COMP 335: Assignement #1 Solution

Winter, 2013

Tuesdays & Thursdays 13:15-14:05

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Solution # 1

Prove the reversal of a string $(uv)^R = v^R u^R$. We are given the recursive definitions

$$\forall a \in \Sigma \Rightarrow a^R = a \qquad \forall w \in \Sigma^*, \forall a \in \Sigma \Rightarrow (wa)^R = aw^R$$

Proof. **Basis:** Pick a length 1 string v so we can use the definitions above. Recall $\forall a \in \Sigma \Rightarrow a \in \Sigma^*$.

$$\forall u, v \in \Sigma^* : |v| = 1 \Rightarrow v = a \Rightarrow (uv)^R = (ua)^R = au^R \text{ (by the definition)}$$

Induction hypothesis: assume it works for length k instances of x, $\forall u, x \in \Sigma^* : |x| = k \Rightarrow (ux)^R = x^R u^R$.

Induction step: pick a string v of length $k+1$ such that $u, v \in \Sigma^* : |v| = k+1$. We will write it as $v = xa$, $x \in \Sigma^*$. Then we have:

$$(uv)^R = \big(u(xa)\big)^R = \big((ux)a\big)^R =_{def.} a(ux)^R =_{I.H.} a(x^R)(u^R) = (ax^R)u^R =_{def.} (xa)^R u^R = v^R u^R$$

Solution # 2

(a) $aaaaaa, bbbbbb, abbaab, baabba \in L$ & $\epsilon, ababab \notin L$
(b) $\epsilon \in L$ & $a, b, ab, ... \notin L$
(c) Strings IN the language:

1. $u = aba$, $v = bab$, $w = a$, $abababa = abababa$
2. $u = b$, $v = a$, $w = b$, $bab = bab$

Strings NOT IN the language:

1. $u = bbb$, $v = aaa$, $bbbaaaw \neq waaabbb$
2. $u = ab$, $v = bb$, $abbbw \neq wbbaa$

(d) Strings IN the language:

1. $w = aa$, $u = aaa$, $aaaaaa = aaaaaa$
2. $u = abbabbabb$, $w = abbabb$, $abbabbabbabbabbabbb = abbabbabbabbabbabb$

Strings NOT IN the language:

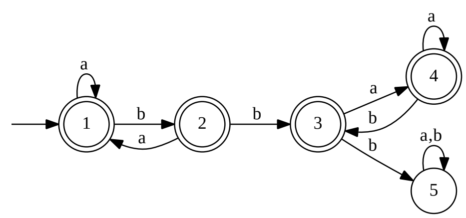
1. $w = aab$, $u = baa$, $aabaabaab \neq baabaabaa$
2. $w = bbb$, $u = bbb$, $bbbbbbbbb \neq bbbbbb$

Solution # 3

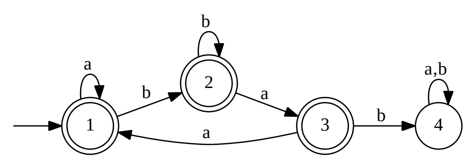
- A1: All the strings that end with zero or have two consecutive zeros.
- A2: All the strings that start with zero.
- A3: All the strings that end with one.

Solution # 4

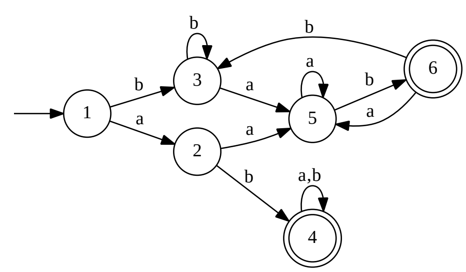
- Note: A string is not accepted if it contains a substring $b^n, n \geq 3$.



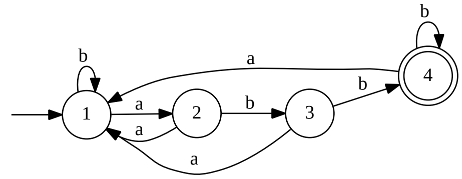
- A string is not accepted if it contains a substring bab



- The set of strings that either begin or end (or both) with ab



- All the accepted strings contain odd number of a's and they end to $b^n, n \geq 2$



Solution # 5

- **Basis**: $n = 0 \Rightarrow \hat{\delta}(q, a^0) = \hat{\delta}(q, \epsilon) = q$
- **Induction hypothesis:** Suppose that for $n = k \to \hat{\delta}(q, a^k) = q$.
- **Induction step:** For $n = k + 1 \to \delta(q, a^{k+1}) =_{def.} \delta(\hat{\delta}(q, a^k), a) =_{I.H.} \delta(q, a) =_{assum.} q$

(b) Let F to be the set of accepted states. If $q \in F$ then $\delta(q, a^k) \in F$ since $\delta(q, a^k) = q$ for all k. This means that $\bigcup_k a^k = \{a\}^*$ is accepted by the automaton which means:

