = 9.1\%$

MEASURING RISK

Risk refers to the variability of the returns on a stock. If the stock's return is very volatile over time, the stock would be considered as a risky stock.

The measures of risk:

Variance of stock returns = $\boldsymbol{\sigma}^2$

Standard deviation of stock returns = $\boldsymbol{\sigma}$

Larger these values, the larger the risk of the financial asset

I) Calculating the variance of returns on an individual security based on historical information

Ex-post Variance = $\boldsymbol{\sigma}^2 = \sum_{i=1}^{n} \frac{(r(i) - r(avg))^2}{n-1}$

Standard deviation = $\boldsymbol{\sigma} = \sqrt{\sigma^2}$

Example (5): Calculate the variance and standard deviation of Stock (A) returns.

Recall that average (arithmetic) return on stock A = 9.6%

Year	Return (A)
2003	10%
2004	12%
2005	15%
2006	3%
2007	8%

$$\boldsymbol{\sigma}^2 = \frac{(0.10-0.096)^2 + (0.12-0.096)^2 + (0.15-0.096)^2 + (0.03-0.096)^2 + (0.08-0.096)^2}{5-1}$$

$\sigma^2 = 0.00203$

$\sigma = 0.04506$

II) Calculating the variance of returns on an individual security based on expectations

En-ante Variance = $\sigma^2 = \sum_{i=1}^{n} (\text{Prob}(i) * (r(i) - E(r))^2)$

Example (6): Calculate the variance of the returns for Stock (A). Recall that the expected return on stock (A) = 9.1%

	Probability	Return (A)
Boom	30%	12%
Normal	50%	9%
Recession	20%	5%

$\sigma^2 = 0.30*(0.12-0.091)^2 + 0.50*(0.09-0.091)^2 + 0.20*(0.05-0.091)^2 = 0.000589$

$\sigma = 0.024269$

EXPECTED RETURN AND RISK FOR PORTFOLIOS

Until now we have only focused on calculating return and risk for individual securities. However no investor invests in only a single financial asset. Investors put their money in to several financial securities. When one invests in more than one financial asset, this person is actually creating a PORTFOLIO.

Weight of an asset with a portfolio: The proportion of the portfolio invested in the asset

Example (7)

Assume that you have invested $80,000 in a portfolio constructed by investing in asset A, B and C.

Stock A investment = $20,000

Stock B investment = $50,000

And the rest of the money invested in stock C

Required: What are the weights of Stock A, B and C within this portfolio?

W_A = 20,000/80,000 = 25%

W_B = 50,000/80,000 = 62.5%

W_C = 10,000/80,000 = 12.5%

Fact 1: Sum of the weights of the assets within a portfolio should always add up to 1!

Expected return on a portfolio = $E(r_p) = \sum_{I=1}^{N} (w(i) * r(i)) = w_1*r_1 + w_2*r_2 + \ldots\ldots\ldots + w_n*r_n$

(Formula above calculates the expected return on a portfolio constructed with n different assets)

Example (8)

Following additional info is given for example (7), please calculate the expected return on the portfolio.

$r_A = 9\%$; $r_B = 5\%$; $r_C = 2\%$

Solution:

$E(r_p) = 25\% * 9\% + 62.5\% * 5\% + 12.5\% * 2\% = 5.625\%$

Caution 1: What does (-) weight mean?

If weight of a stock is negative, the investor is short-selling that stock. In other words the investor is borrowing some shares of that stock and selling those shares in the market right away to raise money. The proceeds from the sales of this stock is used to invest in another stock.

Variance of a portfolio with 2 assets:

$$= \boldsymbol{\sigma}^2(p) = w_a^2 * \sigma_a^2 + w_b^2 * \sigma_b^2 + 2\, w_a * w_b * \sigma_a * \sigma_b * \rho_{a,b}$$

$\mathbf{P}_{a,b}$ = correlation coefficient between stock A and stock B = measures the how dependent stock A and stock B returns are!

$$+\mathbf{1} \geq \boldsymbol{\rho}_{a,b} \geq -1$$

If $\rho_{a,b} = +1$; stock A and stock B are perfectly positive correlated

If the price of stock 1 increases by 10%; price of stock 2 also increases by 10%!

If $\rho_{a,b} = -1$; stock A and stock B are perfectly negatively correlated

If the price of stock A increases by 10%; price of stock B decreases by 10%!

You can create a risk-free portfolio by combining 2 stocks that are perfectly negatively correlated.

$W_a = \frac{\sigma_b}{\sigma_a + \sigma_b}$ and $W_b = 1\text{-}W_a$

Fact 2: $COV_{a,b} = \boldsymbol{\sigma}_{a,b} = \sigma_a * \sigma_b * \rho_{a,b}$

Covariance is a statistical measure of the correlation of the fluctuation of the annual rates of return of different investments.

Covariance can also be calculated by using the following formula:

$$COV_{a,b} = \sum_{i=1}^{n} \text{Prob}(i) * (r_A(i) - E(r_A)) * (r_B(i) - E(r_B))$$

Example (9)

	Probability	Return (A)	Return (B)
Boom	30%	12%	8%
Normal	50%	9%	6%
Recession	20%	5%	4%

Mr. Jones currently has $10,000 in cash. He will be short-selling $5,000 worth of stock B and will invest these proceeds as well as his initial wealth in stock A. What would be the expected return and the variance of Mr. Jones' portfolio?

Solution:

Step 1: Calculate the E(r) for stock A and B

$E(r_A) = 30\% * 12\% + 50\% * 9\% + 20\% * 5\% = 9.1\%$

$E(r_B) = 30\% * 8\% + 50\% * 6\% + 20\% * 4\% = 6.2\%$

Step 2: Calculate the weight of stock A and B within the portfolio

$W_B = -5{,}000/10{,}000 = -50\% = -0.5$

$W_A = 15{,}000/10{,}000 = 150\% = 1.5$

Step 3: Calculate the expected return on the portfolio

$E(r_p) = -0.5 * 6.2\% + 1.5 * 9.1\% = 10.55\%$

Step 4: Calculate the σ^2 of stock A and B

$$\sigma^2_A = 0.30*(0.12-0.091)^2 + 0.50*(0.09-0.091)^2 + 0.20*(0.05-0.091)^2 = 0.000589$$

$$\sigma_A = 0.024269$$

$$\sigma^2_B = 0.30*(0.08-0.062)^2 + 0.50*(0.06-0.062)^2 + 0.20*(0.04-0.062)^2 = 0.000196$$

$$\sigma_B = 0.014$$

Step 5: Calculate the covariance between stock A and B

$\sigma_{A,B}$ = 0.30 * (0.12-0.091) * (0.08-0.062) + 0.50 * (0.09-0.091) * (0.06-0.062) + 0.20 * (0.05-0.091) * (0.04-0.062)

= 0.000338

Step 6: Apply the variance of a portfolio with 2 stocks portfolio and compute the variance of Mr. Jones' portfolio

$\sigma^2(p) = w_a^2 * \sigma_a^2 + w_b^2 * \sigma_b^2 + 2\, w_a * w_b * \sigma_a * \sigma_b * \rho_{a,b}$

$\sigma^2(p) = w_a^2 * \sigma_a^2 + w_b^2 * \sigma_b^2 + 2\, w_a * w_b * COV_{a,b}$

$\sigma^2(p) = 1.5^2 * 0.000589 + (-0.5)^2 * 0.000196 + 2 * (-0.5) * (1.5) * 0.000338$

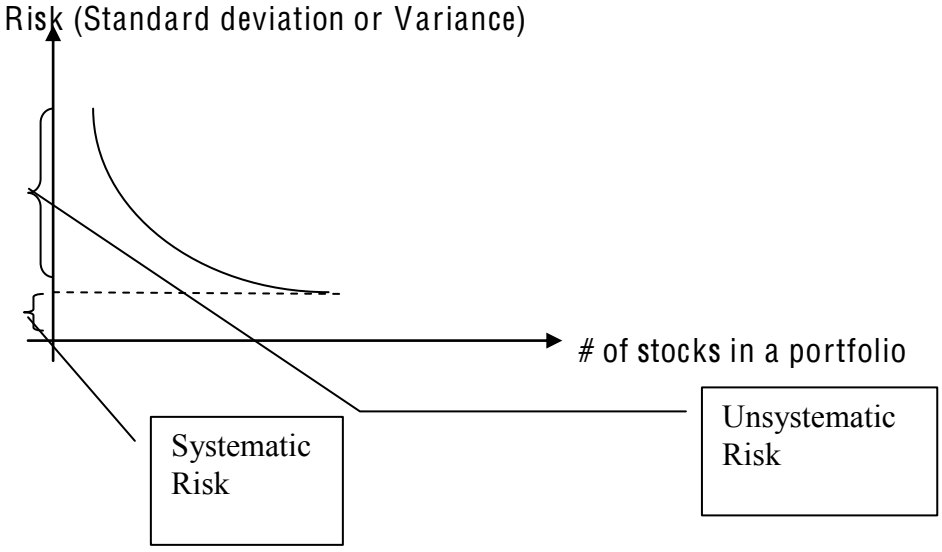
$\sigma^2(p) = 0.00086725$

CHAPTER 9 – DIVERSIFICATION & CAPM

DIVERSIFICATION

The purpose of diversification is to reduce investment risk by investing in many assets.

GRAPH - # OF STOCKS IN A PORTFOLIO VS RISK



IMPORTANT FACTS

- Standard deviation and variance are both measures of total risk
- Total risk = Systematic risk + Unsystematic risk

Definition of systematic risk: Risk factors that affect the whole market.

I.E: Inflation rate, foreign exchange rate, gas price etc.

Definition of unsystematic risk: Firm specific risk; risk factors that affect only the firm itself

I.E: The CEO of IBM is 90 years old

- **Beta of a stock**: Beta is the measure of systematic risk. Measures how risky the stock is relative to the market portfolio

$$\beta(i) = \frac{\sigma(i,m)}{\sigma^2(m)}$$

- **Risk free asset**: The asset that provides certain cash flows
- The std. deviation, variance and the beta of risk free assets equal to zero
- The beta of the market portfolio always equals to 1
- The beta of a stock that is as risky as the market also equals 1
- The beta of a stock that is twice as risky as the market would equal 2
- A well-diversified portfolio doesn't have any unsystematic risk (it has been eliminated)

Beta of the portfolio = $\beta_p = w_1*\beta_1 + w_2*\beta_2 + \ldots\ldots + w_n*\beta_n$

CAPITAL ASSET PRICING MODEL (CAPM)

The model is used to calculate the fair (required) return on an asset based on its particular β.

$k_s = r_f + \beta_s * (r_m - r_f)$

where,

r_f = return on risk-free asset
r_m = return on market portfolio
β_s = Beta of the stock
$r_m - r_f$ = market risk premium

Caution: When the question indicates that value of market risk premium; make sure that you plug in $r_m - r_f$ as the market risk premium (NOT the $r_{m!!!}$)

HOW TO DETERMINE WHETHER AN ASSET IS CORRECTLY PRICED?

Step 1)

Compute the fair (required) return on the stock based on CAPM!

Step 2)

Compare the actual (realized or expected) return on the stock with its fair (required) return computed from CAPM!

If actual return > fair return ------ the asset is underpriced ---- invest!

If actual return < fair return ----- the asset is overpriced ---- do not invest!

Example (10)

Currently the return on T-bills = 6%; the market risk premium is = 8%

	Beta	Realized return
STOCK (A)	1.2	17%
STOCK (B)	0.6	10.8%
STOCK (C)	1.5	15%

Which one these stocks would the investor purchase? Which one would the investor sell? Please explain.

