Université d'Ottawa
Faculté de génie

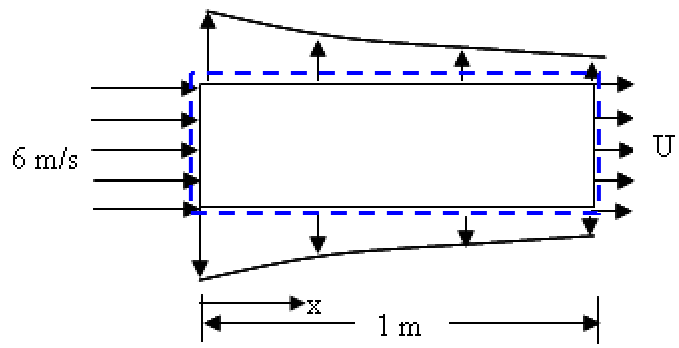
Département de
Génie Chimique et Biologique

University of Ottawa
Faculty of Engineering

Department of
Chemical & Biological Engineering

CHG 2312 – Fluid Flow
Assignment #2 Solution

Question 1:



Given:
D = 0.2 m
V = 6 m/s
$v = U. [2-(x)^{1/2}]/4$

Find: Discharge velocity, U

Assumptions: Steady State, Incompressible Fluid

Solution:
First define a control volume.
Second define the inlet and outlet of the system.
Inlet: left side of the pipe where velocity is at a constant value of 6 m/s.
Outlet: body of the pipe (with variable velocity) and end of the pipe on the right-hand side (with velocity of U).

Mass balacnce:

$$\frac{dm_{C.V.}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0$$

$$\rho \dot{Q}_{in} = \rho \dot{Q}_{out,body} + \rho \dot{Q}_{out,end}$$

$$V_{in} A_{in} = v A_{out,body} + U A_{out,end}$$

$$\left(\frac{\pi D^2}{4}\right) V_{in} = \int_0^1 \left[(\pi D dx)(U \bullet \frac{2 - x^{1/2}}{4}) \right] + U(\frac{\pi D^2}{4})$$

$$DV_{in} = U\left(\int_0^1 [2 - x^{1/2}] dx \right) + DU$$

$$DV_{in} = U[2x - \frac{2x^{3/2}}{3}]_0^1 + DU$$

$$U = \frac{DV_{in}}{D + [2x - \frac{2x^{3/2}}{3}]} = \frac{(0.2m)(6m/s)}{(0.2m) + [2 - 2/3]} = 0.78 m/s$$

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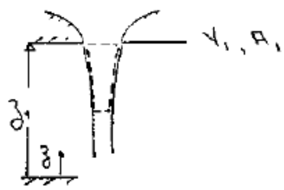
Question 2:

Given:

1- Liquid stream leaving a nozzle pointing downward as shown
2- Frictionless flow
3- Uniform flow

Assumption:

1- Steady flow
2- Incompressible flow
3- Flow along a streamline
4- $P=P_1=P_{atm}$



Find: Variation in jet area for z, z_0

Solution:

From the Bernoulli equation

$$\Delta\left(\frac{p}{\rho}+gz+\frac{V^2}{2}\right)-\frac{dW_{n,f}}{dm}-F$$

=> $V^2 = V_1^2 + 2g(z_1 - z)$

And

$V_1A_1 = VA$

Thus

$$V_1^2\left(\frac{A_1}{A}\right)^2 = V_1^2 + 2g(z_1 - z)$$

Solving for A

$$A = A_1\sqrt{\frac{1}{1+\frac{2g(z_1-z)}{V_1^2}}}$$

Note: Jet area decreases as z decreases, owing to the higher velocity. You can observe such behaviour from any water tap in kitchens or bathrooms.

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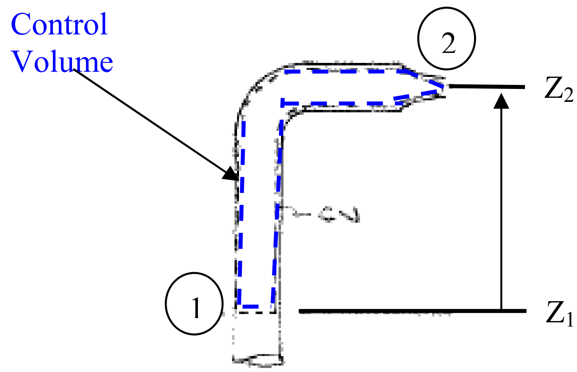
Question 3:



Given:
D_1=0.1 m
D_2=0.05 m
P_2=P_{atm}
V_2=20m/s
Z_1=0 m
Z_2 = 4m

Find:
1) Gauge Pressure, P_1
2) Gauge Pressure, P_1, if device was inverted

Assumptions:
1) Steady Flow
2) Incompressible Fluid
3) Frictionless Flow
4) P_2 gauge = 0
5) z_1=0 m
6) no external work

Solution:
Apply continuity to CV shown to determine V_1; the Bernoulli equation is then applied along a streamline from 1 to 2 to determine P_1.

