ADM 3351 Fixed Income
FINAL EXAM REVIEW FALL 2011

This is a closed book exam. A formula page is provided at the end of the exam question sheet for your convenience, which may not include all formulas.

INSTRUCTIONS:

1. **Books and Cheat Sheets are not permitted**. You must be prepared to participate in solving these **Practice Examination** questions.
2. This **Practice Examination comprises of 7 questions** . Put all your answers in the question booklet.
3. Limit your answer to the space provided. Blank sheets for rough work and supporting calculations are given at the end of each question.
4. You are advised to **show all your work** how you arrive at your solutions for each question as partial marks will be awarded for your work. **State reasonable assumptions, if you feel they are necessary.**

Question		Marks
1	Mortgage-Pass-Through Security	/20
2	Collateralized Mortgage Obligations	/10
3	Planned Amortization Class (PAC) Bond	/10
4	Bond Pricing	/10
5	Current Term Structure of Interest Rate	/15
6	Immunization Portfolio	/20
7	Short Answers	/15
TOTAL		/100

Question 1 (20 points)

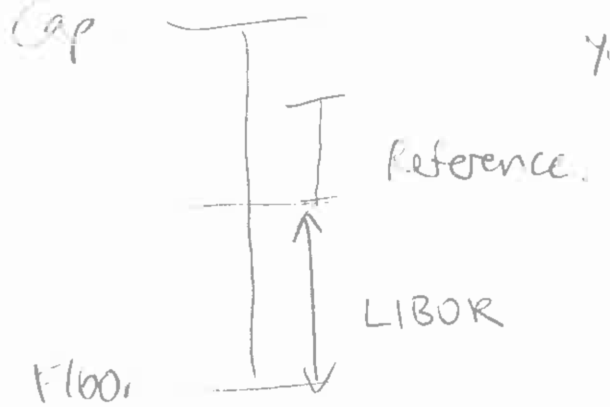
A 6.5% (pass-through rate) 30-year mortgage-pass-through security is already seasoned with a weighted average maturity (WAM) of 332 months left. The current outstanding balance of $200,000,000, and the weighted average coupon (WAC) rate is 8%. The interest rates are quoted as annual percentage rate (APR) based on monthly compounding assumption. Assume that the prepayment rate is 150 PSA.

(a) Calculate the total cash flow of the security to the investors for the next 2 months. Fill out the table on the next page. (15 points)

(b) Home owners do not want to be locked in high mortgage rates. Briefly explain why financial institutions are unwilling to offer fixed rate mortgage for long term (e.g., 30 years), and how this problem has been solved by the federally sponsored agencies such as Fannie Mae and Freddie Mac. (5 points)

Question 2 (10 points)

In a Collateralized Mortgage Obligations (CMO), a 6.5% fixed-rate tranche with \$60,000,000 principal balance is used to create a floating-rate tranche (floater) with \$40,000,000 balance and an inverse-floating-rate tranche (inverse floater) with \$20,000,000 balance. If the floater receives one-month LIBOR + 1%, what interest rate should the inverse floater receive? If the floor rate of the inverse floater is 1.5%, what is the cap rate of the floater?



You never have a situation where you exceed the cap; you'd reset it

BOTTOM LINE, never use anything more than cap.

K−L × LIBOR

$\frac{2}{3}$ $\frac{L}{1+L}$ of fixed rate tranche.

Fixed Rate Tranche 6.5%. \$ principal balance = \$60,000,000

① Floating Rate Tranche \$40,000,000 $\left(\frac{2}{3}\right)$

② Inverse floater Tranche \$20,000,000 $\left(\frac{1}{3}\right)$

$$\frac{2}{3}(\text{LIBOR} + 1\%) + \frac{1}{3}(K - 2\,\text{LIBOR}) = 6.5\%$$

$$\cancel{\frac{2}{3}\text{LIBOR}} + \frac{2}{3}1\% + \frac{1}{3}K - \cancel{\frac{2}{3}\text{LIBOR}} = 6.5\%$$

$$\frac{1}{3}K = 6.5\% - \frac{2}{3}\%$$

$$K = 3 \times 6.5\% - 2\%$$

$$K = 17.5\%$$

K − L × (One-month LIBOR) = 1.5%

17.5 − 2 LIBOR = 1.5%

2 LIBOR = 16%

LIBOR 8%

cap = 9%

Question 3 (10 points)

Assume that a CMO has just one Planned Amortization Class (PAC) bond, and one supporting bond. The PAC bond is created between PSA 90 and PSA 300. Part of the principal payments are calculated and shown in the following table. Assume that there is a sufficient supporting bond at the beginning, but it will be exhausted by Month 210. Given the actual principal payments in the 4th column, fill out the respective principal payments to this PAC bondholders in the last column, then, explain briefly on your results.

