

MECH 325 2012 Midterm
Part 1a
recap

Oct. 23, 2012

1. Which of the following types of gears is appropriate for high-efficiency, smooth (low-noise) transmission of motion between non-parallel shafts with minimal axial loading on either shaft?

- a) Bevel
- b) Spur
- c) Worm
- d) Herringbone
- e) Helical

2. A spur gear has 30 teeth, a 20° pressure angle, and a pitch diameter of 1.5 in. What is the diametral pitch of the gear?

- a) 0.075
- b) 0.667
- c) 1.50
- d) 13.33
- e) 20.0

$$P = \frac{N}{d} = \frac{30 \text{ teeth}}{1.5 \text{ in}} = 20 \frac{\text{teeth}}{\text{inch}}$$

3. The worm gear shown has a 1" pitch diameter and has four threads. The gear has an 8" pitch diameter and 48 teeth. If the worm turns 12 revolutions in the direction indicated, how far does the gear turn?

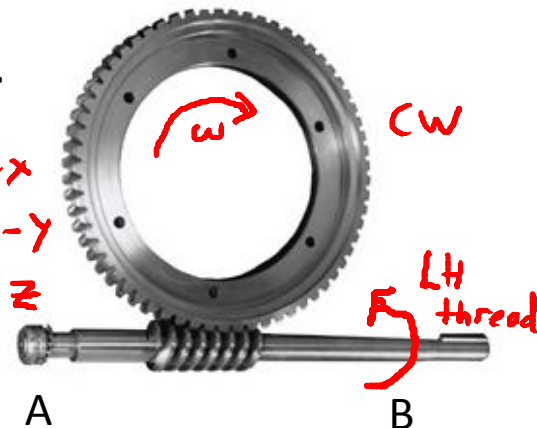
- a) 1 rotation clockwise
- b) 3 rotations clockwise
- c) ½ rotation clockwise
- d) 1 rotation counter-clockwise
- e) 3 rotations counter-clockwise

$$e = \frac{4 \text{ threads}}{48 \text{ teeth}} = \frac{1}{12}$$

4. In the worm from Question 3, the bearing at A is the locating bearing. direction are the forces from the worm shaft onto bearing A?

- a) -x +y -z
- b) +x -y +z
- c) +x -y -z
- d) -x -y +z
- e) -x -y -z

rotation of gear blocked \Rightarrow worm +x
 rotation of worm \Rightarrow force on shaft -y
 rotation of worm \Rightarrow force in -z



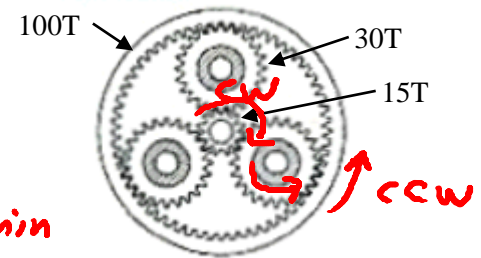
5. A spur gearset has an input gear with 15 teeth and an output gear with 60 teeth. If 4 N·m of torque is supplied to the input gear at 50 rad/sec, approximately how much power is delivered by the output gear to the load?

$$H = T\omega = 4 \cdot 50 = 200 \text{ W}$$

- a) 16 W
 b) 60 W
 c) 75 W
 d) 200 W
 e) 800 W
6. For the planetary gear train shown, the sun gear rotates at 1000 rpm clockwise and the arm is fixed. How fast does the ring gear rotate and in which direction?

$$e = \frac{15}{30} \frac{30}{100} = \frac{15}{100}$$

$$n = \frac{15}{100} \cdot 1000 = 150 \text{ rev/min}$$



7. Comparing a square vs. an ACME power screw, which of the following is true regarding the use of a split-nut:

- a) A split nut reduces friction on both types of power screw
 b) A split nut allows for backlash adjustment on an ACME power screw
 c) A split nut allows for backlash adjustment on a square power screw
 d) A split nut is required when using a bushing for a friction collar on both types of power screw
 e) A split nut is required when an ACME power screw is non-reversing

8. Which of the following components of belt tension present in V-Belts but normally ignored in flat belts that is the main consideration when calculating belt life?

- a) Initial tension (F_i)
 b) Bending tension (F_b)
 c) Centrifugal tension (F_c)
 d) Tight-side tension (F_1)
 e) Slack-side tension (F_2)

V-belt has taller cross-section than flat belts

9. A V-belt drive has a driving pulley operating at 20 rpm, a drivetrain value of $e = 0.25$, a nominal power of $H_{nom} = .75$ hp, a service factor of $K_s = 2.5$, and a design factor of $n_d = 1.5$. TWO belts are to be used in parallel. The following belts are available in the catalogue. Which is the smallest belt that will satisfy the design requirements?