Basic Equations:

$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0 \qquad \text{(Continuity)}$$

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(\Delta z) = \frac{dW}{dm} - F \qquad \text{(Bernoulli)}$$

From the continuity equation:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \Rightarrow V_1 A_1 = V_2 A_2 \text{ or } V_1 = \left(\frac{D_2}{D_1}\right)^2 \cdot V_2$$

From the Bernoulli equation:

$$P_1 = \rho\left(\frac{V_2^2 - V_1^2}{2} + gz_2\right) = \rho\left(\frac{V_2^2}{2}\left(1 - \frac{V_1^2}{V_2^2}\right) + gz_2\right) = \rho\left(\frac{V_2^2}{2}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) + gz_2\right)$$

$$P_1 = (999\frac{kg}{m^3})\left(\frac{(20\frac{m}{s})^2}{2}\left(1-\left(\frac{0.05m}{0.1m}\right)^4\right)+9.81\frac{m}{s^2}\cdot 4m\right) = 227\frac{kN}{m^2} = 227kPa\,(gauge)$$

If the device is inverted, z_2= -4 m with z_1=0 m

$$P_1 = 999\frac{kg}{m^3}\left(\frac{(20\frac{m}{s})^2}{2}\left(1-\left(\frac{0.05m}{0.1m}\right)^4\right)+(9.81\frac{m}{s^2})\cdot(-4m)\right) = 148\frac{kN}{m^2} = 148\,kPa\,(gauge)$$

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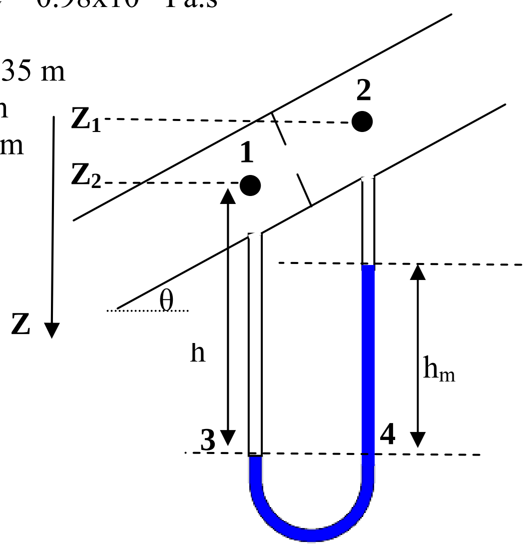
Question 4:

$\rho_w = 999\ kg/m^3$, $\mu_w = 0.98x10^{-3}$ Pa.s
$\rho_m = 13600\ kg/m^3$
$h_m = 135$ mm = 0.135 m
$D_2 = 6$ cm = 0.06 m
$D_1 = 10$ cm = 0.10 m
$\theta = 30°$



Assumptions:
Steady-state flow, Incompressible fluid, inviscid fluid, **Point 2 is very close to the orifice centre**

Mass balance from point 1 to 2:

$$\frac{dm_{C.V.}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0$$

$$\rho V_1 A_1 = \rho V_2 A_2 \qquad \Rightarrow \; V_1 = \frac{A_2}{A_1} V_2 = \frac{D_2^{\,2}}{D_1^{\,2}} V_2$$

Energy balance, B.E.:

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) - W + F = 0 \qquad where\ W = 0,\ F = 0$$

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = 0$$

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^{\,2} - V_1^{\,2}}{2} = 0$$

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^{\,2}}{2}(1 - \frac{D_2^4}{D_1^4}) = 0$$

$$V_2^{\,2} = 2\left(\frac{P_2 - P_1}{\rho} + g(z_2 - z_1)\right) / (1 - \frac{D_2^4}{D_1^4})$$

$$V_2 = \left(\frac{2\frac{P_1 - P_{21}}{\rho} + 2g(z_1 - z_2)}{(1 - \frac{D_2^4}{D_1^4})}\right)^{1/2}$$

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Determine $P_1 - P_2$ from manometer reading:

$$P_1 + \rho_w g h - \rho_m g h_m - \rho_w g (h - h_m + (z_2 - z_1)) - P_2 = 0$$
$$P_1 - P_2 = \rho_m g h_m - \rho_w g h_m + \rho_w g (z_2 - z_1))$$

Mass flow rate in the orifice is:

$$\dot{m} = \rho V_2 A_2 = \rho \pi \frac{D_2^2}{4} C_v \left(\frac{2\frac{P_1 - P_2}{\rho} + 2g(z_1 - z_2)}{(1 - \frac{D_2^4}{D_1^4})} \right)^{1/2} = \rho \pi \frac{D_2^2}{4} C_v \left(\frac{\frac{2\rho_m g h_m - 2\rho g h_m + 2\rho g (z_2 - z_1)}{\rho} + 2g(z_1 - z_2)}{(1 - \frac{D_2^4}{D_1^4})} \right)^{1/2}$$

$$\dot{m} = C_v \rho \pi \frac{D_2^2}{4} \left(\frac{2 g h_m (\rho_m - \rho)}{\rho (1 - \frac{D_2^4}{D_1^4})} \right)^{1/2}$$