Month	At 90% PSA	At 300% PSA	Actual Principal Payment	*Principal Payment to PAC Bondholder*
1	508,169	1,075,931	600,000	*508,169*
2	569,843	1,279,412	800,000	*569,843*
…				
(The supporting ***bond has*** *gone on Month 210)*				
211	949,282	213,309	200,000	200,000
212	946,033	209,409	300,000	300,000
…				
349	613,875	12,314		0
350	612,392	12,008		0

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PAC

Once the support bond is gone, you take on full amount of payment

For months 1 and 2, because the actual principal payments are between the PAC window, and the supporting bond is available, the PAC bondholders will receive the minimum of the PAC window principal payments.

After the supporting bond is gone, there is no more protection on the PAC bond. Hence the PAC bondholders will receive the actual principal payments.

Question 4 (10 points)

Assume that the yield curve is always flat. The current interest rate level is at 7% (BEY). The U.S. Federal Reserve has indicated that it will cut interest rate gradually over the next two years. Based on this information, you have estimated that the interest rate will fall 50 basis points every 6 months for the next 2 years. You have a two-year investment horizon. You are considering investing in an 8% U.S. Treasury bond which has 20 years to maturity. What annualized return (in BEY) can you realize over two years if you invest in this bond?

Year	BEY	r-Semi	# of years	# of periods	Coupon	Price	Interest	Cum Interest
0.0	0.07	0.035	20	40	8%	110.678	0	0
0.5	0.065	0.0325	19.5	39	8%		0.04	0.04
1.0	0.06	0.03	19	38	8%		0.04	0.0813
1.5	0.055	0.0275	18.5	37	8%		0.04	0.12374
2.0	0.05	0.025	18	36	8%	135.334	0.04	0.16714

$$\text{cum interest per 1} \left(1 + r_{\text{-semi}}\right) + r_1$$

$$\text{Cum Int} = 0.04(1 + 0.0325) + 0.04$$

$$\text{Cum Inter}_4 = 0.0813(1 + 0.03) + 0.04$$

$$0.12374$$

$$\text{horizon return} = \sqrt[n]{\frac{P_n + \text{Cum Int}}{P_0}} - 1$$

$$= \sqrt[4]{\frac{135.334 + 16.714}{110.678}} - 1$$

$$= 0.0826$$

$$\times 2 = 0.1652$$

Question 5 (15 points)

The current term structure of interest rate is given as par-yield, i.e., Bond equivalent yield on par valued coupon bonds with maturities greater than 1 year. Some of the zero-coupon spot rates have been identified (also quoted as BEY).

(a) Calculate the missing spot rates (10 points)
(b) Find the price of a 10% coupon bond with 5 years to maturity (5 points).

Maturity (Year)	Par-yield (BEY)	Spot Rate
0.5	5.250	5.250
1.0	5.500	5.500
1.5	5.750	5.769
2.0	6.000	6.0267
2.5	6.250	6.2822
3.0	6.500	6.5494
3.5	6.750	6.8213
4.0	6.800	6.8694
4.5	7.000	7.0947
5.0	7.100	7.2045

Question 6 (20 points)

Assume all the coupon bonds have just paid out their respective semi-annual interest payments today. Assume further that all the bonds have a par value of $100 and are perfectly divisible (i.e., buying or selling any fraction of a bond unit is possible).

You have a fixed liability of $1,000,000 at the end of the **fifth** year. Assume that you can only invest in two bonds: Bond 1 carries 10% coupon with 10 years to maturity and Bond 2 carries 13% coupon with 5 years to maturity.

2 = 5% coupon

(a) (10 points) The term structure is flat and the yield (expressed as bond-equivalent yield) is 8 percent. Construct an immunization portfolio today.

(b) (10 points) Assume the term structure shifts to 9% flat in six months, check if you are able to meet your obligation, then rebalance your portfolio.

