MECH 326 Assignment 2

Due Oct. 19

Solutions

5-3. Do part a) and c) only

5–3 Repeat Prob. 5–1 for a bar of AISI 1030 hot-rolled steel and:
(*a*) $\sigma_x = 25$ kpsi, $\sigma_y = 15$ kpsi
(*b*) $\sigma_x = 15$ kpsi, $\sigma_y = -15$ kpsi
(*c*) $\sigma_x = 20$ kpsi, $\tau_{xy} = -10$ kpsi
(*d*) $\sigma_x = -12$ kpsi, $\sigma_y = 15$ kpsi, $\tau_{xy} = -9$ kpsi
(*e*) $\sigma_x = -24$ kpsi, $\sigma_y = -24$ kpsi, $\tau_{xy} = -15$ kpsi

5-3 From Table A-20, $S_y = 37.5$ kpsi

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$n = \dfrac{S_y}{\sigma'}$

(a) MSS: $\sigma_1 = 25 \text{ kpsi}, \ \sigma_3 = 0 \quad \Rightarrow \quad n = \dfrac{37.5}{25-0} = 1.5 \quad$ *Ans.*

DE: $n = \dfrac{37.5}{\left[25^2 - (25)(15) + 15^2\right]^{1/2}} = 1.72 \quad$ *Ans.*

(c) $\sigma_A, \ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1, \ -4.1 \text{ kpsi}$

$\sigma_1 = 24.1, \ \sigma_2 = 0, \ \sigma_3 = -4.1 \text{ kpsi}$

MSS: $n = \dfrac{37.5}{24.1 - (-4.1)} = 1.33 \quad$ *Ans.*

DE: $\sigma' = \left[20^2 + 3\left(-10^2\right)\right]^{1/2} = 26.5 \text{ kpsi} \quad \Rightarrow \quad n = \dfrac{37.5}{26.5} = 1.42 \quad$ *Ans.*

5-6. Do part a) and c) only

5–6 Repeat Prob. 5–3 by first plotting the failure loci in the σ_A, σ_B plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.

5-6 Note: The drawing in this manual may not be to the scale of original drawing. The measurements were taken from the original drawing.

(a) $\sigma_A = 25$ kpsi, $\sigma_B = 15$ kpsi

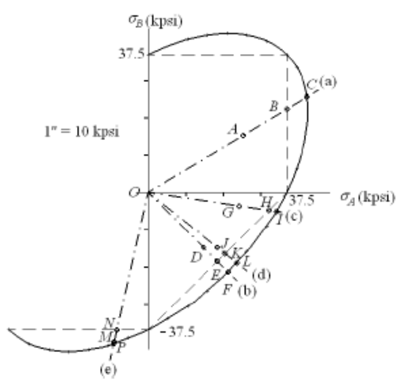
MSS:
$n = \dfrac{OB}{OA} = \dfrac{4.37''}{2.92''} = 1.50 \quad$ *Ans.*

DE:
$n = \dfrac{OC}{OA} = \dfrac{5.02''}{2.92''} = 1.72 \quad$ *Ans.*

(b) $\sigma_A = 15$ kpsi, $\sigma_B = -15$ kpsi

MSS:
$n = \dfrac{OE}{OD} = \dfrac{2.66''}{2.12''} = 1.25 \quad$ *Ans.*

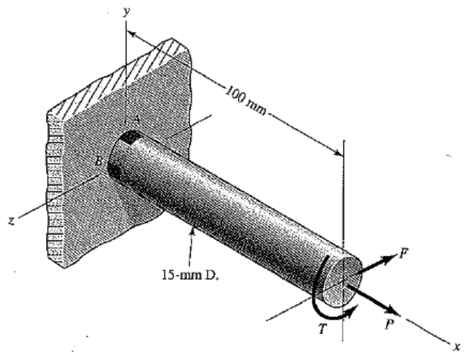
DE:
$n = \dfrac{OF}{OD} = \dfrac{3.05''}{2.12''} = 1.44 \quad$ *Ans.*



