MAT 1330, Fall 2014 Assignment 1
Due Thursday September 25 by 9:00pm.
Late assignments will not be accepted; nor will unstapled assignments. Professors in the math department will not lend you a stapler; do not ask for one.

Instructor (circle one): Frithjof Lutscher   Jason Levy   Farnoosh Babazadeh   Jeff Musgrave

DGD (circle one): 1   2   3   4

Student Name ____________________ Student Number ____________________
By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ____________________

QUESTION 1. Suppose someone has three drinks of alcohol that bring the alcohol content in their body to 42 grams. Each hour, 45% of the alcohol are eliminated from the body. Write the DTDS for the amount of alcohol in the body on an hourly basis. Identify the initial condition and give the general solution. Suppose further that the amount of alcohol in the body has to be below 8 grams before one can drive. Assuming that they have no more drinks, how long does the person have to wait before they can drive?

Answer: The DTDS is $x_{t+1} = \boxed{0.55\, x_t}$

The initial condition is $x_0 = \boxed{42}$

The solution is $x_t = \boxed{(0.55)^t \cdot 42}$

The person has to wait for at least $\boxed{3}$ hours. [2 hours, 50 min]

Show your work here.

Find t such that $x_t < 8$ or $0.55^t \cdot 42 < 8$

$$0.55^t < \frac{8}{42} = \frac{4}{21}$$

$$t \ln(0.55) < \ln\left(\frac{4}{21}\right)$$

$$t > \frac{\ln\left(\frac{4}{21}\right)}{\ln(0.55)} \approx 2.7737$$

note: $\ln(0.55) < 0$!

QUESTION 2. A population of butterflies lives on a meadow, surrounded by forest. We want to investigate the dynamics of the population. We denote the number of butterflies at the beginning of season t by x_t. Over the course of a season, 30% of the butterflies that were there at the beginning die. During each season, 20 new butterflies arrive from other meadows.

a) Write the DTDS for the number of butterflies, i.e. $x_{t+1} = \boxed{0.7 x_t + 20}$

b) The updating function is $f(x) = \boxed{0.7x + 20}$

c) Starting with 40 butterflies in season 0, calculate their number in seasons 1, 2, 3.

$x_1 = \boxed{48}$ $x_2 = \boxed{53.6}$ $x_3 = \boxed{57.52}$

d) The fixed point of the DTDS is $x^* = \boxed{\frac{20}{1-0.7} = 66.67}$

e) The solution of the DTDS in terms of a general initial condition x_0 is

$x_t = \boxed{(0.7)^t (x_0 - x^*) + x^*}$

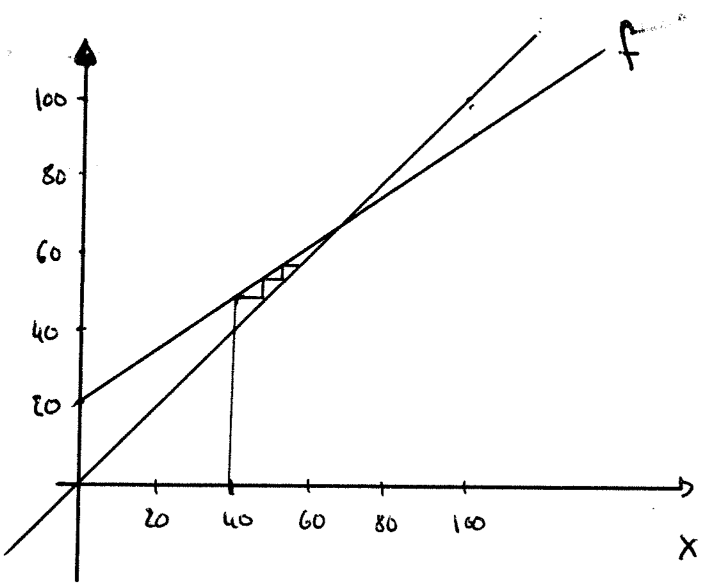
f) In the space below, draw the cobweb for this DTDS, starting at $x_0 = 40$. Also draw the solution as a function of time.

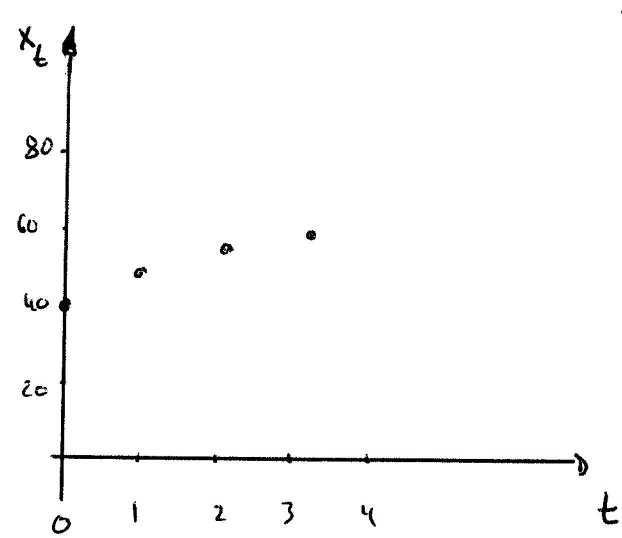
g) Suppose that through some conservation measures, we can improve the quality of the pond and reduce the death rate of the butterflies. To which level do we have to reduce the death rate if we want the steady state butterfly population to be 100? Answer: The death rate should be $\boxed{20}$%. Let d be the death rate.

Draw your cobweb and solution here. The DTDS is $x_{t+1} = (1-d)x_t + 20$

The steady state is $x^* = \frac{20}{1-(1-d)} = \frac{20}{d}$

So $d = \frac{1}{5}$





QUESTION 3. Consider the nonlinear DTDS $x_{t+1} = \frac{rx_t}{1+0.2x_t}$, where $r > 0$ is some parameter.

a) The fixed point(s) of this DTDS is (are) $\boxed{X=0 \text{ and } x = 5(r-1)}$
(Note: the answer may contain parameter r.)

b) Now set $r = 3$. Calculate x_1, x_2, x_3 starting form $x_0 = 8$.

$x_1 = \boxed{9.23}$ $x_2 = \boxed{9.73}$ $x_3 = \boxed{9.91}$

c) Keeping $r = 3$, calculate x_1, x_2, x_3 starting form $x_0 = 12$.

$x_1 = \boxed{10.59}$ $x_2 = \boxed{10.19}$ $x_3 = \boxed{10.06}$

d) Do you observe a trend in these values?

When $x_0 < 10$, the values increase, but stay below 10

When $x_0 > 10$, the values decrease, but stay above 10

The positive fixed point is $x = 10 = 5(3-1)$
So, the fixed point is stable.

Show your work here.

$$X = \frac{rx}{1+0.2x}$$ has solution $x=0$ and $1+0.2x = r$ or $x = 5(r-1)$