Math 200 Section 104 (1 PM) (Peterson)

MATH 200: TEST 2 (12 October 2011)

Q.1	Q.2	Q.3	Q.4	Total

Name: ______________ St#: __________ SOLUTIONS

No books, calculators or notes allowed.

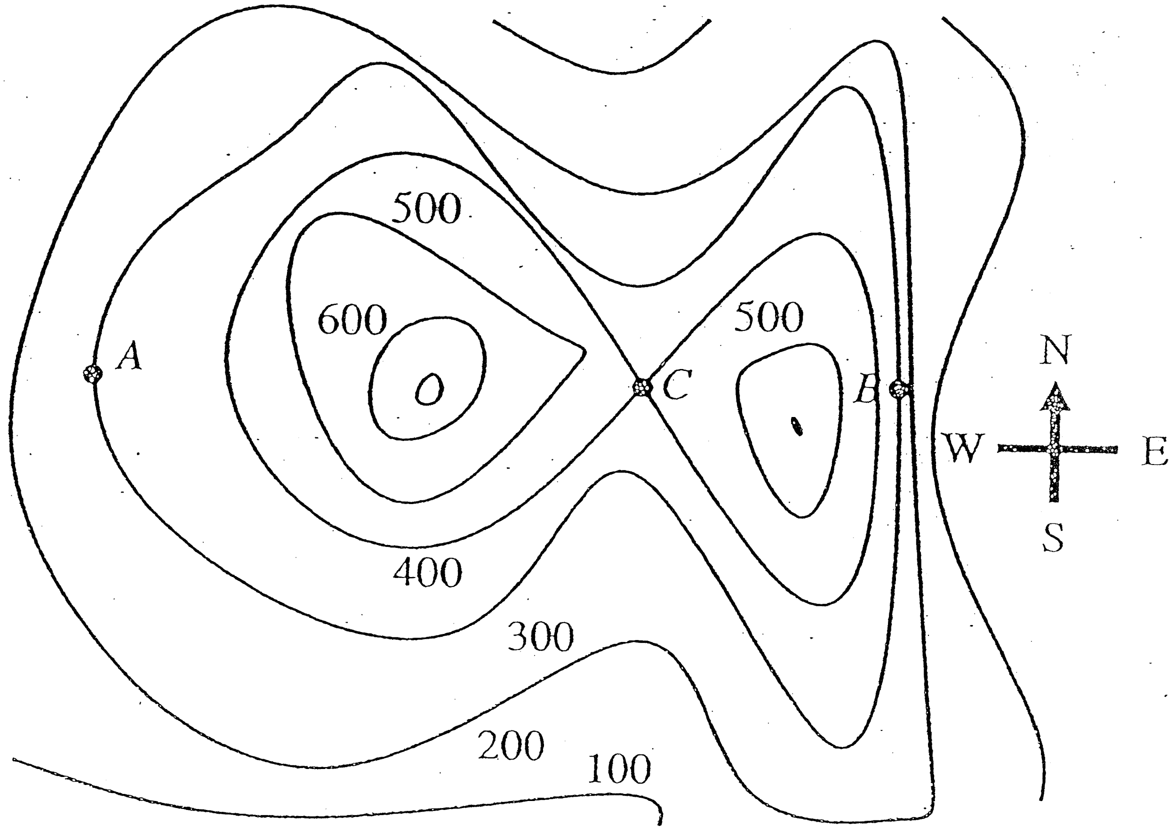
Show enough work to justify your answers.

1. (20 marks)

The figure below shows contours of a hilly region with heights given in metres.

(a) At which of the points A and B is the landscape steeper?

(b) Describe the topography of the region near point C.



(a): steeper near B.

(b): mountain pass or saddle point

2. (20 + 10 marks)

(a) If $z = z(x, y)$, find $\frac{\partial z}{\partial y}$ when $x^2 + 2xz^2 - yz^3 = 1$

$$x^2 + 2xz^2 - yz^3 = 1$$

$$\frac{\partial}{\partial y}: \quad 2x\left(2z\frac{\partial z}{\partial y}\right) - z^3 - y\left(3z^2\frac{\partial z}{\partial y}\right) = 0$$

$$(4xz - 3yz^2)\frac{\partial z}{\partial y} = z^3$$

$$\frac{\partial z}{\partial y} = \frac{z^3}{4xz - 3yz^2}$$

(b) Let $z = f(u) + g(v)$, where $u = 2s+3t$ and $v = s-6t$, and f and g are functions of x satisfying $f''(7) = -2$ and $g''(-4) = 3$. Find the value of $\frac{\partial^2 z}{\partial t^2}$ when $s = 2$ and $t = 1$.