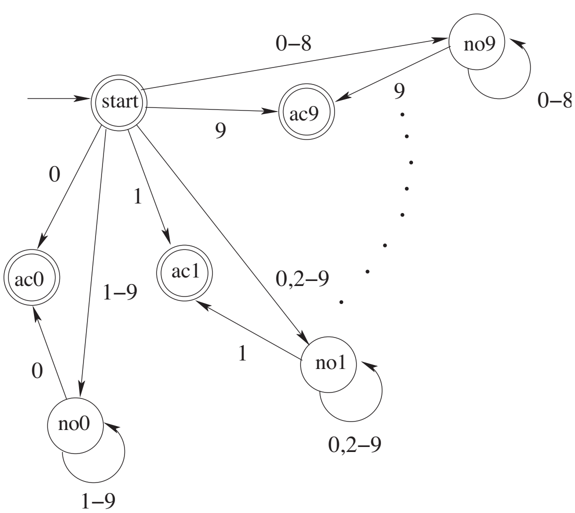
$$\{a\}^* \subseteq L(A)$$

On the other hand, if $q \notin F$ then $\delta(q, a^k) \notin F$ since $\delta(q, a^k) = q$ for all k. This means that $\bigcup_k a^k = \{a\}^*$ is not accepted by the automaton. In other words:

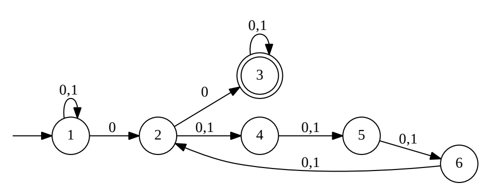
$$\{a\}^* \cap L(A) = \varnothing$$

Solution # 6

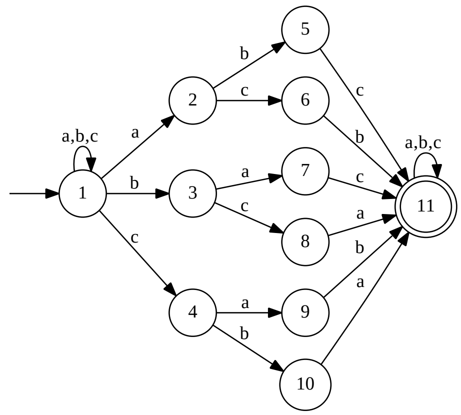
a)



b)



Solution # 7

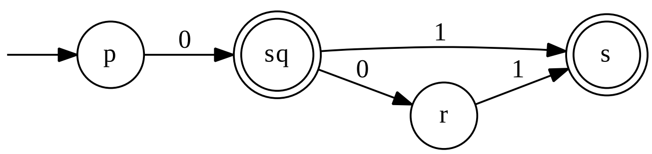


Solution # 8

The tabular representation of the converted DFA is:

DFA1	0	1
$\rightarrow \{p\}$	$\{s,q\}$	$\varnothing$
$*\{s,q\}$	$\{r\}$	$\{s\}$
$\{r\}$	$\varnothing$	$\{s\}$
$*\{s\}$	$\varnothing$	$\varnothing$

This can be visualized as the following graph:

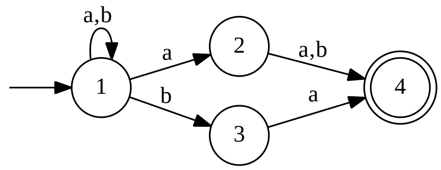


And the following table shows the states equivalency:

state name	equivalent state
p	$\{p\}$
sq	$\{s,q\}$
r	$\{r\}$
s	$\{s\}$

Solution # 9

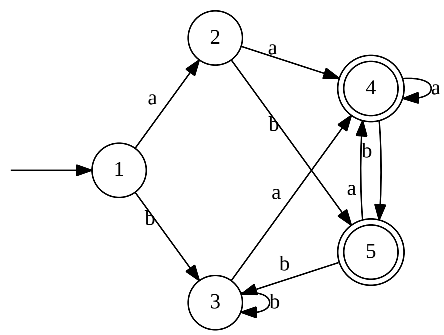
a)



b) Here is tabular representation of the converted DFA:

DFA2	a	b
$\rightarrow \{1\}$	$\{1,2\}$	$\{1,3\}$
$\{1,2\}$	$\{1,2,4\}$	$\{1,3,4\}$
$\{1,3\}$	$\{1,2,4\}$	$\{1,3\}$
$*\{1,2,4\}$	$\{1,2,4\}$	$\{1,3,4\}$
$*\{1,3,4\}$	$\{1,2,4\}$	$\{1,3\}$

The graph representation of the DFA is:



And the following table shows the states equivalency:

state #	equivalent state
1	$\{1\}$
2	$\{1,2\}$
3	$\{1,3\}$
4	$\{1,2,4\}$
5	$\{1,3,4\}$