Step 1 PV $C\left[\frac{1-\frac{1}{(1+y)^n}}{y}\right] + \frac{m}{(1+y)^n}$

par value

$0.05 \times 100 \left[\frac{1 - \frac{1}{(1+0.04)^{20}}}{0.04}\right] + \frac{100}{(1.04)^{20}} = 113.2$

again for Bond 2 $= 120.28$

need for liability $\frac{1000000}{(1.04)^{10}}$ → air term to maturity

step 2 = $MD = \frac{1+y}{y} - \frac{1+y+n(c-y)}{c\left[(1+y)^n - 1\right] + y}$

step 3 = weights $\frac{MD_L^{\text{of liability}} - MD_{B2}}{MD_{B1} - MD_{B2}}$

Step 4 units $\underset{(\text{step } 3)}{\text{weights}} \times \frac{\text{PV of liability}}{\text{Price of Bonds (step 1)}}$

step 5 market value = $\underset{\text{step 4}}{\text{unit}} \times \underset{\text{step 1}}{\text{price}}$

semi-annual *

Step 2: $\frac{1+0.04}{0.04} - \frac{1+0.04+20(0.05-0.04)}{0.05\left[(1+0.04)^{20}-1\right] + 0.04} = 13.54$

for liability it's just the maturity

Step 3: $\frac{10-7.89}{13.4-7.89} = 0.373567$ for Bond 1.

0.626424 for Bond 2.

NOTHING of liability.

Question 7 (15 points)

Briefly answer the follwing short questions (3 points each) :

1. What is meant by on-the-run Treasuries? List all of the on-the-run Treasury securities.

2 What is TIPS? Suppose that the coupon rate for a TIPS is 3% and an investor has purchased $10,000 of par value (initial principal) of this issue today and that the anual inflation rate is 2%. What is the dollar coupon interest that will be paid in cash at the end of the first six months?

3. What is a sinking fund requirement in a bond issue? How does it work?

4. What does the yield spread between commercial paper and Treasury bills of the same maturity reflect?

5. The following excerpt is taken from an article titled "MERUS to Boost Corporates," which appeared in the January 27, 1992, issue of *BondWeek*, p.6:

> MERUS Capital Management will increase the allocation to corporates in its $790 million long investment-grade fixed-income portfolio by $39.5 million over the next six months to a year, according to George Wood, managing director. MERUS will add corporates rated single A or higher in the expectation that spreads will tighten as the economy recovers and that some credits may be upgraded.

What types of active portfolio strategies is MERUS Capital Management pursuing?

Formulas:

Present value interest factor (for $1 at time n): $PVIF(r,n)=\frac{1}{(1+r)^n}$

Present value of interest factor of annuity: PVIFA (r, n) = $\frac{1-\frac{1}{(1+r)^n}}{r}$

Future value interest factor (for $1 at 0): $FVIF(r,n)=(1+n)^n$

Future value interest factor of annuity: $FVIFA(r,n)=\frac{(1+r)^n-1}{r}$

Standard Bond Pricing: $P=\left[\frac{c}{2}\times PVIFA(r_s,n)+\frac{100}{(1+r_s)^n}\right]$

where r_s = BEY/2, since the bond yield is quoted as bond equivalent yield, i.e., yield-to-maturity with semiannual compounding.

$P_{quoted}=P_{invoice}-AI$

$P_{invoice}=(1+r_s)^u\left[\frac{c}{2}\times PVIFA(r_s,n)+\frac{100}{(1+r_s)^n}\right]$

where u = (# of days since last coupon payment)/(# of days between coupon payments)

Accrued Interest AI = $\frac{c}{2}\times u$.

Interest rate conversion: if r is the annualized rate or EAR, R is annualized continuous compounding rate, then, other rates can be solved from the following relationship:

$1+r=(1+r_s)^2=(1+r_q)^4=(1+r_m)^{12}=(1+r_{sm})^{24}=(1+r_{bw})^{26}=(1+r_d)^{365}=e^R$.

If the rate is quoted as APR with m times compounding per year, it must be interpreted as

$r_{period}=APR/m$. Conversely, APR = m*r_{period}.

For example, APR with quarterly compounding must be interpreted as $r_q=APR/4$, and APR = $4r_q$.

Bond Pricing by Spot Rates: $P=\sum_{t=1}^{n}\frac{CF_t}{(1+r_t)^t}$

Bond Equivalent Yield: $Y_{BEY}=2\times\left[\sqrt[t/182.5]{F/P}-1\right]$.

