ADM2302 Assignment 1
(Graphical method for Linear Programming)

Solution and Marking Scheme:

Q1.
An investor has 10 dollars to invest in stock A and stock B. She estimates that every dollar invested in stock A can generate 2 dollars of profit, whereas every dollar invested in stock B generates 1.5 dollars of profit. The investor understands that higher profit always comes with higher risk. She finds in the past records that in a bad market, stock A can potentially lose 30% of its value for every dollar invested, whereas stock B 20% of its value.

The investor would like to know how much money she should invest in each of the stocks so that the total profit can be maximized. She needs to make sure that the total amount she invests does not exceed the money she has, and the total potential loss of the investment cannot exceed 2.5 dollars.

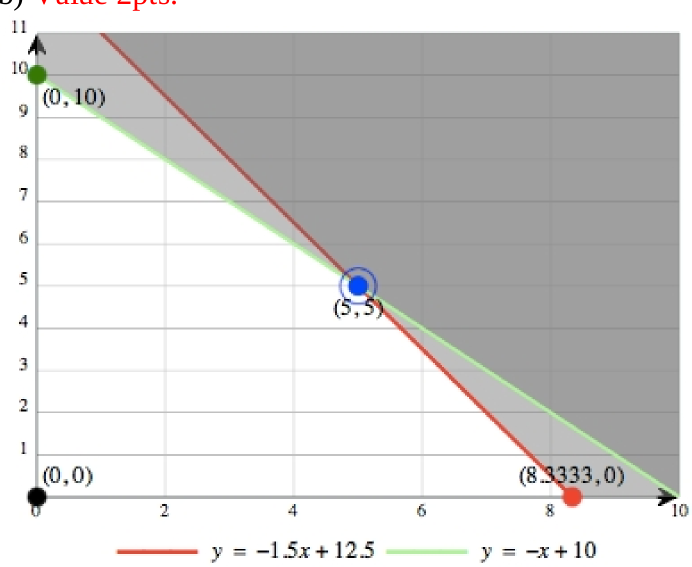
(a) Develop a Linear programming formulation for the above problem. (4 points)

(b) Draw the feasible region and objective function for the formulation. (2 points)

(c) Find the optimal solution(s) and optimal value for the LP model. Describe verbally how the investor should invest. (2 points)

(a) Let x denote the amount invested in stock A, and y denote the amount invested in stock B.
The LP model reads
Maximize 2x+1.5y
Subject to x + y <= 10
0.3x + 0.2y <=2.5
x>=0, y>=0
Value: 4 pts.

(b) Value 2pts.



(c) Value: 2pts.

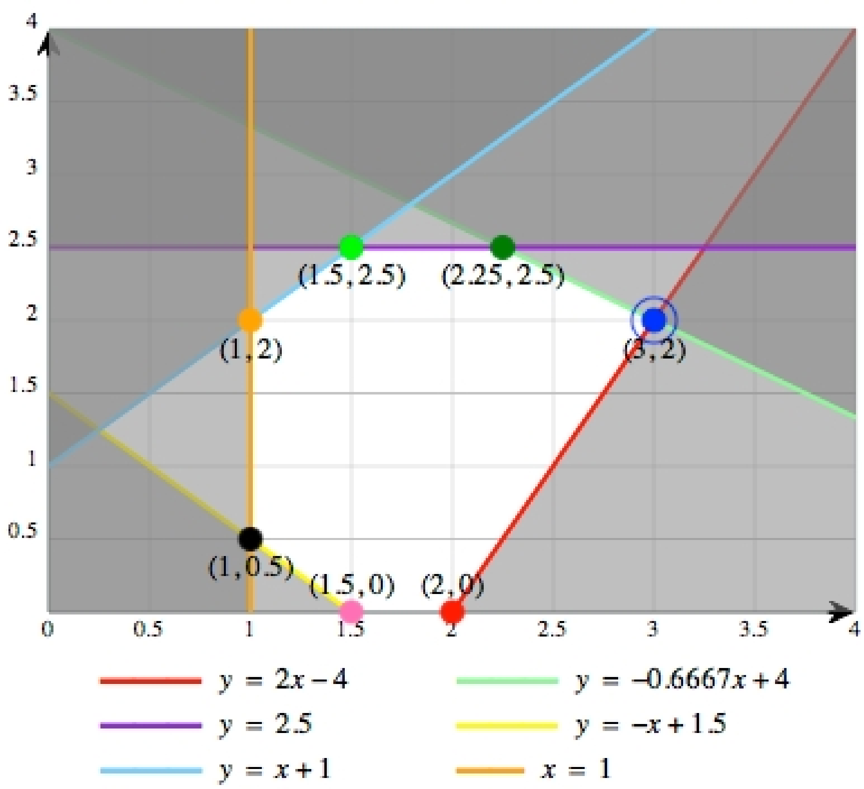
The optimal solution is (5,5) with the optimal value 17.5. Therefore, the investor should invest half of her money, i.e. 5 dollars in stock A and another half, i.e. 5 dollars, in stock B, which will lead to the total profit of 17.5 dolloars.