Solution:

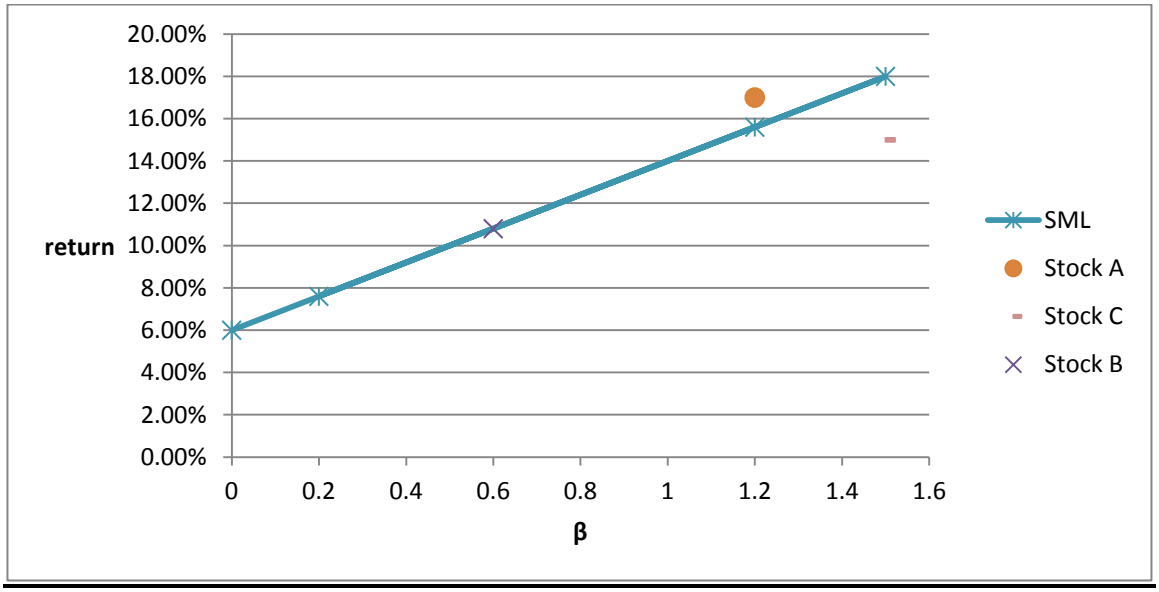
For stock (a): Fair (required) return = 0.06 + 1.2 * (0.08) = 15.6% < 17% realized return therefore Stock A is underpriced. Investors should purchase this stock.

For stock (b): Fair (required) return = 0.06 + 0.6 * (0.08) = 10.8% = 10.8% realized return therefore Stock B is correctly priced.

For stock (c): Fair (required) return = 0.06 + 1.5 * (0.08) = 18% > 15% realized return therefore Stock C is overpriced. Investors should sell this stock.

HOW TO DRAW THE SECURITY MARKET LINE (SML)?

Security Market Line is the line that corresponds to the CAPM formula.



Stocks that plot above the SML would be underpriced; stocks that plot below the SML would be overpriced and the stocks that plot on the SML would be correctly (fairly) priced.

Slope of the Security Market Line = Market risk premium = $r_m - r_f$

REWARD TO RISK RATIO

The additional return expected as a result of being exposed to one more unit of systematic risk

Stock A's reward to risk ratio = $\frac{Risk-premium-on-stockA}{Beta-of-stockA} = \frac{r(a)-rf}{\beta(a)}$

If the stocks are correctly priced their reward to risk ratios would be equal to each other as well as the market risk premium!

Question: What is the Capital Market line? What is the Sharpe Ratio?

If the X-axis (which used to be Beta) is replaced with the standard deviation; the line would be called Capital market line rather than the Security Market Line.

Sharpe ratio of stock A = $\frac{r(a) - rf}{\sigma(a)}$ would also be the slope of the capital market line

CHAPTER 10 – THE EFFICIENT MARKET HYPOTHESIS

The EMH is used to classify markets based on how efficiently market prices respond to the availability of new information. Markets can be classified as being:-

Weak Form Efficient
Market prices only incorporate historical information. The prices in a weak form efficient market only change after the fact i.e. after the event occurs.

Semi Strong Form Efficient
Market prices incorporate historical and public information. The prices in a semi strong form efficient market change after an announcement is made or after information becomes publicly available.

Strong Form Efficient
Market prices incorporate historical, public and private information. The prices in a strong form market change after the insiders of a corporation gain new information. I.e. prices change immediately after inside information becomes available.

Note:

- If a given statement violates the weak form of market efficiency, then by default it violates all forms of efficiency including semi-strong form and strong form. If the statement violates the semi-strong form of efficiency then it violates the strong form as well, and if it violates the strong form of efficiency then it violates the strong form only.

- If a given statement supports the strong form of efficiency, then by default it supports all forms of efficiency. If it supports the semi-strong form then it supports the weak form as well. If the statement supports the weak form then it supports the weak form only.

PRACTICE PROBLEMS

Problem (1)

Suppose you have a level of initial wealth and then borrow an additional 20% of this initial wealth at the risk free rate. You then invest all of your initial wealth plus the amount you borrowed in to a fund that represents the market portfolio. What will your portfolio beta be?

Problem (2)

A portfolio manager is holding the following portfolio:

Stock	Value of holdings	Beta
1	$300,000	0.6
2	300,000	1
3	500,000	1.4
4	500,000	1.8

The risk free rate is 6% and the portfolio's required rate of return is 12.5%. The manager would like to sell all of her holdings in Stock 1 and use the proceeds to purchase more shares of stock 4. What would be the portfolio's required rate of return following this change?

Problem (3)

The risk free rate is 7%, Beta of Itech stock is 1.2 and Beta of LOL stock is 0.8. The expected return on Itech stock is 13.5% and the expected return on LOL Stock is 11%. Further we know that Itech stock is fairly priced and the betas of Itech and LOL are correct. Would you purchase the LOL stock? Why?

Problem (4)

State	Prob	R(A)	R(B)
Boom	30%	18%	14%
Recession	????	10%	8%

What is the standard deviation of the portfolio made up of 20% in stock A and 80% in stock B?

Problem (5)

Mr. Jones is considering investing $400,000 in stock A and B. Stock A return has a variance of 0.0766 and stock B return has a standard deviation of 0.15. Determine how much he should invest in each stock in order to create a zero risk portfolio. Assume a correlation coefficient of -1.

Problem (6)

Using only stock A and B, Mr. Jones has constructed a portfolio with an expected return of 17.5%. The total value of the portfolio is $120,000 and $48,000 of it is in stock B. The T-bill rate is 4.65% and the return on the market is 11.5%. The beta of stock B is 1.45. Assume all stocks are fairly priced. Find the expected return of Stock A.

PRACTICE PROBLEM SOLUTIONS

Problem (1)

W(rf) = -0.20

W(fund) = 1-(-0.20) = 1.20

β(fund)=1 (since it is as risky as the market portfolio)

β(portfolio)=1*1.2+-0.3*0 = 1.2

Problem (2)

Total investment = 300K+300K+500K+500K = $1.6M

Current portfolio weights:

W(1) = 300K/$1.6M = 18.75%

W(2) = 300K/$1.6M = 18.75%

W(3) = 500K/$1.6M = 31.25%

W(4) = 500K/$1.6M = 31.25%

β(current-portfolio)= 0.1875 * 0.6 + 0.1875 * 1 + 0.3125 * 1.4 + 0.3125 * 1.8 = 1.3

12.5% = 6% + 1.3 * (rm – 6%)

Solve for rm = 11%

New portfolio weights:

W(1) = 0%; W(2) = 18.75%; W(3) = 31.25%

W(4) = (300K+500K)/1.6M = 50%

β(new-portfolio)= 0.1875*1 + 0.3125 * 1.4 + 0.50 * 1.8 = 1.525

Required return of the new portfolio = 6% + 1.525 * (11%-6%) = 13.625%

Problem (3)

If Itech is fairly priced, CAPM formula for Itech should give us a return of 13.5%.

k(itech) = 0.135 = 0.07 + 1.2 (rm – 0.07)

Solve for rm = 12.417%

Apply the CAPM for LOL and figure out their required return:

Required return (LOL) = 0.07 + 0.8 (0.12417-0.07) = 11.33% > Expected return (LOL) of 11%

Therefore LOL is overpriced, you shouldn't purchase this stock.

Problem (4)

Prob(recession) = 1-30% = 70%

Since the correlation coefficient is not given, you have two ways of solving for the standard deviation of the portfolio

Method 1: Find the covariance(A,B); Variance (a) and variance (b) then apply the variance of a 2-stock portfolio formula.

However this method is time consuming.

Alternatively, Method 2:

Step 1: Find the return on the portfolio for boom and recession cases.

Boom: rp = 0.20*0.18 + 0.8*0.14 = 14.8%

Recession: rp = 0.2*0.10 + 0.8*0.08 = 8.4%

Step 2: Find the expected return on the portfolio:

E(rp) = Prob(boom) * rp(boom) + Prob(recession)* rp(recession) = 0.30*0.148 + 0.70*0.084 = 10.32%

	Prob	Rp
Boom	30%	14.8%
Recession	70%	8.4%

Step 3: Apply the variance formula

Variance = $0.30 * (0.148-0.1032)^2 + 0.70 * (0.084-0.1032)^2 = 0.00086$

Standard deviation = 0.02933

Problem (5)

W(a) = $\frac{\sigma b}{\sigma a+\sigma b}$ = $0.15/(0.15+0.0766^{1/2}) = 35.148\%$

Stock A investment = 35.148% * 400,000 = $140,591.92

Stock B investment = 400,000 – 140,591.92 = $259,408.08

Problem (6)

W(b) = 48,000/120,000 = 40%

W(a) = 1-40% = 60%

Return(b) = 0.0465 + 1.45(0.115-0.0465) = 14.583%

Rp = 0.60*r(a) + 0.40 * 14.583% = 17.5%

Solve for r(a) = 19.445%

CHAPTER 13 – INVESTMENT CRITERIA

- Refers to different decision rules followed by the company to help them make investment decisions.

Key Concepts for Exam Preparation

1. Types of Investment Criteria
2. Types of Investment Constraints
3. Knowing how to obtain the following for each type of investment criteria:

    i. The Quantitative measure
    ii. The Decision Rule
    iii. Limitations of using the measure

4. Drawing NPV Profiles
5. Computing the Cross Over Rate
6. Computing EANPV and EAC

Types of Investment Criteria

- Payback Period
- Discounted Payback Period
- Internal Rate of Return (IRR)
- Profitability Index
- NPV

Types of Investments Constraints

- Capital Rationing – Is the case when the firm has a fixed investment budget. When a firm has this investment constraint it will undertake all acceptable investments under the given criteria that it can afford

- Mutually Exclusive Investments – Is the case when the firm is only able to undertake one investment. When investments are mutually exclusive the firm will only undertake the best acceptable investment under the given criteria

- Independent Investments – Is the case when the firm is able to undertake more than one investment. When investments are independent the firm will undertake all acceptable investments under the given criteria.

Demonstration Example

ABC Inc. is considering two different investments in projects that will generate the following cash flows each year:

Time	A	B
0	(10000)	(10000)
1	6075	1100
2	4075	2100
3	3075	3100
4	2075	5100
5	1075	8100

The appropriate discount rate for the company is 5%. Additionally, the firm has set a payback cut-off period of 1.5 years and a discounted payback cut-off period of 2.5 years.

- Assume ABC is interested in computing the payback period for project A in order to determine if the investment is acceptable under the firm's policies.

Quantitative Measure

- The payback period measures how long it takes for us to recover the project's initial investment using the projects cash flows. In other words it measures how long it takes for the project to break-even. To compute the payback period for project A we must setup a table as shown below:

Time	Cash Flow	Cumulative Cash Flow
0	(10000)	(10000)
1	6075	(3925)
2	4075	150
3	3075	
4	2075	
5	1075	

- We can stop making the cumulative cash flow column once the sign of the cash flow changes from negative to positive. When this happens it means that the project breaks even somewhere between these two time periods.

- By observing the table above we can see that this project breaks even somewhere between the 1st and the 2nd year of the project's life. To compute the exact fraction of the 2nd year that is needed to break-even we must simply divide the amount that still needs to be recovered at the end of the first year (value in the cumulative cash flow column) by the project's cash flow in the 2nd year, as shown below:

$$payback \quad period = 1 + \frac{3925}{4075} = 1.96319018 \; years$$

Decision Rule

- Under the payback period the decision rule is straightforward as is shown below:

$$If \quad project \quad payback < payback - cutoff - period \longrightarrow accept \quad investment$$
$$If \quad project \quad payback > payback - cutoff - period \longrightarrow reject \quad investment$$

- Using the payback period criteria we accept investments if the project's payback period is less than the firm's target payback period. Since this project takes longer than 1.5 years to payback ABC would not undertake the project.

Limitations

- Timing of Cash Flows (TVM not considered), we are pretending like cash flows which happen in the future are worth the same as if they happened today.
- Cash Flows after the payback period are ignored.
- Payback cut-off period is arbitrarily set; there is no objective way of selecting this number.