Horizon Return = $\sqrt[n]{\frac{P_n+\text{Cumulative Interest}}{P_0}}-1$.

Macaulay Duration for coupon bond:

$$D = \frac{1+y}{y} - \frac{1+y+m(i-y)}{i[(1+y)^m - 1] + y}.$$

In the above formulas, $c = iF$ with i being the semiannual coupon interest rate (i.e., one-half of the quoted annual coupon interest rate), y is the semiannual discount rate, and m is the number of half-years. The result is in unit of half-years. Divide this result by 2 to convert it to number of years.

Modified Duration = Macaulay Duration / (1+y)

Duration Approximation: $\text{Modified D} \approx \frac{P_+ - P_-}{2P_0(\Delta y)} = \frac{P_+ - P_-}{P_0(y_+ - y_-)}$

Convexity measure approximation $\approx \frac{P_+ + P_- - 2P_0}{P_0(\Delta y)^2}$

Approximating the percentage change of bond price:

$$\Delta P / P = -(\text{Modefied D}) \times \Delta y + \left[\tfrac{1}{2}(\text{Convexity}) \times (\Delta y)^2\right]$$

Duration of bond portfolio:

$$D_P = \sum_i w_i D_i \text{ with } \sum_i w_i = 1.$$

The 100 PSA Conditional Prepayment Rate:

$$CPR = \begin{cases} 6\% \times (t/30), & t \le 30 \\ 6\%, & t > 30 \end{cases}$$

Other PSA CPR will be multiple of the above, e.g., the 150 PSA CPR is 1.5 times the above calculation.

Single-month-mortality rate: $SMM = 1 - (1 - CPR)^{1/12}$, where the CPR corresponds to the respective PSA.

Mortgage Payment: $PMT = (MB)\frac{r(1+r)^n}{(1+r)^n - 1}.$

Scheduled Principal Payment = Mortgage PMT – Interest (i.e., r×MB)

$\frac{WAC}{12}$ × out bal

Prepayment = SMM×(MB – Scheduled Principal Payment)

Cash flow = interest + total principal payments.

Inverse Floater receives $K - L \times LIBOR$, where the inverse floater counts for L/(1+L) of the fixed-rate trenche.

The notional amount of x% (0.0x) IO $= \frac{\text{Tranche's Par Value} \times \text{Excess Interest}}{0.0x}.$

Question 8 (15 points)

Consider the following CMO structure backed by 8% collateral:

Tranche	Par Amount (in millions)	Coupon Rate (%)	Excess Interest	Notional Amount
A	$300	6.50%	1.5%	56250
B	$250	6.75%	1.25%	39062.5
C	$200	7.25%	0.75%	18750
D	$250	7.75%	0.25%	7812.5
	1,000,000			121875

Suppose that a client wants a notional IO with a coupon rate of 8%. Calculate the notional amount for this notional IO.

$$\frac{\text{Tranche's Par value} \times \text{Excess interest}}{0.08}$$

$$\frac{300 \times 0.015}{0.08}$$

Question 9 (15 points)

An issuer is considering the following two CMO structures:

STRUCTURE I:

Tranche	Par Amount (in millions)	Coupon Rate (%)
A	$150	6.50%
B	$100	6.75%
C	$200	7.25%
D	$150	7.75%
E	$100	8.00%
F	$500	8.50%

Tranches A to E are a sequence of PAC I's, and F is the support bond.

STRUCTURE II:

Tranche	Par Amount (in millions)	Coupon Rate (%)
A	$150	6.50%
B	$100	6.75%
C	$200	7.25%
D	$150	7.75%
E	$100	8.00%
F	$200	8.25%
G	$300	? 8.6667

$F_I = F_{II} + G_{II}$

$8.5 = \frac{2}{5}\, 8.25 + \frac{3}{5} X$

Tranches A to E are a sequence of PAC I's, F is a PAC II, and G is a support bond without a PAC schedule.

(a) In structure II, tranche G is created from tranche F in structure I. What is the coupon rate for tranche G assuming that the combined coupon rate for tranches F and G in structure II should be 8.5%? weighted average

(b) What is the effect on the value and average life of tranches A to E by including the PAC II in structure II?

(c) What is the difference in the average life variability of tranche G in structure II and tranche F in structure II?