(c) $\sigma_A, \ \sigma_B = \dfrac{20}{2} \pm \sqrt{\left(\dfrac{20}{2}\right)^2 + (-10)^2} = 24.1, \ -4.1 \text{ kpsi}$

MSS: $n = \dfrac{OH}{OG} = \dfrac{3.25''}{2.43''} = 1.34 \quad$ *Ans.* DE: $n = \dfrac{OI}{OG} = \dfrac{3.46''}{2.43''} = 1.42 \quad$ *Ans.*

5–36 This problem illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. Here you are to compute factors of safety, based upon the distortion-energy theory, for stress elements at A and B of the member shown in the figure. This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces $F = 0.55$ kN, $P = 4.0$ kN, and $T = 25$ N · m.



Problem 5–36

5-36 Given: AISI 1006 CD steel, $F = 0.55$ kN, $P = 4.0$ kN, and $T = 25$ N·m. From Table A-20, $S_y = 280$ MPa. Apply the DE theory to stress elements A and B

A:

$$\sigma_x = \frac{4P}{\pi d^2} = \frac{4(4)10^3}{\pi(0.015^2)} = 22.6(10^6) \text{ Pa} = 22.6 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(25)}{\pi(0.015^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)0.015^2}\right] = 41.9(10^6) \text{ Pa} = 41.9 \text{ MPa}$$

$$\sigma' = \left[22.6^2 + 3(41.9^2)\right]^{1/2} = 76.0 \text{ MPa}$$

$$n = \frac{280}{76.0} = 3.68 \quad \textit{Ans.}$$

B:

$$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)10^3(0.1)}{\pi(0.015^3)} + \frac{4(4)10^3}{\pi(0.015^2)} = 189(10^6) \text{ Pa} = 189 \text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(25)}{\pi(0.015^3)} = 37.7(10^6) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2} = \left[189^2 + 3(37.7^2)\right]^{1/2} = 200 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{200} = 1.4 \quad \textit{Ans.}$$

5–84 A plate 100 mm wide, 200 mm long, and 12 mm thick is loaded in tension in the direction of the length. The plate contains a crack as shown in Fig. 5–26 with the crack length of 16 mm. The material is steel with $K_{Ic} = 80$ MPa · $\sqrt{\text{m}}$, and $S_y = 950$ MPa. Determine the maximum possible load that can be applied before the plate (*a*) yields, and (*b*) has uncontrollable crack growth.

5-84 Given: $a = 16$ mm, $K_{Ic} = 80$ MPa·$\sqrt{\text{m}}$ and $S_y = 950$ MPa

(a) Ignoring stress concentration

$$F = S_y A = 950(100 - 16)(12) = 958(10^3) \text{ N} = 958 \text{ kN} \quad \textit{Ans.}$$

(b) From Fig. 5-26: $h/b = 1$, $a/b = 16/100 = 0.16$, $\beta = 1.3$

Eq. (5-37) $\quad K_I = \beta\sigma\sqrt{\pi a}$

$$80 = 1.3\frac{F}{100(12)}\sqrt{\pi(16)10^{-3}}$$

$$F = 329.4(10^3) \text{ N} = 329.4 \text{ kN} \quad \textit{Ans.}$$

5-85 A cylinder subjected to internal pressure p_i has an outer diameter of 14 in and a 1-in wall thickness. For the cylinder material, $K_{Ic} = 72$ kpsi $\cdot \sqrt{\text{in}}$, $S_y = 170$ kpsi, and $S_{ut} = 192$ kpsi. If the cylinder contains a radial crack in the longitudinal direction of depth 0.5 in determine the pressure that will cause uncontrollable crack growth.