Q2.
Consider the following Linear Programming model:
Maximize 3x+y
Subject to 2x-y<=4
2x+3y<=12
x>=1
y<=2.5
x+y>=1.5
y-x<=1
x>=0, y>=0

(a) Draw the feasible region and objective function for the model. Show the optimal solution and optimal value. Justify why the solution is optimal. (2 points)

(b) Change the objective function in the LP model to "minimize 3x+y". Draw the objective function. Report what you find about the optimal solution(s) and the optimal value. Justify your finding. (2 points)

(a) value 2pts.



The optimal solution is (3,2) with the optimal value (11).
Students should show the improving direction of the objective function and provide a short explanation that the solution (3,2) is optimal because it is the last point that the line of objective function touches on before leaving the feasible region when following the improving direction.

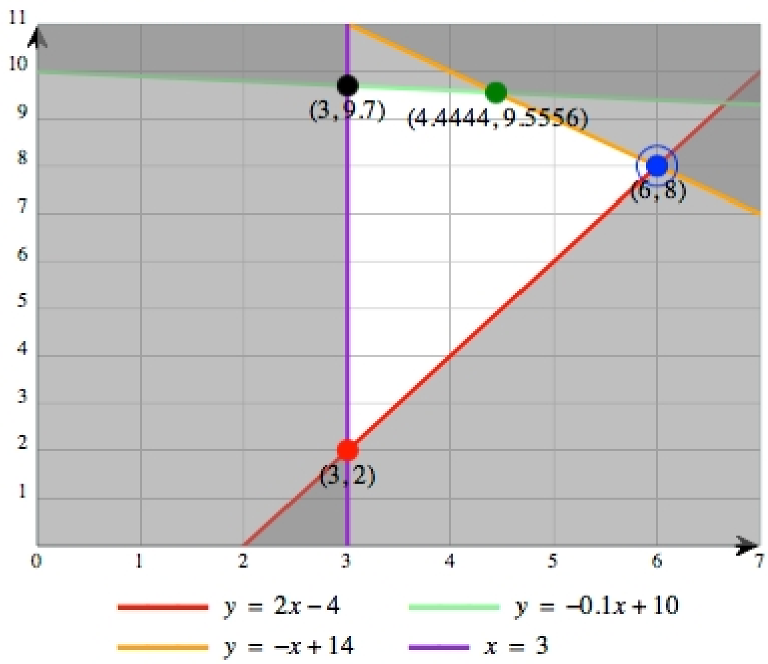
(b) value 2pts.
The optimal solution becomes (1,0.5) with the optimal value (3.5).
Students show the improving direction of the objective function and provide a short explanation that the solution (1,0.5) is optimal because it is the last point that the line of objective function touches on before leaving the feasible region when following the improving direction.

Q3.
Consider the following Linear Programming model:
Maximize 4x+y
Subject to 2x-y<=4
x+10y<=100
x+y<=14
x>=3, y>=0

(a) Draw the feasible region and objective function for the model. Report what you find about the optimal solution(s) and the optimal value. Justify your finding. (2 points)

