

### Assignment 4

Due date: April 12, 2013, by 11:59 p.m. EDT

1. Let  $L = \{a^n b^n : n \geq 0\}$ , and  $M = \{a^{2m} b^{2p} : m \geq 0, p \geq 0\}$ . Construct a PDA for  $L$  and a DFA<sup>1</sup> for  $M$ . Then use the Cartesian construction to obtain a PDA for  $L \cap M$ .

2. Convert the following grammar into Chomsky normal form

$$\begin{aligned} S &\rightarrow aA|aBB \\ A &\rightarrow aaA|\epsilon \\ B &\rightarrow bB|bbC \\ C &\rightarrow C|B \end{aligned}$$

3. Use the Pumping Lemma for CFL's to show that none of the following languages are Context-free.

- (a)  $L_1 = \{ww : w \in \{a, b\}^*\}$   
 (b)  $L_2 = \{a^n b^k : 0 \leq n \leq k^2\}$   
 (c)  $L_3 = \{a^n b^m c^k : 0 \leq n < m, n \leq k \leq m\}$

4. (a) Construct a Turing machine for the language

$$\{xay : x, y \in \{a, b\}^*, |x| = |y|\}.$$

- (b) Construct a Turing machine for the language

$$\{a^n b^n c^n : n \geq 0\}$$

- (c) Construct a Turing machine for the language

$$\{xay : x, y \in \{a, b\}^*, |x| = |y|\}.$$

- (d) Construct a Turing machine that creates a copy of its input string to the right of the input with a blank separating the copy from the original.

5. Give a complete encoding of the TM

$$A = (\{q_1, q_2, q_3\}, \{a_1, a_2\}, \delta, B, \{q_3\}),$$

where the transition function  $\delta$  is given by  $\delta(q_1, a_1) = (q_1, a_1, R)$ ,  $\delta(q_1, a_2) = (q_3, a_1, L)$ ,  $\delta(q_3, a_1) = (q_2, a_2, L)$ .

6. For each of the languages

$$L_i = \{w_i : w_i \notin L(M_{2i})\}$$

and

$$L_{2i} = \{w_i : w_{2i} \notin L(M_i)\}$$

show that the language is not accepted by any TM, using a diagonalization-type argument, as was done for the language  $L_d$ .

7. Prove the following statement:

A language  $L$  is recursive if and only if both  $L$  and  $\bar{L}$  are recursively enumerable.

8. Show that the language of codes for TM's  $M$  that, when started with a blank tape, eventually write a "1" somewhere on the tape is undecidable.

9. For each of the following instances  $(A, B)$  of Post's Correspondence Problem, determine if it has a solution or not. If you think  $(A, B)$  has a solution, give one, and if you think  $(A, B)$  does not have a solution, provide reasoning to justify your claim.

	List A	List B
$i$	$w_i$	$x_i$
(a) 1	11	111
2	100	001
3	111	11

	List A	List B
$i$	$w_i$	$x_i$
(b) 1	00	0
2	001	11
3	1000	011

<sup>1</sup>Leave out the trap state