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial t} f(u) + \frac{\partial}{\partial t} g(v)$$

$$= f'(u)\frac{\partial u}{\partial t} + g'(v)\frac{\partial v}{\partial t}$$

$$= 3f'(u) - 6g'(v)$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t}\left(3f'(u) - 6g'(v)\right)$$

$$= 3\frac{\partial}{\partial t} f'(u) - 6\frac{\partial}{\partial t} g'(v)$$

$$= 3f''(u)\frac{\partial u}{\partial t} - 6g''(v)\frac{\partial v}{\partial t}$$

$$= 9f''(u) + 36g''(v)$$

At $s=2, t=1$, we have $u = 2s+3t = 7$, $v = s-6t = -4$, so

$$\left.\frac{\partial^2 z}{\partial t^2}\right|_{(s,t)=(2,1)} = 9f''(7) + 36g''(-4) = 9\cdot(-2) + 36\cdot 3 = 90$$

Problem 3 (30 marks)

Consider the surface $z = x^2 + xy - y^3 + 4$.

(a) Find an equation for the tangent plane at the point $(1, -1, 5)$.

(b) Find an approximate value for a point y on this surface, given that $x = 0.9$ and $z = 5.2$.

(c) Find all points on this surface where the tangent plane is parallel to the plane

$y + z = 4$.

(a) $\partial z/\partial x = 2x + y$, $\partial z/\partial y = x - 3y^2$, so at $P(1,-1,5)$,

$\partial z/\partial x = 1$, $\partial z/\partial y = -2$, so tangent plane is

$$z - 5 = 1(x-1) - 2(y+1).$$

(b) By tangent approximation, $x = 0.9$ and $z = 5.2$ give

$5.2 - 5 \approx 1(0.9 - 1) - 2(y+1)$, so

$y + 1 \approx -.15$ and $y \approx -1.15$.

(c) $\vec{n}_1 = \langle \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \rangle = \langle 2x+y, x-3y^2, -1 \rangle$

is a multiple of $\vec{n}_2 = \langle 0, 1, 1 \rangle$. $\vec{n}_1 = c\vec{n}_2$ gives

$2x+y = 0$, $x - 3y^2 = c$, $-1 = c$, so $c = -1$ and

$2x+y = 0$, $x - 3y^2 = -1$. $x - 3y^2 = -1$ implies

$x = 3y^2 - 1$. Plugging this into $2x+y = 0$ gives

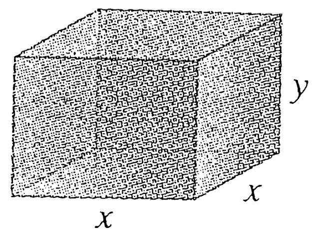
$6y^2 + y - 2 = 0$, so $y = \frac{-1 \pm \sqrt{49}}{12} = \frac{-1 \pm 7}{12}$,

i.e. $y = \frac{1}{2}$, so $\boxed{(x,y,z) = (-\frac{1}{4}, \frac{1}{2}, \frac{61}{16})}$;

or $y = -\frac{2}{3}$, so $\boxed{(x,y,z) = (\frac{1}{3}, -\frac{2}{3}, \frac{113}{27})}$.

4. (20 marks)

Consider a closed rectangular box with a square base as shown in the accompanying figure. If x is measured with error at most 2% and y is measured with error at most 3%, use a differential to estimate the corresponding percentage error in computing the box's volume.



$$V = x^2 y$$

$$dV = 2xy\,dx + x^2\,dy$$

$$\frac{dV}{V} = 2\frac{dx}{x} + \frac{dy}{y}$$

Taking $dx = \Delta x$ and $dy = \Delta y$, we get

$$\left|\frac{\Delta V}{V}\right| \approx \left|\frac{dV}{V}\right| = \left|2\frac{dx}{x} + \frac{dy}{y}\right|$$

$$\leq 2\left|\frac{dx}{x}\right| + \left|\frac{dy}{y}\right|$$

$$= 2(.02) + (.03)$$

$$= .07$$

Maximum possible percentage error is about 7%.