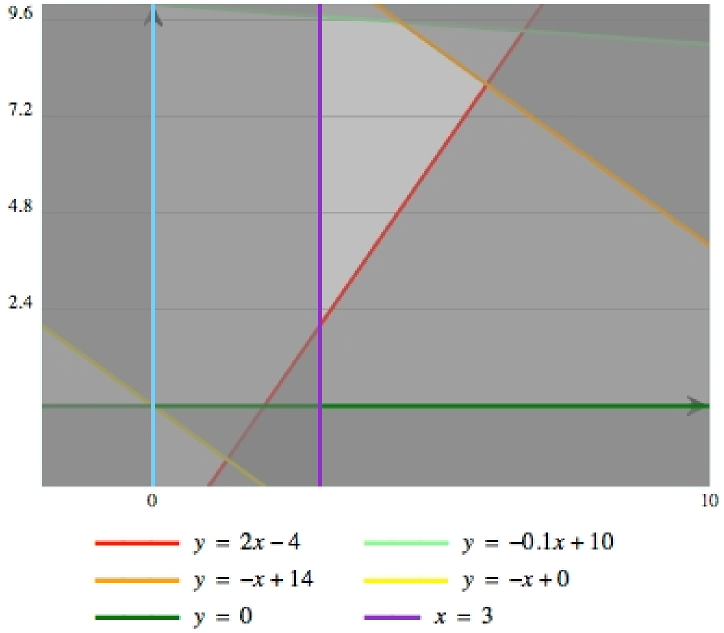
(b) Now, add the constraint "x+y<=0" to the model. Report what you find about the optimal solution(s) and the optimal value. Justify your finding. (2 points)

(a) value 2 pts.



The optimal solution is (6,8) with optimal value (32).
Note: Students could have difficulty of drawing the second constraint, which they need to pick points not on the axis to help draw the constraint.

(b) value 2pts.



The feasible region now becomes empty. No solution exists.

Question 4

Consider the following Linear Programming model:

Minimize 4x+2y

Subject to 2x+y>=3,

y-2x<=1,

x>=1,

y<=4,

y>=0

(a) Draw the feasible region and objective function for the model. Report what you find about the optimal solution(s) and the optimal value. Justify your finding. (2 points)

(b) Change the objective function in the LP model to "maximize 4x+2y". Report what you find about the optimal solution(s) and the optimal value. Justify your finding. (2 points)

(a) value 2pts.

There are multiple (infinitely many) optimal solutions. Any point on the red line between (1,1) and (1.5,0) are optimal solutions with the same optimal value (6). (Students should report more than one optimal solutions exist).

(b) value 2pts.
There is no optimal solution as the improving direction of the objective function heads to the part of the region that is unbounded.
(Students should report the finding of unboundness).

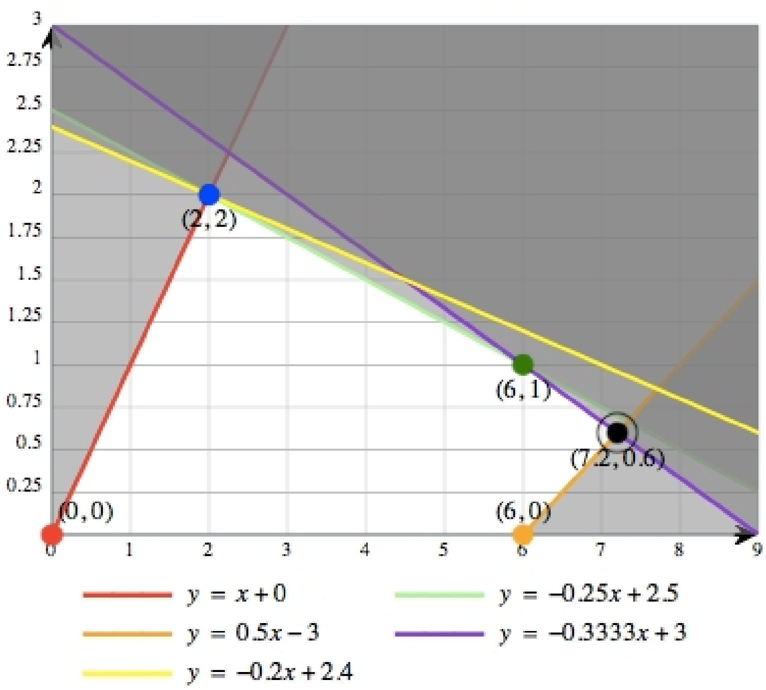
Q5.
Consider the following Linear Programming model:
Maximize 3x-y
Subject to x-y>=0,
x+4y<=10,
x-2y<=6,
x+3y<=9,
x+5y<=12.
x>=0, y>=0.

(a) Draw the feasible region for the model, but DO NOT draw the objective function. Without graphing the objective function, find the optimal solution(s) and the optimal value. Justify your method and why the solution(s) you obtain is (are) optimal. (2 points)

(b) Is there any redundant constraint? Which one(s) and why? (1 point)

(c) Is there any constraint that can be removed without changing the optimal solution(s) you obtained in (a) and why? (2 points)

(a) value 2pts.