5-85 Given: $a = 0.5$ in, $K_{Ic} = 72$ kpsi$\cdot\sqrt{\text{in}}$ and $S_y = 170$ kpsi, $S_{ut} = 192$ kpsi

$$r_o = 14/2 = 7 \text{ in}, \ r_i = (14-2)/2 = 6 \text{ in}$$

$$\frac{a}{r_o - r_i} = \frac{0.5}{7-6} = 0.5, \qquad \frac{r_i}{r_o} = \frac{6}{7} = 0.857$$

Fig. 5-30: $\beta \doteq 2.4$

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a} \quad \Rightarrow \quad 72 = 2.4\sigma\sqrt{\pi(0.5)} \quad \Rightarrow \quad \sigma = 23.9$ kpsi

Eq. (3-50), p. 113, at $r = r_o = 7$ in:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2) \quad \Rightarrow \quad 23.9 = \frac{6^2 p_i}{7^2 - 6^2}(2) \quad \Rightarrow \quad p_i = 4.315 \text{ kpsi} \qquad \textit{Ans.}$$

6-5 A steel rotating-beam test specimen has an ultimate strength of 230 kpsi. Estimate the fatigue strength corresponding to a life of 150 kcycles of stress reversal.

6-5 $S_{ut} = 230$ kpsi, $N = 150\ 000$ cycles

Fig. 6-18, point is off the graph, so estimate: $f = 0.77$

Eq. (6-8): $S_{ut} > 200$ kpsi, so $S_e' = S_e = 100$ kpsi

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.77(230)]^2}{100} = 313.6 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(230)}{100}\right) = -0.08274$$

Eq. (6-13): $$S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0 \text{ kpsi} \qquad \textit{Ans.}$$

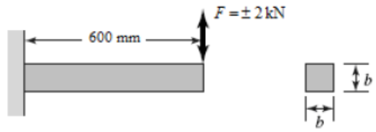
6–13 A solid square rod is cantilevered at one end. The rod is 0.6 m long and supports a completely reversing transverse load at the other end of ±2 kN. The material is AISI 1080 hot-rolled steel. If the rod must support this load for 10^4 cycles with a factor of safety of 1.5, what dimension should the square cross section have? Neglect any stress concentrations at the support end.

6-13 $L = 0.6$ m, $F_a = 2$ kN, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770$ MPa, $S_y = 420$ MPa (Table A-20)

First evaluate the fatigue strength.

$$S'_e = 0.5(770) = 385 \text{ MPa}$$

$$k_a = 57.7(770)^{-0.718} = 0.488$$



Since the size is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

Eq. (6-18): $S_e = 0.488(0.85)(385) = 160$ MPa

In kpsi, $S_{ut} = 770/6.89 = 112$ kpsi

Fig. 6-18: $f = 0.83$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.83(770)]^2}{160} = 2553 \text{ MPa}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.83(770)}{160}\right) = -0.2005$$

Eq. (6-13): $$S_f = aN^b = 2553(10^4)^{-0.2005} = 403 \text{ MPa}$$

Now evaluate the stress.

$$M_{max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_a = \sigma_{max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6(1200)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary b.

$$n = \frac{S_f}{\sigma_a} = \frac{403(10^6)}{7200/b^3} = 1.5$$

$b = 0.0299$ m Select $b = 30$ mm.

Since the size factor was guessed, go back and check it now.

Eq. (6-25): $$d_e = 0.808(hb)^{1/2} = 0.808b = 0.808(30) = 24.24 \text{ mm}$$

Eq. (6-20): $$k_b = \left(\frac{24.2}{7.62}\right)^{-0.107} = 0.88$$

Our guess of 0.85 was slightly conservative, so we will accept the result of

$b = 30$ mm. *Ans.*

Checking yield,

$$\sigma_{max} = \frac{7200}{0.030^3}(10^{-6}) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{420}{267} = 1.57$$

Do Problems 1 and 2

1) Calculate the maximum permissible edge-crack size in a panel subjected to a uniform stress of 20 ksi; b = 6 inches given that the fracture toughness is 50 ksi(in$^{-1/2}$) and the yield is 60 ksi.