- Assume that ABC wants to compute the discounted payback period for project A, given the firm's discount rate of 5%

Quantitative Measure

- The discounted payback period measures how long it takes for us to recover the project's initial investment using the project's discounted cash flows. In other words it measures how long it takes for the project to break-even when incorporating time value money. To compute the discounted payback period for project A we must setup a table as shown below:

Time	Cash Flow	Discounted Cash Flow	Cumulative Discounted Cash Flow
0	(10000)	(10000)	(10000)
1	6075	$\frac{6075}{1.05} = 5785.7143$	(4214.2857)
2	4075	$\frac{4075}{(1.05)^2} = 3696.1451$	(518.1406)
3	3075	$\frac{3075}{(1.05)^3} = 2656.3006$	2138.16

- We can stop making the cumulative discounted cash flow column once the sign of the cash flow changes from negative to positive. When this happens it means that the project breaks even somewhere between these two time periods.

- By observing the table above we can see that this project breaks even somewhere between the 2nd and 3rd year of the project's life. To compute the exact fraction of the 3rd year that is needed to break-even we must simply divide the amount that still needs to be recovered at the end of the second year (value in the cumulative discounted cash flow column) by the project's discounted cash flow in the 3rd year, as shown below:

$$discounted \quad payback \quad period = 2 + \frac{518.1406}{2656.3006} = 2.19506098 \, years$$

Decision Rule

- Under the discounted payback period the decision rule is the same as with the payback period as is shown below:

$$If \quad project's \quad discounted \quad payback < discounted - payback - cutoff \longrightarrow accept - investment$$

$$If \quad project's \quad discounted \quad payback > discounted - payback - cutoff \longrightarrow reject - investment$$

- Using the discounted payback period criteria we accept investments if the project's discounted payback period is less than the firm's target discounted payback period. Since this project has a discounted payback period of less than 2.5 years, ABC would undertake the project.

Limitations

- Cash Flows after the discounted payback cut-off year are ignored.
- Discounted payback cut-off period is arbitrarily set; there is no objective way of selecting this number.

Method 3: NPV

- The net present value criteria is measure of how profitable an investment is while taking into account time value of money. For example assume that ABC wanted to measure the NPV of project B at the given discount rate of 5%

Quantitative Measure

- The net present value measures the difference between the present value of a project's inflows and its initial outflow. To compute the NPV for a given project we must simply enter all of the projects cash flows into the CF button. Then press the NPV button, enter the discount rate for I, press the down arrow and then press CPT.

$$NPV_B @ 5\% = 6172.6221$$

Decision Rule

- According to the NPV criterion we accept any investment that has a positive NPV and we reject any investment that has a negative NPV. Using this criteria ABC would accept project B since its NPV is greater than 0.

Limitations

- Although, NPV is the most widely used criteria, it cannot be used directly when we are dealing with mutually exclusive investments which have unequal lives and are going to be repeated over time. When we deal with this type of a problem (the scale problem) we must compute the EAC or EANPV as will be discussed next.

Computing the EANPV and EAC

- Whenever we are dealing with mutually exclusive projects which have different lives that will be repeated over time, we must compute the equivalent annual net present value (EANPV) when dealing with a project that generates inflows or the equivalent annual cost (EAC) when dealing with a project that requires outflows.

- Use the following steps to compute the EANPV or the EAC

    1. Compute the NPV for each project
    2. Compute the EAC or the EANPV
    3. Make the Decision

How to find the EANPV (EAC)?

2^{nd} FV
2^{nd} CE/C
NPV = PV
Useful life = N
Discount rate = I/Y

CPT PMT = EANPV (or EAC)

EXAMPLE:

ITECH is evaluating two potential projects. These projects have the following cash-flows:

PERIOD	CF(A)	CF(B)
0	-1000	-1000
1	800	1500
2	900	1600
3	1200	

The appropriate discount rate for the projects would be 10%.

i) If the selected project will be used only once, which project should be selected?
ii) If the selected project will be repeated forever, which project should be selected?

Solution:

(i) NPV (A) = $1,372.65; NPV(B) = $1,685.95
Select project B since NPV (B) > NPV (A)

(ii) Decision must be based on EANPV

To find EANPV (A):
1,372.65 PV; 3 N; 10 I/Y CPT PMT = EANPV (A) = $551.96/year

To find EANPV (B):
1,685.95 PV; 2 N; 10 I/Y CPT PMT = EANPV (B) = $971.43/year

Since EANPV (B) > EANPV (A), pick project B.

Method 4: Internal Rate of Return

- Recall that as interest rates rise, present values fall. The IRR measures the highest discount rate that will still provide the project with an NPV that is not negative. In other words it is the discount rate which equates the project's NPV to 0.

- At any rate higher than the IRR the project's NPV will be negative, whereas at any rate that is lower the project's NPV would be positive.

- Assume that ABC wants to determine if both the projects are acceptable using the IRR criteria. Further assume that the two projects are independent of each other.

Quantitative Measure

- Computing an IRR is only possible using either a financial calculator or by trial-and-error. To compute the IRR for a given project we must simply enter all of the project's cash flows into the CF button and then press IRR followed by compute. To compute the IRR for project B we would simply enter the following into the CF button:

CF	A	B
CF0	(10000)	(10000)
C01	6075	1100
C02	4075	2100
C03	3075	3100
C04	2075	5100
C05	1075	8100

- Once we have entered a given project's cash flows into the CF button we must simply press on IRR followed by compute. The IRR's for each project are shown below:

$$IRR_A = 26.5662\%$$
$$IRR_B = 19.6082\%$$

Decision Rule

- Since the IRR represents the rate which equates the project's NPV to 0, the decision rule is to accept any investment which has an IRR that is greater than the project's discount rate.

- Since ABC appropriate discount rate for the two projects (5%) is less than the IRR of each respective projects, and because the projects are independent they are both acceptable. If they were mutually exclusive projects, the project with the larger IRR would have been accepted.

Limitations

- Multiple IRR values can be computed with non-conventional cash flows. Non-conventional cash flows are simply cash flows which change sign from negative to positive or from positive to negative more than once.

- What is important for us to remember is that with non-conventional cash flows the number of IRR's that exist is equal to the number of times that the sign of the cash flows would change. For the following project; 3 different IRRs would exist:

CF	Project A
CF0	(10000)
C01	6075
C02	(4075)
C03	3075
C04	2075
C05	1075

- Problem with Mutually Exclusive Investments may arise with some projects when using the IRR criterion. Sometimes when dealing with mutually exclusive projects (the case when the firm must select the best alternative) the IRR and NPV criteria may provide us with conflicting results.

- According to the IRR criterion, with mutually exclusive projects we would choose the project that has the largest IRR. For example with our example, if project's A and B were mutually exclusive, the IRR criterion would dictate to undertake project A since project A has the larger IRR. This decision does not necessarily imply that project A also has a higher NPV than project B at the company's given discount rate.

- To better visualize the problem with the IRR and NPV criteria providing us with conflicting results, we can construct NPV profiles for both projects A and B to determine how the value of a given project changes when we change the discount rate. This will be discussed next.

Drawing NPV Profiles

- Assume that projects A and B are mutually exclusive and that the appropriate discount rate is 5%. The projects have the following cash flows:

Time	A	B
0	(10000)	(10000)
1	6075	1100
2	4075	2100
3	3075	3100
4	2075	5100
5	1075	8100

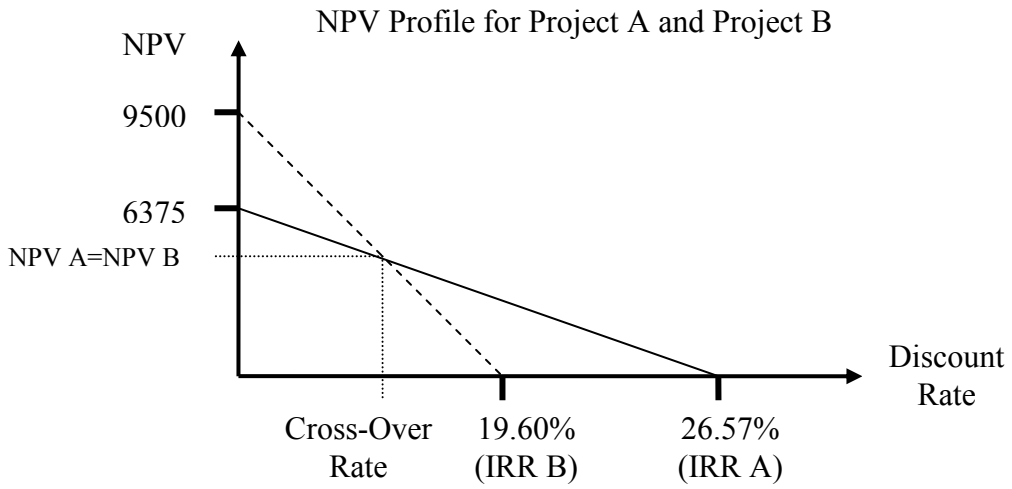
- To construct an NPV profile for each project use the following steps:
  - Compute the IRR for each project. To do this we must simply enter the project's cash flows in the CF button and then press IRR compute just as we did before:

$$IRR_A = 26.5662\%$$
$$IRR_B = 19.6082\%$$

  - Compute the NPV for each project assuming a discount rate of 0. To do this we simply enter the project's cash flows in the CF button and then press NPV. Enter 0 for I, press the down arrow, then press CPT:

$$NPV_A @ 0\% = 6375$$
$$NPV_B @ 0\% = 9500$$

- Use the previous answers as the x and y intercept respectively of the graph and construct the profile as is shown below:



- By looking at the graph above we can see that anywhere on the right of the cross over rate we prefer project A to project B since A has a higher NPV, whereas anywhere on the left of the cross over rate we prefer project B to project A since B has a higher NPV.

Computing the Cross-Over Rate

- The cross-over rate is the discount rate where the NPV's of both the projects is the same. To compute this rate simply use the steps below:

1. Compute the Incremental Cash Flow (A-B) or (B-A)

Time	A	B	(A-B)
0	(10000)	(10000)	0
1	6075	1100	4975
2	4075	2100	1975
3	3075	3100	(25)
4	2075	5100	(3025)
5	1075	8100	(7025)

2. Enter the Incremental Cash Flow's in the CF button

3. Compute the IRR → 11.5057%. Since 5% is less than the cross over rate, ABC would prefer project B over project A even though A has a higher IRR.

Method 5: Profitability Index

- The profitability index is also known as the benefit to cost ratio since it tells you how much money do you get back in terms of present value for every dollar that you invest in undertaking the project (also in terms of present value).
- Assume ABC wants to compute the profitability index for project B.

Quantitative Measure

- To compute the profitability index we must simply divided the present value of a project's cash flows by the absolute value of the initial investment. In general the profitability index is computed as shown below:

$$\text{Profitability Index} = \frac{PV(\text{Inflows})}{|PV(\text{Outflows})|}$$

- To obtain the answer for the numerator we must simply enter all of the cash flows for project B except for the initial investment into the CF button and compute the present value at a discount rate of 5%. We can then divide this number by the initial investment needed for project B as shown below:

$$\text{Profitability Index} = \frac{16172.6221}{|-10000|} = 1.6173$$

Decision Rule

- When using the profitability index criterion, the decision rule is to accept investments if the profitability index is greater than 1 and to reject any investment where the profitability index is less than 1.
- Based on the profitability index for project B, ABC would accept the project since the profitability index is 1.6173 which is bigger than 1.

Limitations

- Since the profitability index is a ratio, it can't be used to decide between mutually exclusive investments. Even if two projects have the same profitability index, this does not necessarily mean that they both have the same NPV.

Investment Criteria Examples

Example 1

A project has a required return of 15% and a five-year life. The value of which one of the following criteria is inconsistent with the other four?

A. The discounted payback is 5 years.
B. PI (profitability index) = 0
C. NPV = $0
D. IRR = 15%
E. The present value of the future cash flows equals the initial investment

Solution:

If NPV = 0, this would indicate:
The discounted payback period = useful life of the project = 5 years
IRR = the current discount rate
PI = 1
And the PV of cash in-flows = PV of cash out-flows

Therefore the odd answer is B.