The solution will appear below.

Vertex	Lines through vertex	Value of objective
● (2,2)	$x-y=0$ $x+4y=10$	4
● (0,0)	$x-y=0$ $x=0$	0
● (6,1)	$x+4y=10$ $x+3y=9$	17
● (7.2,0.6)	$x-2y=6$ $x+3y=9$	21 Maximum
● (6,0)	$x-2y=6$ $y=0$	18

Students should show the calculations of objective function value for every vertex, and argue that due to the Corner Point Property it suffices to find the optimal solutions by comparing only vertices of the region.

(b) value 1pt.
The constraint x+5y<=12 is redundant because removing it won't change the feasible region.

(c) value 2pts.
The removal of constraints "x-y>=0", "x+4y<=10", and " x+5y<=12" and "y>=0" won't change the optimal solution. Since they do not intersect at the optimal solution

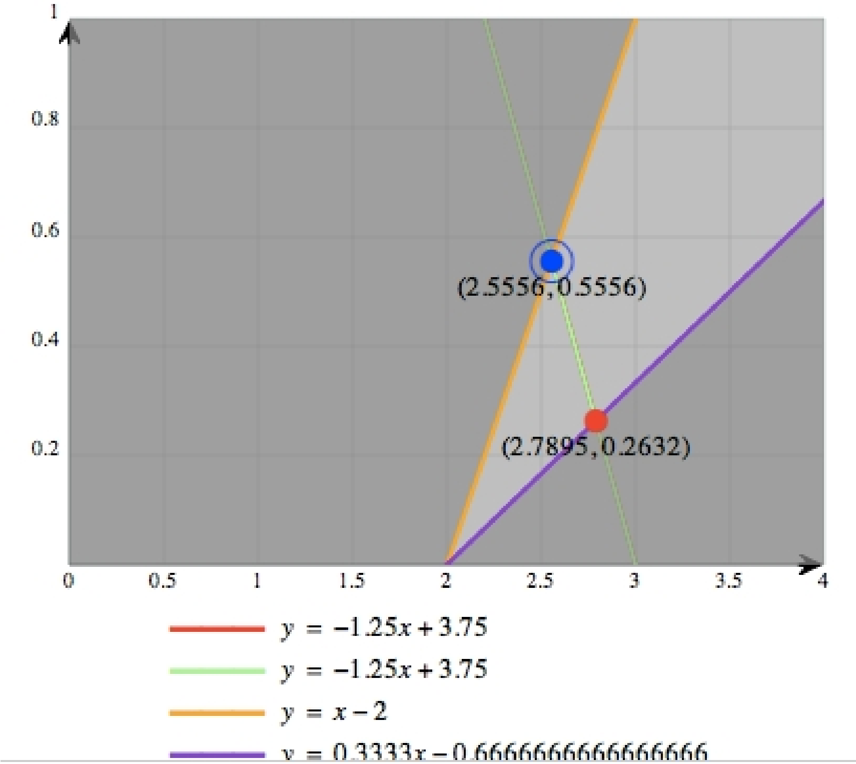
Q6.
Consider the following Linear Programming model:
Maximize 2x+6y
Subject to x-y>=2
x-3y<=2,
5x+4y = 15,
x>=0, y>=0.

(a) Draw the feasible region for the model, but DO NOT draw the objective function. Highlight the feasible region precisely by filling it with a dark color. (2 points)

(b) Without graphing the objective function, find the optimal solution(s) and the optimal value. Justify your method and why the solution(s) you obtain is (are) optimal. (2 points)

(c) Change the objective function to "minimize 2x+6y". Solve (b) again. (1 point)

(a) value 2pts.



The feasible region is only the part of line segment connecting two highlighted points. Students need to be precise on identifying the line segment rather than a 2-dimensional area as the feasible region.

(b) value 2pts.

The solution will appear below.

Vertex	Lines through vertex	Value of objective
● (2.5556, 0.5556)	$5x + 4y = 15$ $x - y = 2$	8.4444 Maximum
● (2.7895, 0.2632)	$5x + 4y = 15$ $x - 3y = 2$	7.1579

(c) value 1pt.

The solution will appear below.

Vertex	Lines through vertex	Value of objective
● (2.5556, 0.5556)	$5x + 4y = 15$ $x - y = 2$	8.4444
● (2.7895, 0.2632)	$5x + 4y = 15$ $x - 3y = 2$	7.1579 Minimum