Economics 301
Midterm Examination
Wednesday February 12
1:15 Hall 459

Answer any two questions.

1. A consumer has a utility function $u(x_1, x_2) = \ln(a_1 x_1 + a_2 x_2)$ and a budget constraint $p_1 x_1 + p_2 x_2 = m$. 1. Are the preferences of this individual homothetic? 2. How much of each good will the individual consume? 3. Comment on the appropriateness of this utility function as a reasonable representation of consumer preferences.

2. For a consumer whose preferences are represented by the utility function $u(x_1, x_2) = x_1^{0.5} + x_2$ and a budget constraint $p_1 x_1 + p_2 x_2 = m$ how much consumer surplus does she lose if p_1 is increased from 1 to 1.5?

3. For a consumer with a utility function $u(x_1, x_2) = \ln(x_1) + x_2$ compute the substitution effect on good 2 if p_2 doubles.

Answers

1. Preferences are homothetic since $u(x_1, x_2)$ is a monotonic transformation of a linear function which is homogeneous of degree 1. Indifference curves are straight lines so that the optimal consumption choice will be be described by a corner solution. This means that

$$x_1 = \frac{m}{p_1} \text{ if } \frac{a_1}{a_2} > \frac{p_1}{p_2} \quad (1)$$

$$= 0 \text{ if } \frac{a_1}{a_2} < \frac{p_1}{p_2} \quad (2)$$

and can be any point on the budget line if $\frac{a_1}{a_2} = \frac{p_1}{p_2}$. Similarly for x_2. Preferences here describe two goods which are perfect substitutes. Most goods are not perfect substitutes so this representation is not accurate for most goods.

2. The demand function for good 1 for this person satisfies a first order condition

$$\frac{0.5 x_1^{-0.5}}{1} = \frac{p_1}{p_2} \quad (3)$$

This makes the demand function for good 1

$$x_1(p_1, p_2, m) = [\frac{p_2}{2p_1}]^2 \quad (4)$$

The loss in consumer surplus is

$$\Delta CS = [\frac{p_2}{2}]^2 \int_1^{1.5} p_1^{-2} dp_1 \quad (5)$$

$$= -[\frac{p_2}{2}]^2 p_1^{-1}]_1^{1.5} \quad (6)$$

$$= 0.33[\frac{p_2}{2}]^2 \quad (7)$$

3. To get the substitution effect the Hicksian demand function for good 2 is needed. To obtain this solve the first order condition and utility constraint equations which are

$$h_1 = \frac{p_2}{p_1} \tag{8}$$

$$u^0 = \ln(h_1) + h_2 \tag{9}$$

This gives

$$h_2 = u^0 - \ln(p_2/p_1) \tag{10}$$

The substitution effect is

$$\Delta h_2 = h_2(p_1, 2p_2, u^0) - h_2(p_1, p_2, u^0) \tag{11}$$

$$= -\ln(2) \tag{12}$$

Notice in this case it was not necessary to compute u^0.