Example 2

XYZ Inc. has imposed the following policies for new investments:

- Discount rate: 18%
- Payback cut-off period: 2.85 years
- Discounted payback cut-off period: 3.25 years
- Capital Rationing: $300000

The firm is considering one of three possible independent investments (A, B or C), which cost $160000, $180000 and $140000 respectively. Based on this information complete the following table:

Method	Project A	Project B	Project C	Choose
Payback	**1.60**	**1.8**	**2.76**	
Discounted Payback	**3.35**	**3.4**	**2.95**	
NPV @ 12%	**?**	**100800**	**?**	
IRR	**20.65**	**23.65**	**17.45**	
Profitability Index	**1.25**	**?**	**1.54**	

Solution:

For project A: PV of in-flows = 1.25 * 160,000 = $200,000
Therefore NPV (A) = 200,000 – 160,000 = $40,000

For project B: PV of in-flows = $180,000 + $100,800 = $280,800
So the profitability index = 280,800/180,000 = 1.56

For project C: PV of in-flows = 1.54 * 140,000 = $215,600
Therefore NPV (C) = 215,600 – 140,000 = $75,600

Method	Project A	Project B	Project C	Choose
Payback	**1.60**	**1.8**	**2.76**	**A,C**
Discounted Payback	**3.35**	**3.4**	**2.95**	**C**
NPV @ 12%	**$40,000**	**$100,800**	**$75,600**	**A,C**
IRR	**20.65**	**23.65**	**17.45**	**B**
Profitability Index	**1.25**	**1.56**	**1.54**	**A,C**
Cost	**$160,000**	**$180,000**	**$140,000**	

CHAPTER 14- CAPITAL BUDGETING

In this chapter, we will focus on how to make decisions in the following cases:

Case 1: Whether to accept or reject a project
Case 2: Whether to keep or replace an existing machine that is not at the end of its useful life yet (Replacement questions)

In both of these cases, we will run an **NPV analysis** based on **RELEVANT CASHFLOWS**!
What are the relevant cash flows?
The relevant cash flows are the cash flows that would only exist if you were to accept the project. If you reject the project these cash flows wouldn't take place.

*** Incremental cash flows:** The difference between the cash flows of the firm when they take on the project and the cash flows of the firm when they do not take on the project. Assume that the current annual operating expenses of the firm are $100,000. When the firm takes on the new project the annual operating expenses go up to $250,000. So when we are evaluating whether or not to accept the new project, should we consider the $250,000 as the annual expense?

The answer is no! Without the project you are already incurring $100,00 in annual expenses so the project is only increasing your annual expense by $150,000. Therefore one should only consider the $150,000 as the appropriate annual operating expense that can be assigned to the new project.

*** Opportunity cost:** The lost profit because of the missed opportunity when you invest in a project
*** Erosion cost:** If you have a coffee shop on corner of Peel and Sherbrooke; and decide to open another coffee place a block away; the new coffee place might cause a decline in the sales of your previous coffee shop. This sales decline would be referred to as an "erosion cost".

Some examples of irrelevant cash flows:
***Sunk cost:** The cost that has already been incurred regardless of investing in the project or not

***Financing cost:** The interest expense or any other type of financing cost that might be involved with the project is never taken in to account
during our NPV analysis. The discount rate that we use for NPV analysis is assumed to include the cost of financing within.

CASE 1 ANALYSIS: WHETHER TO ACCEPT OR REJECT A PROJECT

Step 1:
Calculate the initial investment (C) amount for this project!
C = Cost of acquiring the machine + Cost of installation + Cost of transportation

Step 2:
If the depreciation method is straight-line to zero:
Calculate the annual incremental operating cash flow! (**OCF(i))**

If there is a CCA rate provided within the question:
Calculate the annual incremental operating cash flow excluding the depreciation tax shield! (**OCF* (i))**

Operating cash flow is an annual figure. This cash flow is assumed to take place at the end of every year.

OCF(i) = Operating cash flow (i)
(Before-tax incremental oper. Rev.(i) - Before-tax incremental oper. exp(i)) * (1-tc)
+ Annual depreciation(i) * tc

OCF* (i) = Operating cash flow excluding depreciation tax shield (i) =
(Before-tax incremental oper. rev(i) - Before-tax incremental oper. Exp.(i)) * (1-tc)

Depreciation (CCA) tax shield (i) = Annual depreciation(i) * tc

What is depreciation (CCA) tax shield?

Depreciation expense or CCA (capital cost allowance) refers to the $ value that your asset would lose over 1 year time. This annual expense can be listed∆ in your income statement. Once you account for this expense, the income before tax would look less, leading the government to charge you less in taxes. The annual tax savings achieved as a result of the depreciation expense is called the depreciation tax shield.

Additional formula: After-tax CF = Before-tax CF * (1-tc)

Step 3:

Calculate the Change in Net Working Capital for every year!

In the exam you might come across three different situations where you'd have to follow a different approach to calculate the Change in Net Working Capital:

Situation a) "Initially the net working capital investment requirement is $20,000. At the end of year 2, the new working capital is reduced by $15,000. Project has 5-yr life"

If the word "requirement" or "increase" is used; the Change in NWC is +
If the word "reduction" or "decrease" is used; the change in NWC is -

∆NWC(0) = + 20,000
∆NWC(1) = 0 (nothing has been mentioned about the end of yr 1 in the question)
∆NWC(2) = -15,000
∆NWC(3) = 0 (nothing has been mentioned about the end of yr 3 in the question)
∆NWC(4) = 0 (nothing has been mentioned about the end of yr 4 in the question)

VERY IMPORTANT FACT: THE CHANGE IN NET WORKING CAPITAL AT THE END OF PROJECT'S LIFE

The ∆NWC at the end of the last year of the project = ∆NWC (n)
= - (Change in NWC (0) + Change in NWC (1) + ... + Change in NWC (n-1))

ΔNWC(5) = -(20,000-15,000) = -$5,000

Situation b) " At the beginning of the project inventory goes up by $20,000 and accounts payable by $8,000. At the end of year 2, there is a decrease of $3,000 in accounts payable.
By the end of year 3, the accounts receivable decreases by $6,000. Project has 5-yr life"

ΔNWC(i) = Change in currents assets (i) - Change in current liabilities (i)

Current assets: Accounts receivable; inventory
Current Liabilities: Accounts payable

ΔNWC(0) = $20,000 - $8,000 = $12,000
ΔNWC(2) = 0 - (-$3,000) = $3,000
ΔNWC(3) = -$6,000 - 0 = -$6,000
ΔNWC(5) = - (12,000+3,000-6,000) = -$9,000

Situation c) NWC is a % of the sales revenue!
E.g.
The NWC is equal to 10% of the same year's sales revenue. There is an initial NWC requirement of $20,000. The sales price per unit is $20.

Year 1 sales: 50,000 units
Year 2 sales: 70,000 units
Year 3 sales: 40,000 units
Year 4 (last year) sales: 80,000 units

Solution:
Year 1 sales revenue = 20 * 50,000 = $1,000,000
Year 2 sales revenue = 20 * 70,000 = $1,400,000
Year 3 sales revenue = 20 * 40,000 = $800,000

	0	1	2	3	4
Sales Rev.	N/A	$1,000,000	$1,400,000	$800,000	$1,600,000
NWC	$20,000	10%*1M = $100,000	$140,000	$80,000	0
ΔNWC	$20,000	$80,000	$40,000	-$60,000	-$80,000

Please note that: **ΔNWC(i) = NWC(i) - NWC(i-1)**

Step 4: This step is only applied if there is a CCA rate provided within the question!

Calculate the PV of CCATS (Present Value of CCA tax shield) using the following formula:

$$\frac{C*d*tc}{k+d}*\frac{1+0.5*k}{1+k}-\frac{S*d*tc}{k+d}*\frac{1}{(1+k)^n}$$

Where

C = Initial investment calculated in Step 1(would exclude the land cost)
d = CCA rate
tc = corporate tax rate
S = Salvage Value
n = useful life
k = discount rate

Step 5: If the depreciation method is straight-line to zero; create the following CF table:

Time Period	CF
0	- C - ΔNWC(0)
1	OCF(1) - ΔNWC(1)
2	OCF(2) - ΔNWC(2)
.	.
.	.
n-1	OCF(n-1) - ΔNWC(n-1)
n	OCF(n) - ΔNWC(n) + Salvage Value * (1-tc)

Calculate the NPV at the given discount rate. If NPV > 0, accept the project!

If the CCA rate is provided within the question; create the following CF table:

Time Period	CF
0	- C - ΔNWC(0) + PV of CCATS
1	OCF*(1) - ΔNWC(1)
2	OCF*(2) - ΔNWC(2)
.	.
.	.
n-1	OCF*(n-1) - ΔNWC(n-1)
n	OCF*(n) - ΔNWC(n) + Salvage Value

Calculate the NPV at the given discount rate. If NPV > 0, accept the project!

Caution: If there is an indication of opportunity or erosion cost in the question; these costs should be subtracted from the appropriate period's cash flow!

Note: Salvage value is the $ sales price of an asset at the end of its useful life.

CASE 2 ANALYSIS: REPLACEMENT QUESTIONS

Step 1:

Calculate the initial investment (C) amount for this project!

C = Cost of acquiring the new machine – Proceeds from the sale of the existing machine today

Step 2:

Calculate the annual incremental operating cash flow excluding the depreciation tax shield! (**OCF^* (i))**

OCF^* (i) = Operating cash flow excluding depreciation tax shield (i) =

(Before-tax incremental operating revenue(i) - Before-tax incremental operating expense(i)) * (1-tc)

Step 3:

Calculate the Change in Net Working Capital for every year!

Step 4: This step is only applied if there is a CCA rate provided within the question!

Calculate the PV of CCATS (Present Value of CCA tax shield) using the following formula:

$$\frac{C*d*tc}{k+d}*\frac{1+0.5*k}{1+k}-\frac{\Delta S*d*tc}{k+d}*\frac{1}{(1+k)^n}$$

Where

C = Initial investment calculated in Step 1(would exclude the land cost)
d = CCA rate
tc = corporate tax rate
ΔS = Salvage Value of new machine – Salvage value of the existing machine
n = useful life
k = discount rate

Step 5:

Time Period	CF
0	- C - ΔNWC(0) + PV of CCATS
1	OCF*(1) - ΔNWC(1)
2	OCF*(2) - ΔNWC(2)
.	.
.	.
n-1	OCF*(n-1) - ΔNWC(n-1)
n	OCF*(n) - ΔNWC(n) + ΔSalvage Value

Calculate the NPV at the given discount rate. If NPV > 0, replace the machine!

CHAPTER 14 – DEMONSTRATION EXAMPLES

Example #1:

ABC Inc. is looking into acquiring a new machine which can boost the annual revenues by $170,000 for the next 4 years. IF the company goes ahead with the project, payables will increase by $42,000 at time 0. Receivables will increase by $11,000 initially and again by $11,000 at the end of year 2.

The machine costs $472,000 to purchase and $21,000 to install. Maintaining the system requires the firm to hire a part-time technician at an after-tax cost of $20,000 per year. The equipment will be depreciated at a rate of 30% and can probably be salvaged for $47,931 at the end of year 4. Tax rate = 40% and required return is 10%.

Should the company invest in this machine?

SOLUTION:

This is a Case 1 question.
Deciding whether to invest in a machine or not!

Step 1:

C = 472,000 + 21,000 = $493,000

Step 2:

CCA rate is provided in the question therefore OCF*(i) should be calculated for each of the 4 years.

Before-tax incremental annual operating revenues = $170,000
Before-tax incremental annual operating expenses = $20,000 / (1-0.40) = $33,333.33

OCF*(i) = (170,000 – 33,333.33) * (1-0.4) = $82,000 per year

Step 3:

ΔNWC will be calculated based on situation b since current asset and current liability info has been provided.

ΔNWC(0) = 11,000 – 42,000 = -31,000
ΔNWC(1) = 0
ΔNWC(2) = 11,000
ΔNWC(3) = 0
ΔNWC(4) = - (-31,000+11,000) = 20,000

Step 4:
CCA rate has been provided in the question therefore this step has to be applied!