Determine C_v:

1. find V_2 without C_v

$$V_2 = \left(\frac{\frac{2\rho_m g h_m - 2\rho g h_m}{\rho}}{(1 - \frac{D_2^4}{D_1^4})} \right)^{1/2} = \left(\frac{2(9.8)(0.135m)(13600 - 999)}{(999)(1 - (\frac{0.06}{0.1})^4)} \right)^{1/2} = 6.2 m/s$$

2. find Reynolds:

$$\mathrm{Re} = \frac{\rho D_2 V_2}{\mu} = \frac{(999)(0.06)(6.2)}{(0.89x10^{-3})} = 417559$$

3. read C_v from graph
   $C_v = 0.61$
4. Calculate new V_2 by including C_v, and then new Re number:
   $V_2 = 3.8$ m/s
   Re = 243249
   New $C_v = 0.61$
   Since new Cv and old Cv are similar then no more calculation is required and $V_2 = 3.8$ m/s

Determine mass flow rate:

$$\dot{\boldsymbol{m}} = \rho \pi \frac{\boldsymbol{D}_2^2}{4} \boldsymbol{V}_2 = (999)\, \pi\, (0.06)^2 (3.8)/4 = 10.73 \boldsymbol{kg/s}$$

Question 5

Given:

Fluid: Air

$D_2 = 3$ inch

$D_1 = 6$ inch

$P_1 = 60$ psi

$T_1 = T_2 = 68$ ˚F = 20˚C

Assumptions:

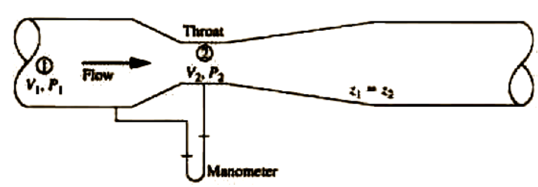
1- Density change is negligible
2- Ideal gas
3- Steady state system
4- Frictionless Flow

Find: Maximum flow rate for incompressible flow; Pressure reading

Solution:

In order to determine the mass flow rate of the air in the venturi meter, we could apply the B.E. equation. However, for any gas to be counted as an incompressible flow (important criteria for using B.E. equation), its velocity has to be low. In your text book this limit has been set at 200 ft/s = 63 m/s, whereas other literature use velocity of 100 m/s. The later number has been used here.

Looking at a venturi meter arrangement and again for applying B.E., then the maximum gas velocity in the system will be 100 m/s. But now the question would be where would the gas has a maximum velocity (i.e., at upstream of piping or at the throat).



As we know the highest velocity will be at the throat of the venturi since it has the smallest diameter. Therefore, $V_2 = 100$ m/s. Therefore, we can easily calculate the maximum mass flow rate as:

$$\dot{m} = \rho V_2 A_2$$

$$\rho = \frac{PM}{RT} = \frac{(60\,psi\,\frac{101325\,Pa}{14.7\,psi})(29\,kg/kmol)}{(8.314\times10^3\,\frac{m^3 Pa}{K.kmol})(20+273)K} = 4.9\,kg/m^3$$

$$\dot{m} = (4.9\,kg/m^3)(100\,m/s)(\pi\times(3inch\,\frac{1m}{39.37inch})^2/4) = 2.23\,\frac{kg}{s}$$

The pressure drop (i.e. reading on the manometer) can be found from the velocity equation of the venturi meter:

$$V_2 = C_v\sqrt{\frac{2(P_1-P_2)/\rho_{air}}{\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}}$$

$$(P_1-P_2) = \frac{\rho_{air}V_2^2\left(1-\left(\frac{A_2}{A_1}\right)^2\right)}{2C_v^2}$$

Since we assumed air is incompressible flow in this problem, then its density will stay constant at what was calculated above.

We need to find the C_v factor, in which Reynolds number needs to be calculated. Viscosity of air at 68 ˚F = 20 ˚C is found to be about 1.9×10^{-5} Pa.s.

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$$\mathrm{Re} = \frac{\rho V_1 D_1}{\mu} = \frac{\rho(\frac{V_2 A_2}{A_1})D_1}{\mu} = \frac{(4.9\frac{kg}{m^3})(100m/s(\frac{3inch}{6inch})^2(6inch\frac{1m}{39.37inc})}{1.9\times10^{-5}\,Pa.s} = 9.8\times10^5$$

From charts of C*v* vs. Reynolds number, you can find the Cv to be about 0.985.

$$(P_1 - P_2) = \frac{(4.9\frac{kg}{m^3})(100\frac{m}{s})^2\left(1-\left(\frac{3inch}{6inch}\right)^4\right)}{2(0.985)^2} = 23673.63Pa = 23.7kPa$$

Therefore, the manometer reading (Δh) will be :

$$\Delta \mathrm{P} = \rho_{\mathrm{Hg}} g \Delta h$$

$$\Delta h = \frac{\Delta P}{\rho_{\mathrm{Hg}} g} = \frac{23673.63Pa}{(13600kg/m^3)(9.8m/s^2)} = 0.177m$$