Concordia University

Department of Computer Science & Software Engineering

COMP 335/4 — Theoretical Computer Science — Winter 2013

Assignment 1 Due date: February 1, 2013

1. The *reversal* of a string w, denoted by w^R, is the string "spelled backwards". For example $cat^R = tac$. Reversal is defined inductively as follows:

$$a^R = a, (wa)^R = aw^R.$$

(Here a is a symbol in the alphabet Σ.) Let $u, v \in \Sigma^*$. Prove that

$$(uv)^R = v^R u^R.$$

Hint: Use induction on $|v|$.

2. Let $\Sigma = \{a, b\}$. For each of the languages below, give an example of a string in the language, and a string not in the language.

   (a) $\{w \in \Sigma^* : w = uu^R u, \text{ for some } u \in \Sigma^2\}$

   (b) $\{w \in \Sigma^* : ww = www\}$

   (c) $\{w \in \Sigma^* : uvw = wvu, \text{ for some } \{u, v\} \subset \Sigma^*\}$.

   (d) $\{w \in \Sigma^* : www = uu, \text{ for some } u \in \Sigma^*\}$.

3. For each of the DFA's below (in tabular form), describe in English the strings the DFA accepts.

A_1	0	1
$\rightarrow p$	q	p
$\star q$	r	p
$\star r$	r	r

A_2	0	1
$\star p$	p	p
$\rightarrow q$	p	r
r	r	r

A_3	0	1
$\rightarrow p$	p	q
$\star q$	p	q

4. Construct a DFA for each of the following languages.

   (a) $\{w \in \{a, b\}^* : bb \text{ appears at most once as a substring of } w\}$

   (b) $\{w \in \{a, b\}^* : bab \text{ is not a substring of } w\}$

   (c) The set of strings that either begin or end (or both) with ab.

   (d) $\{w \in \{a, b\}^* : w \text{ contains an odd number of } a's \text{ and ends in at least two } b's\}$

5. Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

   (a) Show by an induction on n, that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

   (b) Show that either $\{a\}^* \subseteq L(A)$, or $\{a\}^* \cap L(A) = \emptyset$.

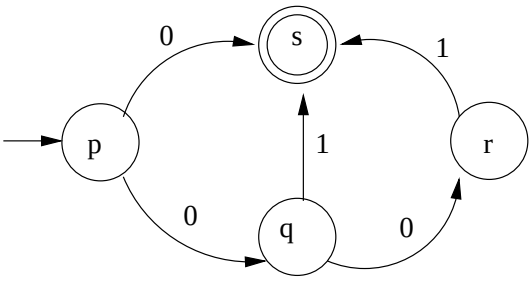
6. Construct an NFA for each of the following languages.

   (a) The set of strings over $\{0, 1, \ldots, 9\}$, such that the final digit has not appeared before

   (b) The set of strings over $\{0, 1\}$, such that there are two 0's separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.

7. Construct an NFA for the set of strings over alphabet $\{a, b, c\}$ that have a substring of length 3 containing each of the symbols.

8. Consider the following NFA A:



   Write down the tabular representation of A, and convert A to a DFA using the subset construction. Give the DFA both in tabular form and as a transition diagram.

9. Let $\Sigma = \{a, b\}$.

   (a) Construct an NFA that accepts the strings in Σ^* where at least one of the last two symbols is an a.

   (b) Convert your NFA to a DFA using the subset construction. Give the DFA both in tabular form and as a transition diagram.