C = $493,000
d = 30%
tc = 40%
S = $47,931
n = 4
k = 10%

PV of CCATS =

$$\frac{493{,}000*0.30*0.40}{0.10+0.30}*\frac{1+0.5*0.10}{1+0.10}-\frac{47{,}931*0.30*0.40}{0.10+0.30}*\frac{1}{(1+0.10)^4}$$

= $131,356.02

Step 5:

Time Period	CF
0	-493,000 – (-31,000) + 131,356.02
1	82,000
2	82,000 – 11,000
3	82,000
4	82,000 – 20,000 + 47,931

CPT NPV @ 10% = -$60,728.67 since < 0; **Do not purchase the machine!**

In the same question; additional questions that you could have been asked are:

a) **Find the depreciation tax shield for year 2**
b) **Find the OCF for year 3**
c) **Find the PV of the OCF excluding the depreciation tax shield**
d) **Find the Ending UCC for the project at the end of year 4**
e) **Find the PV of the cash flows due to the acquisition and sale of the fixed assets over the life of the project**
f) **Find the PV of cash flows due to changes in NWC**

Solutions for questions (a-f):

Period	BUCC	CCA	EUCC
1	493,000	73,950	419,050
2	419,050	125,715	293,335
3	293,335	88,000.50	205,334.50
4	205,334.50	61,600.35	143,734.15

BUCC (1) = C = $493,000
CCA(1) = 493,000 * 30%/2 = $73,950 (half-year rule applied)
EUCC(1) = 493,000 – 73,950 = $419,050

CCA(2) = 419,050 * 30% = $125,715
EUCC(2) = 419,050 – 125,715 = $293,335

CCA(3) = 293,335 * 30% = $88,000.50
EUCC(3) = 293,335 - 88,000.50 = $205,334.50

CCA(4) = 205,334.50 * 30% = $61,600.35
EUCC(4) = 205,334.50 - 61,600.35 = $143,734.15 (**answer to d**)

a)
Depreciation tax shield for year 2: CCA(2) * tc = 125,715 * 40% = $50,286

b)
OCF*(3) = 82,000
OCF (3) = 82,000 + 88,000.50 * 40% = $117,200.20

c)

Time Period	CF
0	0
1	82,000
2	82,000
3	82,000
4	82,000

CPT NPV @ 10% = $259,928.97

e)

Time Period	CF
0	-493,000
1	0
2	0
3	0
4	47,931

CPT NPV @ 10% = -$460,262.48

f)

Time Period	CF
0	-(-31,000)
1	0
2	-11,000
3	0
4	-20,000

CPT NPV @ 10% = $8,248.82

Example #2:

A company is considering the replacement of its refrigeration system. The old system can currently be sold for $100,000. The relevant CCA rate is 20% on a declining balance basis. The capital budgeting director estimates that if the old system is held for another four years it can be sold for $20,000. The new system will cost $335,000 and it is estimated that this system can be resold in four years for $50,000. Tax rate is 40% and the cost of capital is 12%. The additional end of year revenues before tax that result from the acquisition:

Year	1	2	3	4
CF	$52,500	$55,064	$57,696	$60,398

Assume that the operating expenses stay constant.
There is an initial NWC requirement of $2,000 and NWC each year would equal 5% of the additional revenue earned during that year.

SOLUTION:

This is a replacement question (Case 2)

Step 1:
C = $335,000 - $100,000 = $235,000

Step 2:

OCF*(1) = ($52,500 – 0) * (1-0.40) = $31,500
OCF*(2) = ($55,064 – 0) * (1-0.40) = $33,038.40
OCF*(3) = ($57,696 – 0) * (1-0.40) = $34,617.60
OCF*(4) = ($60,398 – 0) * (1-0.40) = $36,238.80

Step 3:

	0	1	2	3	4
Sales Rev.	N/A	$52,500	$55,064	$57,696	$60,398
NWC	$2,000	$2,625	$2,753	$2,885	0
ΔNWC	$2,000	$625	$128	$132	-$2,885

NWC (i) = Sales Rev(i) * 5%

Step 4:

Calculate the PV of CCATS (Present Value of CCA tax shield) using the following formula:

$$\frac{C * d * tc}{k + d} * \frac{1 + 0.5 * k}{1 + k} - \frac{\Delta S * d * tc}{k + d} * \frac{1}{(1 + k)^n}$$

Where

C = $235,000
d = 20%
tc = 40%
ΔS = 50,000 – 20,000 = $30,000
n = 4
k = 12%

$$\frac{235{,}000 * 0.20 * 0.40}{0.12 + 0.20} * \frac{1 + 0.5 * 0.12}{1 + 0.12} - \frac{30{,}000 * 0.20 * 0.40}{0.12 + 0.20} * \frac{1}{(1 + 0.12)^4}$$

= $55,602.68 - $4,766.39 = $50,836.29

Step 5:

Time Period	CF
0	-235,000 – 2,000 + 50,836.29
1	31,500 – (625)
2	33,038.40 – 128
3	34,617.60 – 132
4	36,238.80 – (-2,885) + 30,000

Calculate the NPV @ 12% = -$63,885.18 < 0 so do not replace!

CHAPTER 20 – COST OF CAPITAL

Three ways to raise capital: Issuing debt, common equity, and preferred equity

CASE 1: ISSUING DEBT (bonds)

Cost of debt before-tax = k_d = YTM on the bond

Cost of debt after-tax = $k_i = k_d * (1\text{-}tc)$

D = Market value of debt = # of bonds outstanding * current bond price

How to find the YTM on a bond?

Using the financial calculator:

- Price of the bond today = PV

Coupon rate * Face value of the bond / # of coupon payments per year = PMT

# of remaining years in bond's life * # of coupon payments per year = N

Face Value = FV

CPT I/Y

YTM = I/Y * # of coupon payments per year

Important Note: According to some professors the before-tax cost of debt = Effective YTM on the bond = $(1+\frac{YTM}{m})^m - 1$ where m = # of coupon payments per year

Example:

Itech has just issued 5,000 10% coupon bonds that make quarterly coupon payments. The bonds mature in 5 years and are currently sold at 90% of par value. Tax rate is 40%

Required:

i) Compute the before and after-tax cost of debt

ii) Compute the market value of debt

Solution:

(i) Current bond price = 90% * 1,000 = $900

-900 PV; 5*4 = 20 N; 1,000 FV; (10%*1,000)/4 = $25 PMT CPT I/Y = 3.18%

YTM = 4 * 3.18% = **12.72% = before-tax cost of debt**

(If your professor argues the effective YTM should be the before-tax cost of debt; the answer would be $(1+\frac{0.1272}{4})^4 - 1 = 13.36\%$)

After-tax cost of debt = 12.72% * (1-0.4) = 7.63%

(ii)

MV of debt = D = 5,000 * 900 = $4,500,000

CASE 2: ISSUE COMMON EQUITY (common stock)

Two ways to identify cost of common equity (r_e)

- Using CAPM model

$k_e = r_f + \beta_e * (r_m - r_f)$

- Using Dividend growth model (applicable if there is a constant growth rate forever!

$$k_e = \frac{D1}{P0} + g$$

Where D1 = Dividend payment at the end of year 1 = Expected dividend payment

S = Market value of common equity = # of common shares outstanding * Current common share price

Example:

Itech Inc. has 5,000,000 common shares outstanding that will pay an annual dividend of $2 for the next two years. After this point dividends will grow at 10% for 2 years and 5% thereafter. The return on the market is 15% and the risk-free rate is 5%. Itech common stock is twice as risky as the market.

Required:

i) What is the cost of equity for the firm?

ii) What is the market value of common equity for Itech

Solution:

i) β(Itech) = 2; k(Itech) = 0.05 + 2 * (0.15 – 0.05) = 0.25 = 25%

ii) Step 1: t = 4 (the moment in time when the last growth rate change takes place)

Step 2:

D_1 = \$2; D_2 = \$2; D_3 = 2 * (1+10%) = \$2.2; D_4 = 2.2 * (1+10%) = \$2.42

D_5 = 2.42 * (1+5%) = \$2.541

Step 3:

$P_4 = \frac{2.541}{0.25-0.05}$ = \$12.705

Step 4:

$P_0 = \frac{2}{(1+0.25)^1} + \frac{2}{(1+0.25)^2} + \frac{2.2}{(1+0.25)^3} + \frac{(2.42+12.705)}{(1+0.25)^4}$ = \$ 10.20

MV of common equity = S = 5,000,000 * 10.20 = \$51,000,000

CASE 3: ISSUE PREFERRED EQUITY (preferred stock)

$k_p = \frac{D}{P0}$

where D = constant dividend payment

P0 = current preferred stock price

P = Market value of preferred equity = # of preferred shares outstanding * Current preferred share price

Dividend payment = Dividend rate * Par Value = Dividend yield (t) * Price (t)

Example:

Itech preferred stock has a dividend rate of 10%. Par value is $10. Currently shares trade at $12 per share. There are 1,000,000 preferred shares outstanding in the market.

Required:

i) Cost of preferred equity for Itech?

ii) The market value of preferred equity for Itech?

Solution:

i) Dividend payment = 10% * 10 = $1

k_p = $1/$12 = 8.33%

ii) MV of preferred equity = P = 1,000,000 * $12 = $12,000,000

OVERALL COST OF CAPITAL (WEIGHTED AVERAGE COST OF CAPITAL)

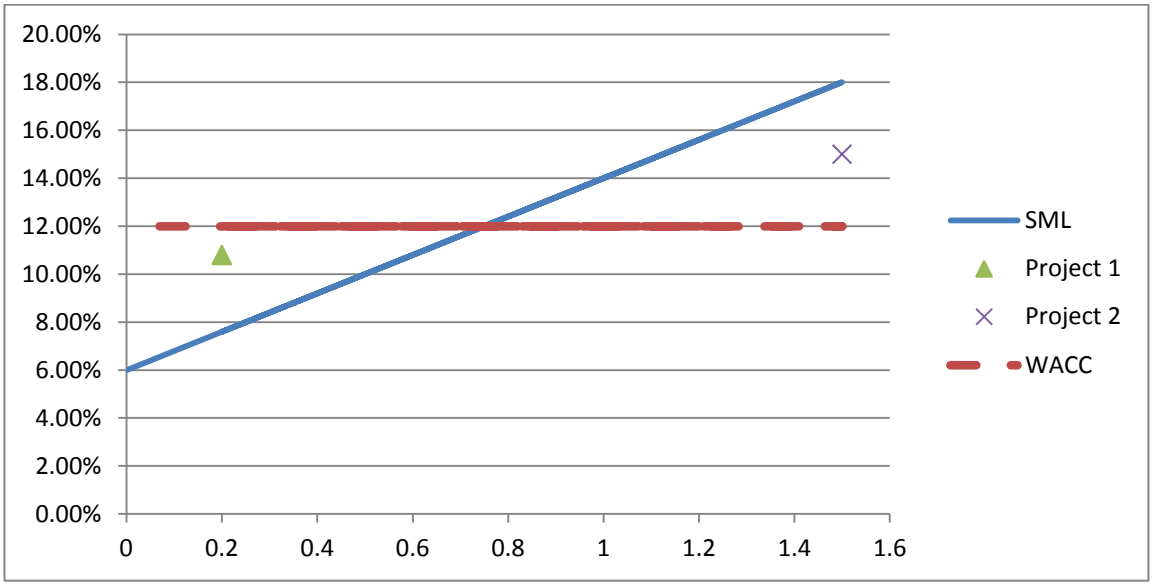
Market value of the firm = V = D + S + P

WACC = Weighted average cost of capital

$$WACC = \frac{D}{V} * k_i + \frac{S}{V} * k_e + \frac{P}{V} * k_p$$

Important Note: If a project is as risky as a firm's typical project, WACC should be used as the discount rate when NPV is computed!

SHALL WE USE WACC OR SML WHEN DETERMINING WHETHER TO ACCEPT OR REJECT A PROJECT?



If WACC is used as the decision criteria:

Project 2 would be accepted (since return on project 2 is > WACC)

Project 1 would be rejected (since return on project 1 is < WACC)

However this approach fails to take the risk factor in to account. WACC can only be used as an appropriate hurdle rate if the project is as risky as firm's typical project otherwise SML should be the used as the appropriate benchmark.

If the SML is used as the decision criteria:

Project 1 would be accepted since project 1 plots above the SML (return earned from project 1 is > the required return that was supposed to be earned for selecting a project that is as risky as project 1)

Project 2 would be rejected since project 2 plots below the SML (return earned from project 2 is < the required return that was supposed to be earned for selecting a project that is as risky as project 2)

Using WACC rather than SML leads to

- accepting certain projects that should have been rejected
- rejecting certain projects that should have been accepted
- firm being riskier in the long-run

PRACTICE PROBLEMS

Problem (1)

Balance Sheet Information

Short-term Liabilities	$500,000
Debentures (3,000 bonds, 6% coupon rate, maturing in 8 years)	$3,000,000
Common stock (500,000 shares outstanding)	$5,000,000

The firm's debentures currently trade at a 10% discount from their par value. Common shares are currently priced at $15 per share. Four years ago, the company paid an annual dividend of $0.51 per share. Dividends have been growing and will be growing at a constant rate. The company recently paid an annual dividend of $0.62. What is the WACC of the firm is there is 40% tax?

