COMP 335: Assignement #2 Solution

Winter, 2013

Tuesdays & Thursdays 13:15-14:05

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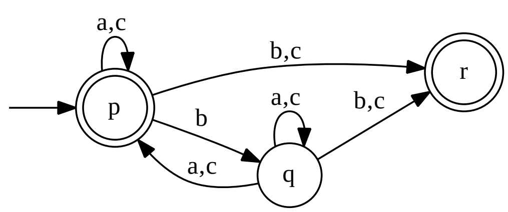
Solution # 1

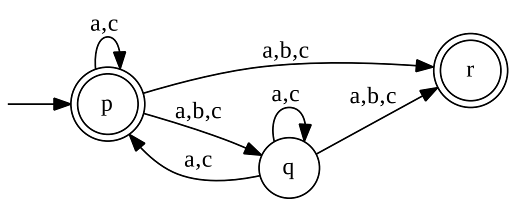
A) The tabular representation of the converted $\epsilon - NFA$ is:

NFA	a	b	c
$\rightarrow *p$	$\{p\}$	$\{q,r\}$	$\{p,r\}$
q	$\{p\}$	$\{r\}$	$\{p,r\}$
$*r$	$\varnothing$	$\varnothing$	$\varnothing$

This can be visualized as the following graph:



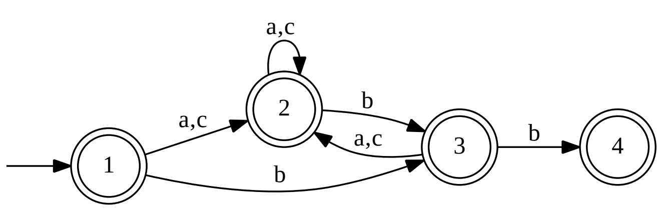
The following solution is also correct:



B) The tabular representation of the converted DFA is:

DFA	a	b	c
$\rightarrow *\{p\}$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$
$*\{p,q,r\}$	$\{p,q,r\}$	$\{q,r\}$	$\{p,q,r\}$
$*\{q,r\}$	$\{p,q,r\}$	$\{r\}$	$\{p,q,r\}$
$*\{r\}$	$\varnothing$	$\varnothing$	$\varnothing$

The DFA can be represented as follows:



And the following table shows the states equivalency:

state name	equivalent state
1	$\{p\}$
2	$\{p, q, r\}$
3	$\{q, r\}$
4	$\{r\}$

Solution # 2

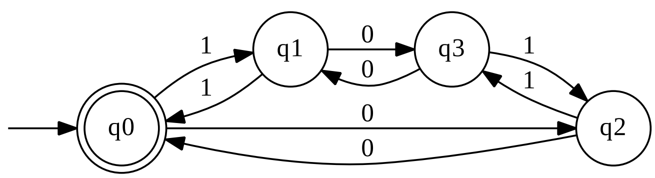
- a) $00 \in L(R) \backslash L(S)$
- b) $01 \in L(S) \backslash L(R)$
- c) $1 \in L(R)$ & $1 \in L(S) \Rightarrow 1 \in L(R) \cup L(S)$
- d) $010 \notin L(R), L(S) \Rightarrow 010 \notin L(R) \cup L(S) \Rightarrow 010 \in \overline{L(R) \cup L(S)}$

Solution # 3

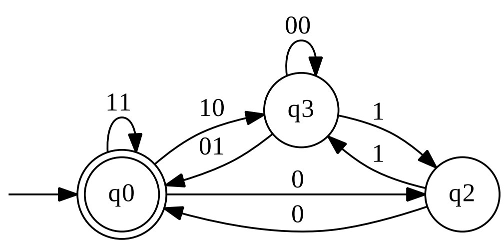
- a) $ab + ba$
- b) $a(aa)^*b(bb)^*c(cc)^* + a(aa)^*(bb)^*(cc)^* + (aa)^*b(bb)^*(cc)^* + (aa)^*(bb)^*c(cc)^*$
- c) $(\epsilon + c)(\rho c)^*(\epsilon + \rho)$ where $\rho = a(ba)^*(\epsilon + b) + b(ab)^*(\epsilon + a)$
- d) $(\epsilon + 1)(01^+)^*(\epsilon + 00)(1^+0)^*1(\epsilon + 00)(1^+0)^*(\epsilon + 1) + (01^+)^*(\epsilon + 000)(1^+0)^*(\epsilon + 1)$

Solution # 4

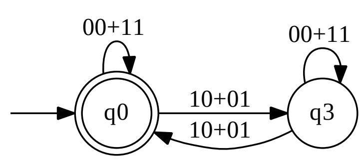
A) The DFA we have is represented with the following graph:



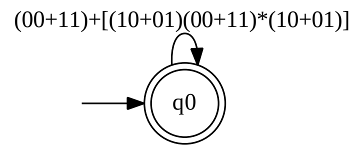
By eliminating the state q_1 we have:



By eliminating the state q_2 we have:



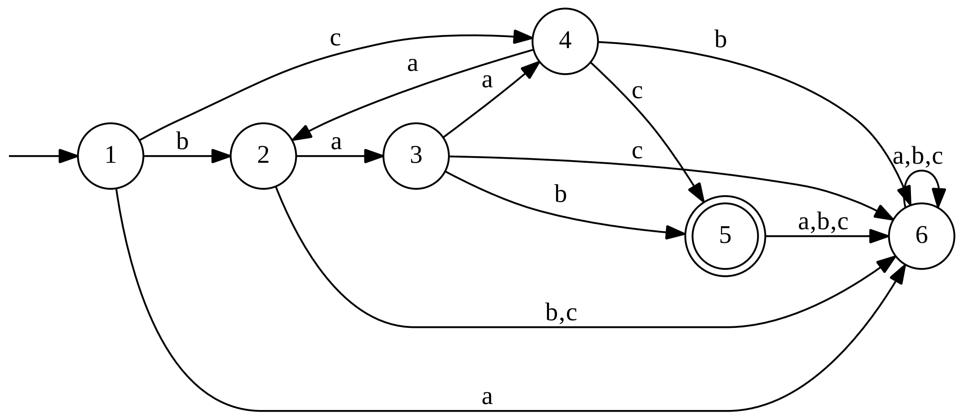
And finally by eliminating q_3 we have:

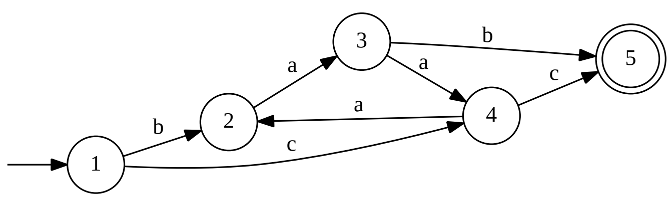


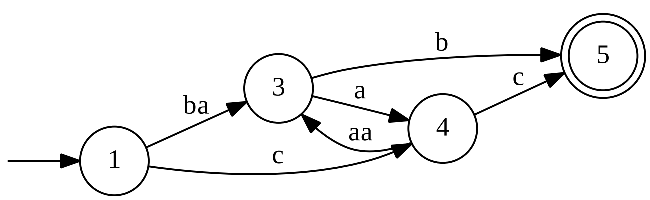
this means that the regular expression is:

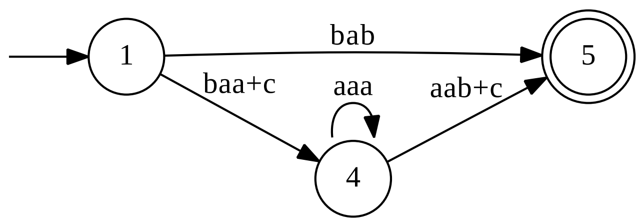
$$R = \left[(11+00)+(10+01)(00+11)^*(01+10)\right]^*$$

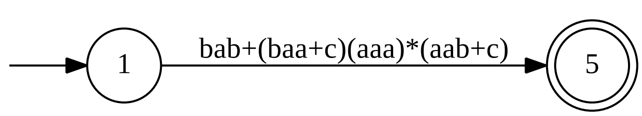
B)









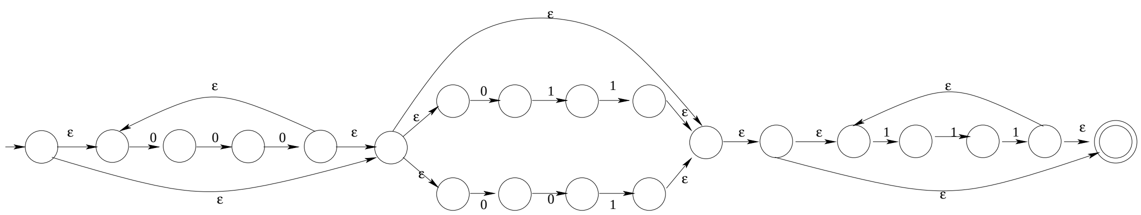


So, the regular expression is:

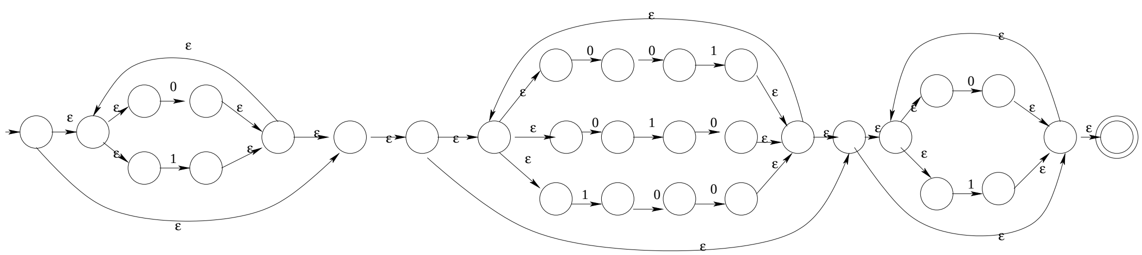
$$R = bab + (baa + c)(aaa)^*(aab + c)$$

Solution # 5

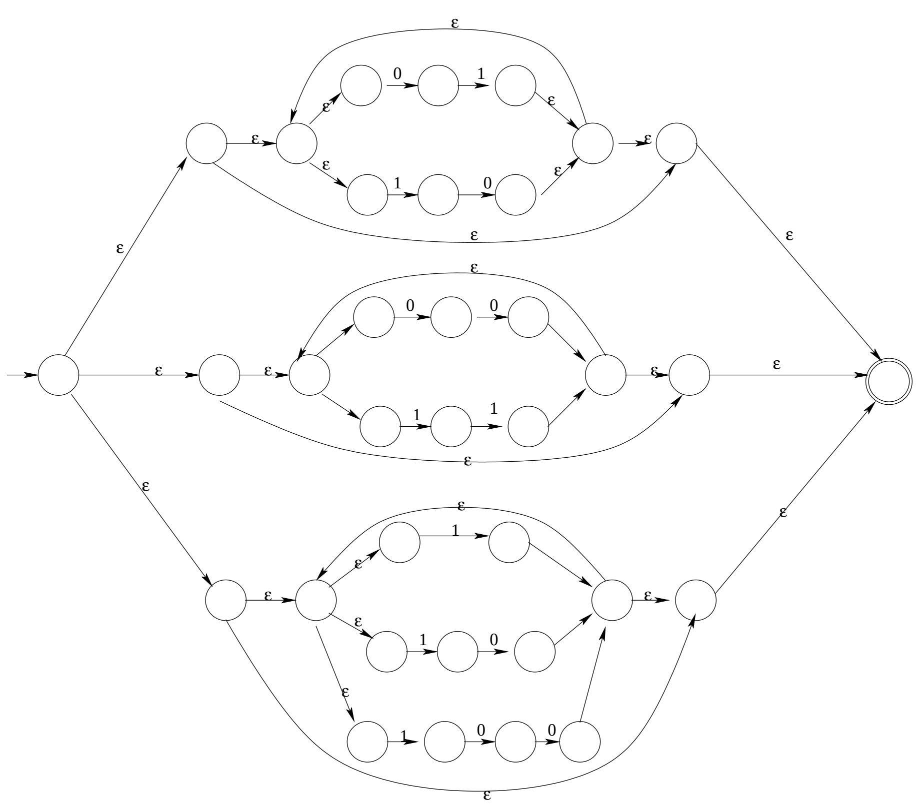
A)



B)



C)



Solution # 6

- Let $L = \{a\}$, $M = \{b\}$ then $aba \in L((\mathbf{a}+\mathbf{b})^*)$ but $aba \notin (L^*M)^* \Rightarrow$ The statement is false.
- True. The proof is provided in the Lecture slides.
- Let $L = \{a\}$, $M = \{b\}$ then $\varepsilon \in (L + ML + MML)^*M^*$ but $\varepsilon \notin L^*(M(M^* + L^*)L)M^* \Rightarrow$ The statement is false.

Solution # 7

- a) Assume L is regular. Let m be the number provided by the pumping lemma. Choose $w = a^{m-k}a^k b^{2^m}$. If $w = xyz$, where $|xy| \le m$ and $|y| \ge 1$, then $y = a^k$ for some $|k| \ge 1$. Now string $a^{m-k}a^{2k}b^{2^m} = a^{m+k}b^{2^m} \notin L$. This contradicts the first assumption that L is regular and therefore, L is not regular.
- b) Assume L is regular. Let m be the number provided by the pumping lemma. Choose $w = a^m b a^m = a^{m-k}a^k b a^m$. If $w = xyz$, where $|xy| \le m$ and $|y| \ge 1$, then $y = a^k$ for some $|k| \ge 1$. Now string $a^{m-k}a^{2k}ba^m = a^{m+k}ba^m \notin L$. This contradicts the first assumption that L is regular and therefore, L is not regular.
- c) Assume L is regular. Let m be the number provided by the pumping lemma. Choose $w = a^m b^m c^m d^m = a^{m-k}a^k b^m c^m d^m$. If $w = xyz$, where $|xy| \le m$ and $|y| \ge 1$, then $y = a^k$ for some $|k| \ge 1$. Now string $a^{m-k}a^{2k}b^m c^m d^m = a^{m+2k}b^m c^m d^m \notin L$. This contradicts the first assumption that L is regular and therefore, L is not regular.