You needs to make an assumption of h/b. Choose either 1.0 or 0.5 (no wrong answer)
Recall $\beta = K_I/K_o$

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ME 351 - SOLUTION TO RECOMMENDED PROBLEMS ON CRACKED BODY DESIGN

1) $\sigma = 20$ ksi
$b = 6$ in
$K_c = 50$ ksi $\sqrt{in}$
$\sigma_y = 60$ ksi

$$K_c = \sigma\sqrt{\pi a}\left(\frac{K_I}{K_o}\right)$$

$$= 20\sqrt{\pi}\sqrt{a_c}\left(\frac{K_I}{K_o}\right) = 50 \text{ ksi} \sqrt{in}$$

$$\therefore a_c = \left(\frac{50}{\left(\frac{K_I}{K_o}\right)20\sqrt{\pi}}\right)^2$$ where $\frac{K_I}{K_o}$ is evaluated at a_c

$\therefore$ estimate a_c → find $\left(\frac{K_I}{K_o}\right)_{a_c}$ from Figure 5-22 → solve for a_c (iterate)

a_c (est)	a/b	$\frac{K_I}{K_o}$ $(h/b = \infty)$	a_c (cal)
1.4	.233	1.4	1.015
1.1	.183	1.15	1.504
1.15	.192	1.18	1.428
1.3	.216	1.23	1.315

$\therefore a_c \simeq 1.3$ in

(Note: $\frac{K_I}{K_o}$ estimates very crude from Figure).

2) A large removable beam used as a main beam to support equipment being delivered to an off-shore oil platform has been inspected and has been found to contain a 0.1 inch edge crack located as shown in Figure 1. The beam is made from a tough steel that has a yield strength of 195 ksi and a fracture toughness of 160 ksi-in$^{1/2}$. Assuming pin jointed ends and the load application points as noted, generate a residual strength diagram and determine if failure will occur (ignore shear effects) when:

a) the force F = 25,000 lbs.
b) the force is increased to F = 100,000 lbs.

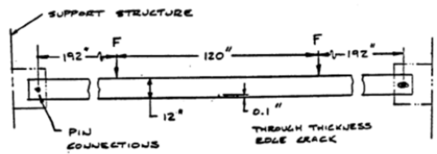

FIGURE 1

NOTE: RECTANGULAR SECTION THICKNESS t = 4"

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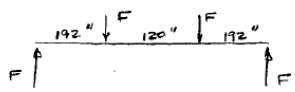
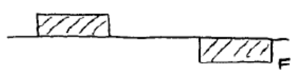
2) FORCE BALANCE

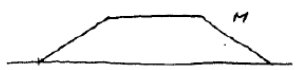


$h = 12"$ $t = 4"$
$F = 25{,}000$ lb or $100{,}000$ lb
$\sigma_y = 195$ ksi
$K_c = 160$ ksi $\sqrt{in}$
$a_i = 0.1"$

SHEAR FORCE



BENDING MOMENT



$M_1 = 25{,}000 \times 192"$
$= 4.8 \times 10^6$ in lbs

FOR $a_i = 0.1$

$a/h = \frac{0.1}{12} = 0.0083\bar{3}$

$M_2 = 100{,}000 \times 192"$
$= 19.2 \times 10^6$ in lbs

STRESSES $\sigma_1 = \frac{M_1 c}{I} = \frac{4.8 \times 10^6 \cdot 6}{\frac{1}{12}(4)(12)^3} = \underline{50000 \text{ psi}}$

$\sigma_2 = \frac{M_2 c}{I} = \underline{200{,}000 \text{ psi}}$

$K = \sigma\sqrt{\pi a}\left(\frac{K_I}{K_o}\right)$ $\frac{K_I}{K_o} \cong 1.12$

1) $K_1 = 50{,}000\sqrt{\pi(.1)}(1.12) = 31{,}387.94$ psi $\sqrt{in} < K_c$
$\therefore$ no fracture
$\sigma_1 < \sigma_y$ $\therefore$ no yielding

2) $K_2 = 4 \times K_1 = 125{,}551.75$ psi $\sqrt{in} < K_c$ $\therefore$ no fracture
$\sigma_2 > \sigma_y$ $\therefore$ yielding occurs!