Problem (2)

Itech Inc. is considering a $12,000,000 project. The firm has just reported net earnings of $20M. The dividend pay-out ratio is 15%. The firm has 2 million shares of stock. The firm's capital structure is as follows:

Long-term debt (MV) = $37.8M
Common Equity (MV) = $25.2M
Preferred Equity (MV) = $17M

The company expects its earnings to continue to grow at 4% per year forever. The current risk free rate is 4.5% and the market risk premium is 7.5%

The firm has 1 million preferred shares that were originally issued at $20 with a dividend yield of 9%

The firm also has 50,000 bonds that were issued 5 years ago at a coupon rate of 8%. The bond will mature in 10 years. The tax rate is 40%.

Required:

What is the WACC of the firm?

Problem (3)

The firm has a debt to equity ratio of 0.25.
If the before-tax cost of debt is 8% and the cost of common equity is 15%. What is the WACC if the tax rate is 40%?

PRACTICE PROBLEM SOLUTIONS

Problem (1)

Very important note: Short-term liabilities are never taken in to account when the weight of debt and equity is being calculated.

D = MV of debt = # of bonds * current bond price = 3,000 * 900 = $2.7M

S = MV of Equity = # of shares * current share price = 500,000 * 15 =$7.5M

V = D + S = $2.7M + $7.5M = $10.2M

To find WACC, you have to also come up with:

K_d = YTM of the bond

and k_e = cost of common equity

To find YTM:

1,000 FV; 60 PMT; 8 N; -900 PV CPT I/Y = 7.72% = YTM = k_d

To find re:

Re = (D1/P0) + g

In order to find g:

-0.51 PV; 0.62 FV; 4 N CPT I/Y = g = 5%

D1 = D0 * (1+g) = 0.62 * 1.05 = $0.651

k_e = 0.651/15 + 0.05 = 9.34%

WACC = (2.7M/10.2M) * 0.0772 * (1-0.4) + (7.5M/10.2M) * 0.0934 = 8.094%

Problem (2)

V(firm) = 37.8M + 25.2M + 17M = $80M

To find k_e:

EPS(0) * dividend pay-out ratio = D(0)

EPS(0) = $20M/2M = $10 per share

D(0) = $10 * 0.15 = $1.5 per share

D(1) = 1.5 * 1.04 = $1.56

P(0) for common equity = MV of common equity/# of common shares = $25.2M/2M = $12.6

Therefore k_e = 1.56/12.6 + 0.04 = 16.38%

To find k_p:

Dividend pmt = $20 * 9% = $1.8

P(0) for preferred equity = MV of preferred equity/ # of preferred shares = $17M/1M = $17 per share

k_p = 1.8/17 = 10.59%

To find k_d:

Current price of the bond = MV of debt/# of bonds outstanding = $37.8M/50,000 = $756

-756 PV; 1000 FV; 10 N; 80 PMT CPT I/Y = 12.39% = YTM = k_d

WACC = (37.8M/80M) * 0.1239 * (1-0.4) + (25.2M/80M) * 0.1638 + (17M/80M) * 0.1059 = 10.92%

Problem (3)

Please assume that tax rate of 40% is provided in the question

D/S = 0.25

Therefore D/V = 0.25/1.25 and S/V = 1/1.25

WACC = 0.25/1.25 * 0.08 * (1-0.40) + 1/1.25 * 0.15 = 12.96%

CHAPTER 21 – CAPITAL STRUCTURE

The firms are always in a constant search for the optimal capital structure that would maximize the value of the firm (which is the same thing as maximizing the value for the shareholders)

Firm's capital structure has two main components: Debt and Equity

Value of the firm = Market value of Equity + Market value of Debt

V = S + B

- If a firm finances itself only with equity; this firm would be called an UNLEVERED or ALL-EQUITY firm
- If the firm finances itself with equity and debt; this firm would be called a LEVERED firm

Several parameters that a firm would use to measure its performance are:

$$\text{ROA: Return on Assets} = \frac{Net\_Income}{Value\_of\_the\_assets}$$

$$\text{ROE: Return on Equity} = \frac{Net\_Income}{Book\_Value\_of\_the\_equity}$$

$$\text{EPS: Earnings per share} = \frac{Net\_Income\_available\_to\_common\_shareholders}{\#\_of\_common\_shares\_outstanding}$$

Net Income available to C/S

= (EBIT – Interest Payment) * (1-tc) – Total Pref. Share Dividend Payments

Interest Payment = Value of Debt * Interest rate on debt

Example (1):

Assume the following values are both the book and the market values

	CURRENT	PROPOSED
ASSETS	$16,000	$16,000
DEBT	N/A	$8,000
EQUITY		
INTEREST RATE	10%	10%
PRICE/SHARE	$20	$20
# OF SHARES		

a) Calculate the missing values

For the current structure: Equity = Assets – Debt = $16,000 - $0 = $16,000

# of shares = Equity / price per share = $16,000/$20 = 800 shares

For the proposed structure: Equity = Assets – Debt = $16,000 - $8,000 = $8,000

# of shares = Equity / price per share = $8,000/$20 = 400 shares

b) With the help of the following table, please calculate the ROA; ROE and EPS values under the proposed structure and booming economy assumption. Assume that tax rate = 40%

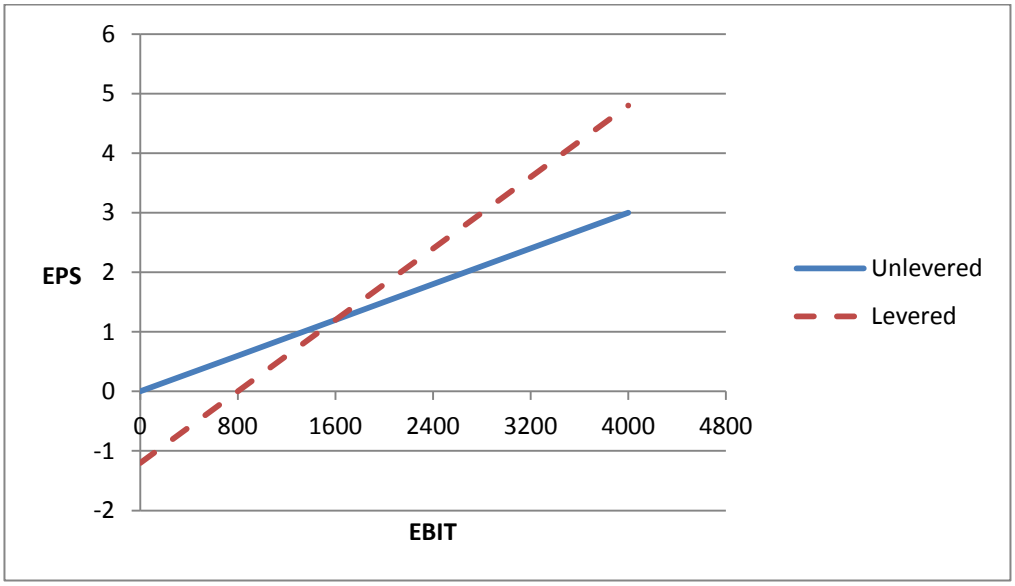
	Recession	Normal	Boom
EBIT	$800	$2,400	$4,000

ROA = {(4,000 – 8,000*10%) * (1-0.40)}/16,000 = 12%

ROE = {(4,000 – 8,000*10%) * (1-0.40)}/8,000 = 24%

EPS = {(4,000 – 8,000*10%) * (1-0.40)}/400 = $4.8/share

c) Graph the relationship between EBIT and EPS values



For the unlevered firm the EBIT-EPS line would always start from the origin whereas for the levered firm the same line would start from the negative section of the y-axis. However the line for the levered firm is steeper therefore the EPS values would increase faster for the levered firm.

The advantage of the levered firm is the interest tax shield benefit that it would enjoy. Interest tax shield is the tax savings generated due to the interest expense incurred by the levered firm. The more debt a company has (the more leverage a firm has); the larger this advantage (and the steeper the line) would be.

Note:

Break-even EBIT (indifference EBIT) = The EBIT value that would make the EPS under both capital structures equal to each other

Example (2):

ABC Inc. is currently an all-equity firm with 1,000 common shares outstanding and the current common share price is $45 per share. The firm is evaluating two alternative capital structures:

Proposed Plan: Firm will raise $9,000 through debt and use the proceeds to repurchase their common stock from the market. Assume that after the share repurchase the price of the common share doesn't change.

Interest rate on debt is 10%; tax rate is 40%

Find the break-even EBIT between current structure and the proposed structure

Solution:

Step 1: Find the number of remaining common shares outstanding after the stock repurchase:

# of shares repurchased by the firm = Amount of debt/current common share price

= $9,000/$45 = 200 shares

# of remaining shares outstanding = 1,000 – 200 = 800 shares

Step 2: Write down the EPS formula for both capital structures

$EPS_{current}$ = {(EBIT – 0*10%) * (1-0.40)}/1,000

$EPS_{proposed}$ = {(EBIT – 9,000*10%) * (1-0.40)}/800

Step 3: Set the two EPS values equal to each other and solve for the EBIT. The answer would be the break-even EBIT

$EPS_{current} = EPS_{proposed}$

{(EBIT – 0*10%) * (1-0.40)}/1,000 = {(EBIT – 9,000*10%) * (1-0.40)}/800

Break-even EBIT = $4,500

MM PROPOSITIONS

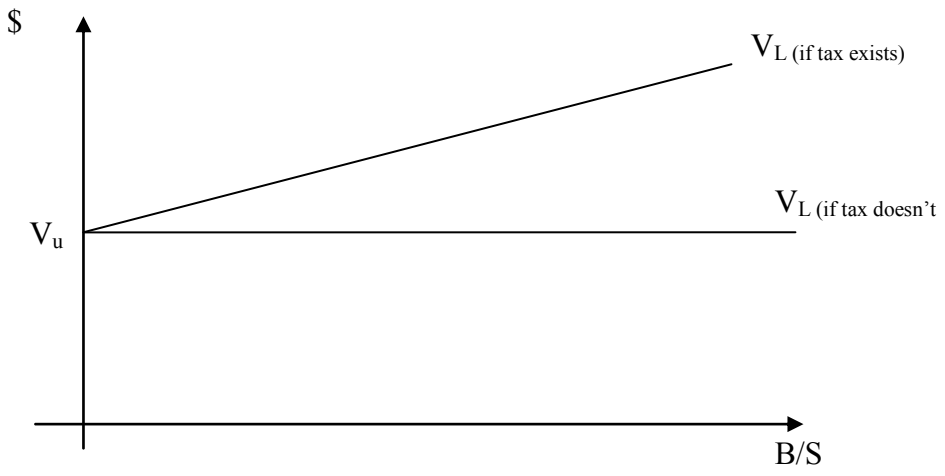
MM PROPOSITION-I: $V_l = V_u + B * tc$

V_l = Value of the levered firm

V_u = Value of the unlevered firm

The second half of the formula (B*tc) calculates the Present value of all interest tax shield benefits.

- Due to the interest tax shield benefits → The larger the value of debt (the more levered the firm is); the larger the value of the firm
- However we have to realize that the MM-I proposition doesn't take the bankruptcy and financial distress costs in to account.



Other useful formulas:

Interest tax shield is the tax savings created by the interest expense.

Interest tax shield per year = Interest payment * tc

$$V_u = \frac{EBIT(1-tc)}{ro} = S_u = \#\_of\_shares * Share\_price\_today$$

$$V_l = \frac{EBIT(1-tc)}{WACC_L} = B + S_L$$

$$WACC_L = \frac{B}{V_L} * r_b * (1\text{-}tc) + \frac{S_L}{V_L} * r_s$$

r_o = cost of equity for the unlevered firm = WACC of the unlevered firm

S_L = Market value of equity for the levered firm

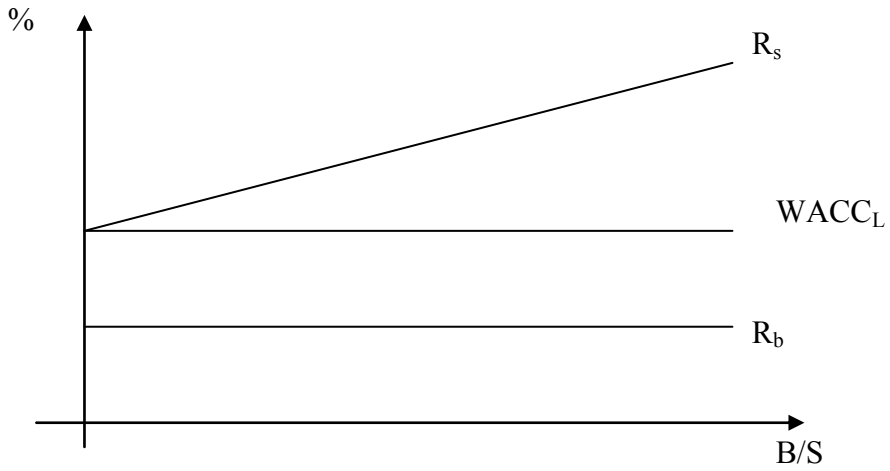
S_u = Market value of equity for the unlevered firm

MM PROPOSITION-II : $r_s = r_o + (r_o - r_b) * (1\text{-}tc) * (\frac{B}{S})$

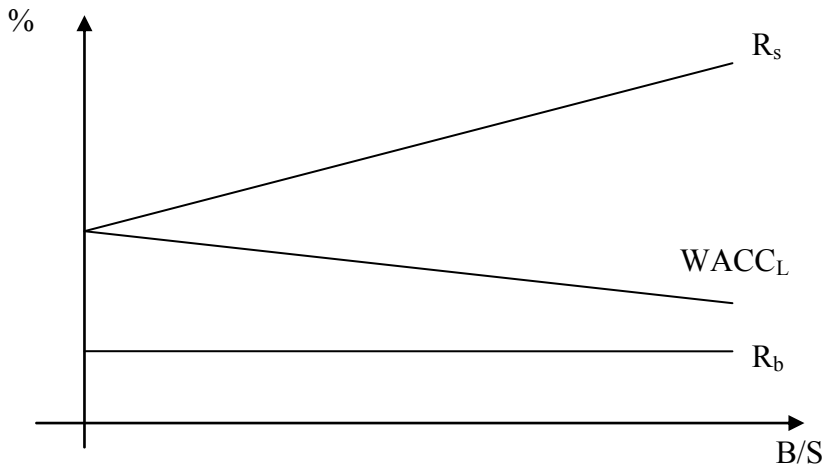
r_s = cost of equity for the levered firm

- The larger the B/S (Debt to Equity) ratio; the larger the cost of equity for the levered firm but Why?
- Cost of equity for the levered firm = Required return demanded by the shareholders of the levered firm. Since levered firm has debt; it is riskier. Therefore the shareholders are exposed to more risk. Because they are exposed to more risk, they demand higher return.

In a world with no taxes:



In a world with taxes:



Example (3):

Market value of a firm with $500,000 of debt is $1.7 million. The pre-tax interest rate on debt is 10%. Tax rate = 34%. Expected EBIT = $306,000.

a) What would be the firm value if it was entirely financed with equity?
b) What amount of the firm's annual earnings is available to shareholders?

Example (4):

An all-equity firm has 175,000 shares of stock, each worth $20. Its equity holders demand 20% return. The firm decides to issue $1,000,000 of 10% debt and use the proceeds the repurchase common stock. Tax rate is 40%. What is the value of firm's equity after the repurchase?

Example (5):

The firm had a debt-to-equity ratio of 2.5. Its WACC is 15% and its pre-tax cost of debt is 10%. Tax rate = 35%

a) What is the firm's cost of equity capital?
b) What is the firm's unlevered cost of equity capital?

Example (6):

EBIT: $4M; firm's unlevered cost of equity capital is 15%. Tax rate = 35%. Pretax cost of debt is 10%. The firm has $10M worth of debt.

a) What is the firm value?
b) What is the cost of equity capital?
c) What is the firm's WACC?

Example (7):

Consider two firms that are identical in every respect except for their capital structures. The unlevered firm (U) has a WACC of 16%. Both firms have a perpetual EBIT of $80,000 per year. The levered firm (L) pays $10,000 in interest every year to bondholders. Its cost of debt is 5%.

i) What is the cost of equity for the levered firm?

ii) Assume corporate taxes of 40%. What is the value of the levered firm?

iii) Assume tc = 40%. What is the WACC of the levered firm?

SOLUTIONS FOR EXAMPLE (3)-(7)

Example (3):

a) $V_L = \$1.7M = V_u + 500{,}000 * 34\%$ therefore $V_u = \$1{,}530{,}000$

b) Earnings available to S/H = $(306{,}000 - 500{,}000 * 10\%)*(1-0.34) = \$168{,}960$

Example (4):

$V_u = S_u = 175{,}000 * 20 = \$3.5M$

$V_L = 3.5M + 1{,}000{,}000 * 40\% = \$3.9M$

$S_L = V_L - B = \$3.9M - \$1M = \$2.9M$

Example (5):

a) B/S = 2.5 therefore if $S = X \rightarrow B = 2.5X \rightarrow V_L = B + S = 2.5X + X = 3.5X$

$$WACC_L = \frac{B}{V_L} * r_b * (1\text{-tc}) + \frac{S_L}{V_L} * r_s \rightarrow 15\% = \frac{2.5X}{3.5X} * 0.10 * (1\text{-}0.35) + \frac{X}{3.5X} * r_s$$

Solve for $r_s = 36.25\%$

b) $r_s = r_o + (r_o - r_b) * (1\text{-tc}) * (\frac{B}{S}) \rightarrow 0.3625 = r_o + (r_o - 0.10) * (1\text{-}35) * (2.5)$

Solve for $r_o = 20\%$

Example(6):

a) Vu = (4M*(1-0.35))/0.35 = $17.33M
   VL = 17.33M + 10M*0.35 = $20.83M

b) Rs = 0.15 + (0.15-0.10) * (1-0.35) * (10M/18.33M) = 18%
c) VL = 20.83M = (4M*(1-0.35))/WACC(L)

   WACC(L) = 12.48%

Example (7):

a)

If interest pmt is $10,000 = Debt * 0.05

Db = B = $200,000

Ro = 16%

Vu = 80,000 * (1-0)/0.16 = $500,000

Since tc= 0; VL = Vu = $500,000

S(L) = 500,000-200,000 = $300,000

Rs = 0.16 + (0.16-0.05) * (1-0) * (200,000/300,000) = 23.33%

b)

Vu = 80,000 * (1-0.40)/0.16 = $300,000

Since tc= 0.40; VL = 300,000 + 200,000*0.4 = $380,000

c)

VL = 380,000 = (80,000*(1-0.40))/WACC(L)

WACC(L) = 12.63%

CHAPTER 22 – DIVIDENDS AND OTHER PAYOUTS

Dividend usually refers to the cash distribution of the earnings to shareholders.

Most common dividend payment is the regular cash dividend. These payments are usually made every quarter of the year.

Another type of dividend is the stock dividend is the stock dividend. The dividend payment is actually made in shares of stock. This is not a true dividend since no cash leaves the firm.

X for Y stock Split: The firm issues X number of new shares for every Y number of share they collect. If X>Y, it is a normal stock split. If X<Y, it is a reverse stock split.

The decision to whether to pay a dividend is made by the board of directors of the corporation. A dividend is distributable to shareholders of record on a specific date.

There are four crucial dates for the dividend payment:

1. Declaration date: The day when the board of directors announce the amount of dividend to be paid as well as the date.

2. Date of record: The corporation would only pay dividends to those shareholders that are on record on this day.

3. Ex-dividend date: The second business day before the date of record is called the ex-dividend date. Before this date the stock trades "with dividend". If you purchase the stock on the ex-dividend date or after, you will not be entitled to the upcoming dividend payment.

4. Date of payment: The date when the dividend payment is placed.

If the stock price before the ex-dividend date is called P(0); the stock price on the ex-dividend date P(X) = P(0) – cash dividend payment/share

When a cash dividend payment is made, the number of shares in the market would not change.

If there is an X% stock dividend declaration:

Price after the stock dividend payment

= Price before the stock dividend payment / (1+X%)

# of shares after the stock dividend payment

= # of shares before the stock dividend payment * (1+X%)

If there is an X for Y stock split:

Price after the split = Price before the split * Y/X

# of shares after the split = # of shares before the split * X/Y

If there is a normal stock split, # of shares would increase but stock price would decrease.

If there is a reverse stock split, # of shares would decrease but stock price would increase.

Example (1):

ABC Inc. is considering a 5 for 3 stock split to keep its shares trading in an affordable range. What will be the %change in share price due to the split?

Solution:

Let initial stock price be $60; the new stock price after the split = 60 * 3/5 = $36

% change in price = (36-60)/60 = - 40%

Example (2):

ABC Inc. is considering stock dividend payment as an alternative to the stock split. What will be the %change in share price if the firm issues 30% stock dividend?

Solution:

Let initial stock price be $60; the new stock price after the stock dividend payment

= 60 / (1+30%) = $46.15

% change in price = (46.15-60)/60 = - 23.08%

Call Option: Gives the holder the right to purchase the underlying stock at a predetermined (exercise) price on a predetermined (exercise date) date. Each contract would have 100 options

Example: $140 IBM August 2011 Call option

The buyer of this call option will have the right to purchase the IBM stock for $140 (the exercise price = X) on the third Friday of August 2011.

* Holder of a call option would only uses (exercises) his/her right when the stock price is larger than the exercise price. Pay-off to the buyer of a call option = Max (S_T – X; 0)

Where S_T = Stock price on the exercise date

* In the mean time the seller of the option would only have the obligation to do what the buyer wants. Pay-off to the seller of a call option = - Pay-off to the buyer of a call

* Sum of the gain to the seller and the buyer always equals zero!!!

Put Option: Gives the holder the right to sell the underlying stock at predetermined (exercise) price on a predetermined (exercise date) date. Each contract would have 100 options.

* Holder of a put option would only uses (exercises) his/her right when the stock price is less than the exercise price. Pay-off to the buyer of a put option = Max (X - S_T; 0)

* Pay-off to the seller of a put option = - Pay-off to the buyer of a put option

Profit from the option = Pay-off from the option + CF(initial)

For the buyer → CF(initial) = - Price paid to purchase the option

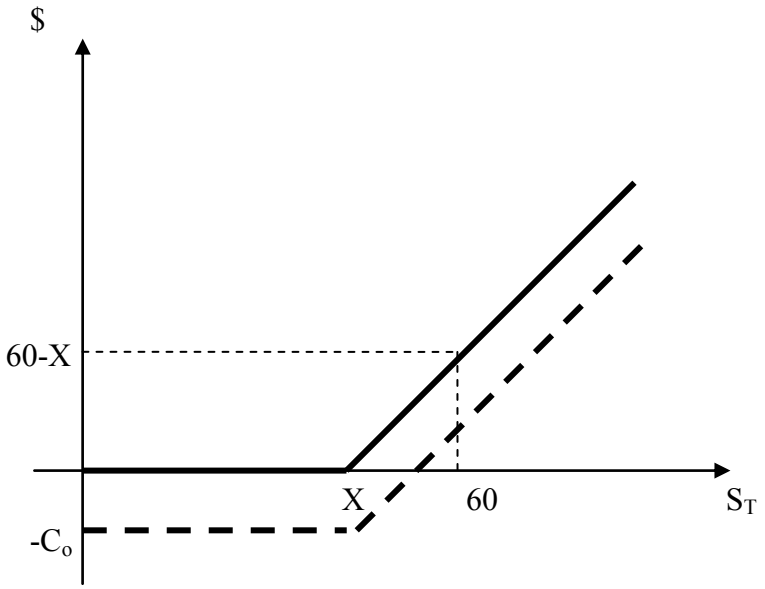
For the seller → CF(initial) = Price paid to purchase the option

American options: The option can be exercised any time until the exercise date (inclusive)

European options: The option can be exercised only on the exercise date.

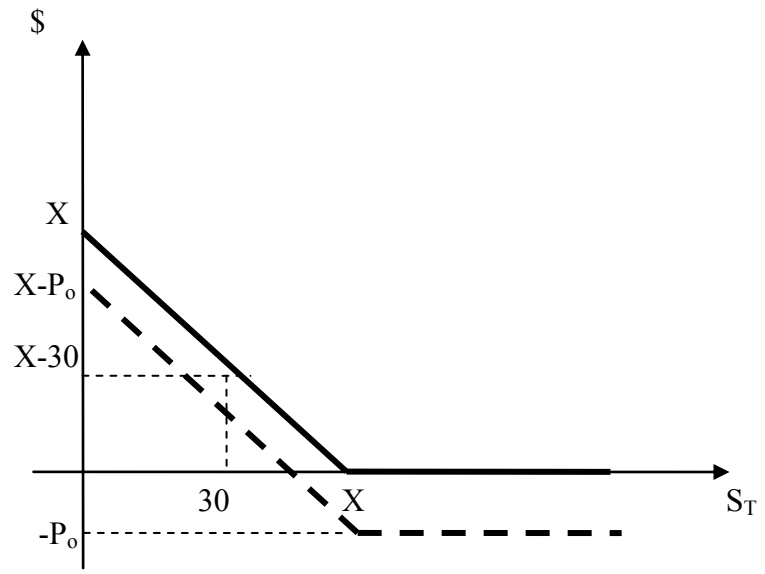
Pay-off & Profit Graphs for buyers of Call and Put options

Call option:



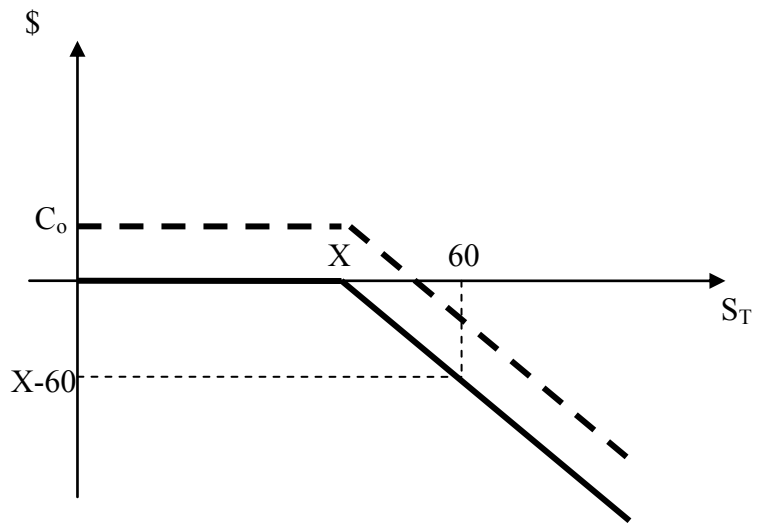
X = Exercise Price = Strike Price; C_o = Value of the call option today
S_T = Stock price at the exercise date; P_o = Value of the put option today

Put Option:



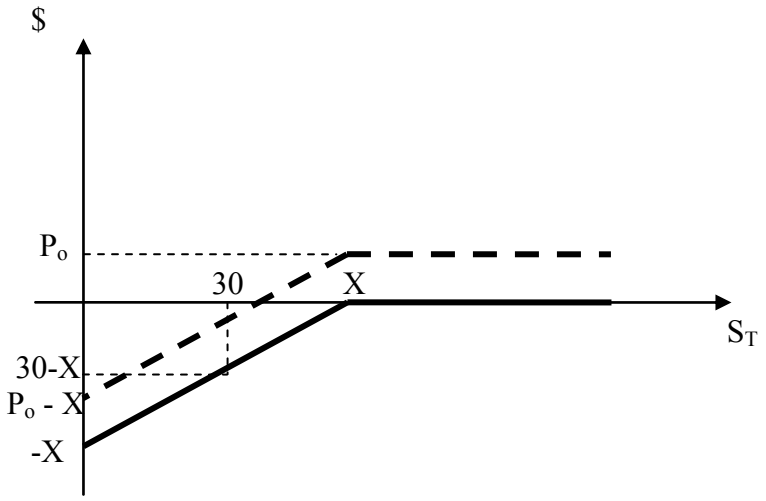
Pay-off & Profit Graphs for the sellers of Call and Put options

Call option:



X = Exercise Price = Strike Price; C_o = Value of the call option today
S_T = Stock price at the exercise date; P_o = Value of the put option today

Put Option:



Put-Call Parity

$S_0 + P_0 = C_0 + PV(X)$

The following conditions have to be satisfied to be able to apply the Put-Call Parity equation:

- Options must be European
- The call and the put option must have the same underlying asset
- The call and the put option must have the same exercise price
- The call and the put option must have the same exercise date

Option Premium (Price) = Intrinsic Value + Time Value = IV + TV

Intrinsic Value (IV) = The pay-off from the option if the option is to be exercised today

Intrinsic Value of the call option: Max (So-X; 0)
Intrinsic Value of the put option: Max (X-So; 0)

If IV = + → The option is ***in-the-money***
If $S_0 = X$ → The option is ***at-the-money***
If IV = 0 and $S_0 \neq X$ → The option is ***out-of-the-money***

Time value = Premium amount that is charged because of the possibility of the stock price moving in the preferred direction. The longer time there is until the exercise date, the larger the time value should be.

HOW CAN YOU TELL IF AN OPTION IS MISPRICED?

(i) The comparison of the option premium and the IV: The option premium must be > than the IV. If not, the option is mispriced.

Example: Current tock price = S_0 = \$50 Exercise price of the 1-year call = X = \$45
Current call option price = C_0 = \$3

IV = Max (50-45; 0) = \$5 C_0 = \$3

One can buy the call option for \$3 today and exercise it immediately to gain \$5. This risk-free profit is called arbitrage. The financial markets must price the investments in an accurate way to avoid such opportunities. Hence the call option premium should have been more than \$5

(ii) The comparison of the time value for the options with different exercise dates. The TV for the option with the longer life should always be greater.

Example:

- 1-year, IBM, X = $140, call option is currently priced at $8
- 6-month, IBM, X = $140, call option is currently priced at $10

The situation above also represents an option mispricing. You have two identical options (except for the length of their life), for which the option with the shorter life is more expensive than the option with the longer life.

Factors that affect the option premium:

Increase in	Call option value	Put Option Value
Stock price	+	-
Exercise Price	-	+
Stock Volatility	+	+
Time to exercise date	+	+
Interest rate	+	-
Dividend rate	-	+

If you own a call option, you'd want the stock price to exceed the exercise price. Thus, higher the stock price, the more attractive the option. The more attractive the option, the more valuable it is. Also the exercise price is the threshold that you are trying to exceed. When that threshold is pulled higher, the option would be less attractive.

If you own a put option, you'd want the stock price to stay below the exercise price. Thus, higher the stock price, the less attractive the option. The less attractive the option, the less valuable it is. Also the exercise price is the threshold that you are trying to stay under. When that threshold is pulled higher, the option would be more attractive y.

When you own an option, your losses are limited but your gains are not. Therefore, higher volatility would be preferred. Higher the volatility, the more valuable the options are.

If there is more time to the exercise date, the time value of the option would be larger. There is more time for the stock price to move in the direction you want it to move.

If own a call option, the exercise price would be considered as an expense (if you exercise the option). Higher interest rates would make the PV of this expense lower. Thus, the call option would be more attractive and valuable.

If own a put option, the exercise price would be considered as a revenue (if you exercise the option). Higher interest rates would make the PV of this revenue lower. Thus, the put option would be less attractive and valuable.

The increase in the dividend rate causes the stock price to decline more on the ex-dividend date. Lower future stock prices would lower the potential pay-off from the call option. Hence the call option would be less attractive and less valuable. On the other hand, lower future stock prices would increase the pay-off from the put option. Hence the put option would be more attractive and valuable.

Example (1):

ABC's stock is currently selling for $82. In one year from now, you expect the stock to be selling either at $85 or $93. Today you will sell 3 call option contracts with an exercise price of $88. The value of a call option today is $5.95. The exercise date is a year from now.

i) What would your profit from this investment be if $93 price is reached?

ii) What would your profit from this investment be if $85 price is reached?

iii) At what stock price on the exercise date would you break-even?

iv) At what stock price on the exercise date would you profit $600?

Solution:

i)

Pay-off to the buyer of a call option = Max ($S_T - X$; 0) = Max (93-88;0) = $5/option
Pay-off to the seller of a call option = - Pay-off to the buyer of a call option = -$5/option

3 contracts → 3*100 = 300 options therefore total pay-off to the seller = -5*300 = -$1,500

Total profit to the seller = Pay-off to the seller + CF(initial)

CF(initial) = 3 * 100 * 5.95 = $1,785; Total profit to the seller = -1,500 + 1,785 = $285

ii)

Pay-off to the buyer of a call option = Max ($S_T - X$; 0) = Max (85-88;0) = $0/option
Pay-off to the seller of a call option = - Pay-off to the buyer of a call option = -$0/option

3 contracts → 3*100 = 300 options therefore total pay-off to the seller = -0*300 = -$0

Total profit to the seller = Pay-off to the seller + CF(initial)

CF(initial) = 3 * 100 * 5.95 = $1,785 Total profit to the seller = -0 + 1,785 = $1,785

iii)

Break-even → Total profit = 0 = pay-off to the seller + 1,785

Therefore the total pay-off to the seller = -1,785 for 300 options

Per option pay-off to the seller = -1,785/300 = -$5.95

Per option pay-off to the buyer = - (-5.95) = $5.95 = Max ($S_T$ – X; 0) = Max (S_T-88;0)

S_T = 88 + 5.95 = $93.95

iv)

Total profit = $600 = pay-off to the seller + 1,785

Therefore the total pay-off to the seller = -1,185 for 300 options

Per option pay-off to the seller = -1,185/300 = -$3.95

Per option pay-off to the buyer = - (-3.95) = $3.95 = Max ($S_T$ – X; 0) = Max (S_T-88;0)

S_T = 88 + 3.95 = $91.95

Example (2):

ABC's stock is currently selling for $82. In one year from now, you expect the stock to be selling either at $85 or $93. Today you will sell 3 put option contracts with an exercise price of $88. The value of a put option today is $3.95. The exercise date is a year from now.

i) What would your profit from this investment be if $93 price is reached?

ii) What would your profit from this investment be if $85 price is reached?

iii) At what stock price on the exercise date would you break-even?

iv) At what stock price on the exercise date would you profit $600?

Solution:

i)

Pay-off to the buyer of a put option = Max ($X - S_T$; 0) = Max (88-93;0) = $0/option
Pay-off to the seller of a put option = - Pay-off to the buyer of a put option = -$0/option

3 contracts → 3*100 = 300 options therefore total pay-off to the seller = -0*300 = -$0

Total profit to the seller = Pay-off to the seller + CF(initial)

CF(initial) = 3 * 100 * 3.95 = $1,185

Total profit to the seller = -0 + 1,185 = $1,185

ii)

Pay-off to the buyer of a put option = Max ($X - S_T$; 0) = Max (88-85; 0) = $3/option
Pay-off to the seller of a put option = - Pay-off to the buyer of a put option = -$3/option

3 contracts → 3*100 = 300 options therefore total pay-off to the seller = -3*300 = -$900

Total profit to the seller = Pay-off to the seller + CF(initial)

CF(initial) = 3 * 100 * 3.95 = $1,185

Total profit to the seller = -900 + 1,185 = $285

iii)

Break-even → Total profit = 0 = pay-off to the seller + 1,185

Therefore the total pay-off to the seller = -1,185 for 300 options

Per option pay-off to the seller = -1,185/300 = -$3.95

Per option pay-off to the buyer = - (-3.95) = $3.95 = Max (X - S_T; 0) = Max (88 - S_T; 0)

S_T = 88 - 3.95 = $84.05

iv)

Total profit = $600 = pay-off to the seller + 1,185

Therefore the total pay-off to the seller = -585 for 300 options

Per option pay-off to the seller = -585/300 = -$1.95

Per option pay-off to the buyer = - (-1.95) = $1.95 = Max (X – S_T; 0) = Max (88 - S_T; 0)

S_T = 88 - 1.95 = $86.05

Example (3):

You have just established the following position: sold short a share of ABC stock, currently trading at $70, bought a 1-year call option on ABC stock with an exercise price of $75 and wrote a 1-year put option on ABC stock with an exercise price of $65.

(a) Please draw the pay-off graph on your newly established position.

(b) A 1-year put option on ABC stock with an exercise price of $75 trades at $7, while a 1-year call option on ABC stock with an exercise price of $65 trades at $9. If the risk-free rate is 4% per year, what is the maximum profit you can expect from this position?