- a) Allowable power $H_a = 0.7$ hp
- b) Allowable power $H_a = 1.1$ hp
- c) Allowable power $H_a = 1.5$ hp
- d) Allowable power $H_a = 2.9$ hp
- e) Allowable power $H_a = 5.7$ hp

$$H_d = K_s \cdot n_d \cdot H_{nom} = 2.5 \cdot (1.5) (0.75) = 2.81 \text{ HP}$$

$$H_{all} = \frac{H_d}{N_{belts}} = \frac{2.81}{2} = 1.405 \text{ HP}$$

10. A timing belt has a driving pulley with pitch diameter of 20 cm and 40 teeth, and a driven pulley with 80 teeth. The wrap angle is 120° . The tight side tension is 100 N, slack side tension is 40 N, and the driving pulley rotation rate is 100 rad/sec. What is the torque transmitted from the motor shaft to the driving pulley?

- a) 6.0 N·m
- b) 9.0 N·m
- c) 60.0 N·m
- d) 100.0 N·m
- e) 600 N·m

$$T = (F_1 - F_2) \frac{d}{2} = (100 - 40) \left(\frac{0.2}{2} \right) = 6 \text{ Nm}$$

11. Which of the following design or operating parameters should be increased in order to be able to reduce the required bushing length of a boundary-lubricated bearing in service? (assume all other parameters remain fixed)

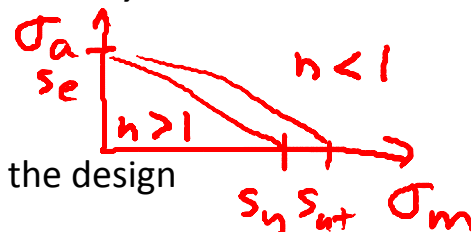
- a) Operating rotation speed, N
- b) Applied force, F
- c) Coefficient of friction, f_s
- d) (PV) rating of the bushing
- e) Wear factor, K

12. An angular contact ball bearing is best suited for:

- a) Only large radial loads
- b) Only large axial loads
- c) Small radial loads, plus large axial loads in both directions
- d) Large radial loads, plus large axial loads in both directions
- e) Large radial loads, plus large axial loads in one direction only**

13. Which of the following statements is true for the Soderberg, Gerber, Modified Goodman and ASME-elliptic fatigue failure criteria:

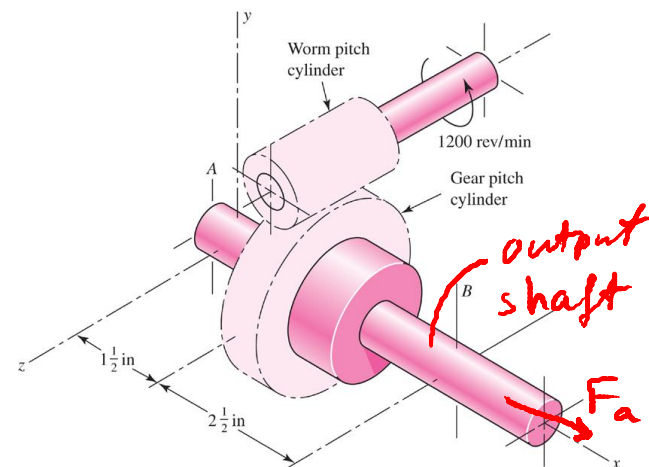
- a) All criteria predict fatigue failure will occur before yielding
- b) Only the Modified Goodman criterion is valid for tensile stresses
- c) The zone to the left of each of the fatigue criteria lines represents the design domain where $n < 1$
- d) When $\sigma_a = 0$, the safety factor $n \geq 1$ when $\sigma_m \geq S_{ut}$ for all criteria**
- e) When $\sigma_m = 0$, the safety factor $n = 1$ when $\sigma_a = 0.5 * S_e$ for all criteria



Freebie

14. A motor drives a gearbox input shaft (top shaft in figure to right) at 1200 rpm. The gearbox uses a worm gear to drive a constant torque load attached to the output shaft via a flexible coupling. In addition to torsional shear and direct shear due to the worm gearset, the output shaft is subject to stresses best characterized by:

- a) steady bending stress plus alternating axial stress
- b) steady bending stress plus steady axial stress
- c) zero midrange stress and steady axial stress**
- d) partially but not fully-reversed bending stress only
- e) alternating bending stress in the z-axis only, plus alternating axial stress



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15. The larger helical gear shown has a normal pressure angle of 20° , a helix angle ψ of 30° , $N = 35$ teeth, and a pitch diameter of $5''$. What is the normal pitch in inches? Show your calculations. (2 marks)

$$P_n = P_t \cos \psi ; P_t P_t = \pi$$

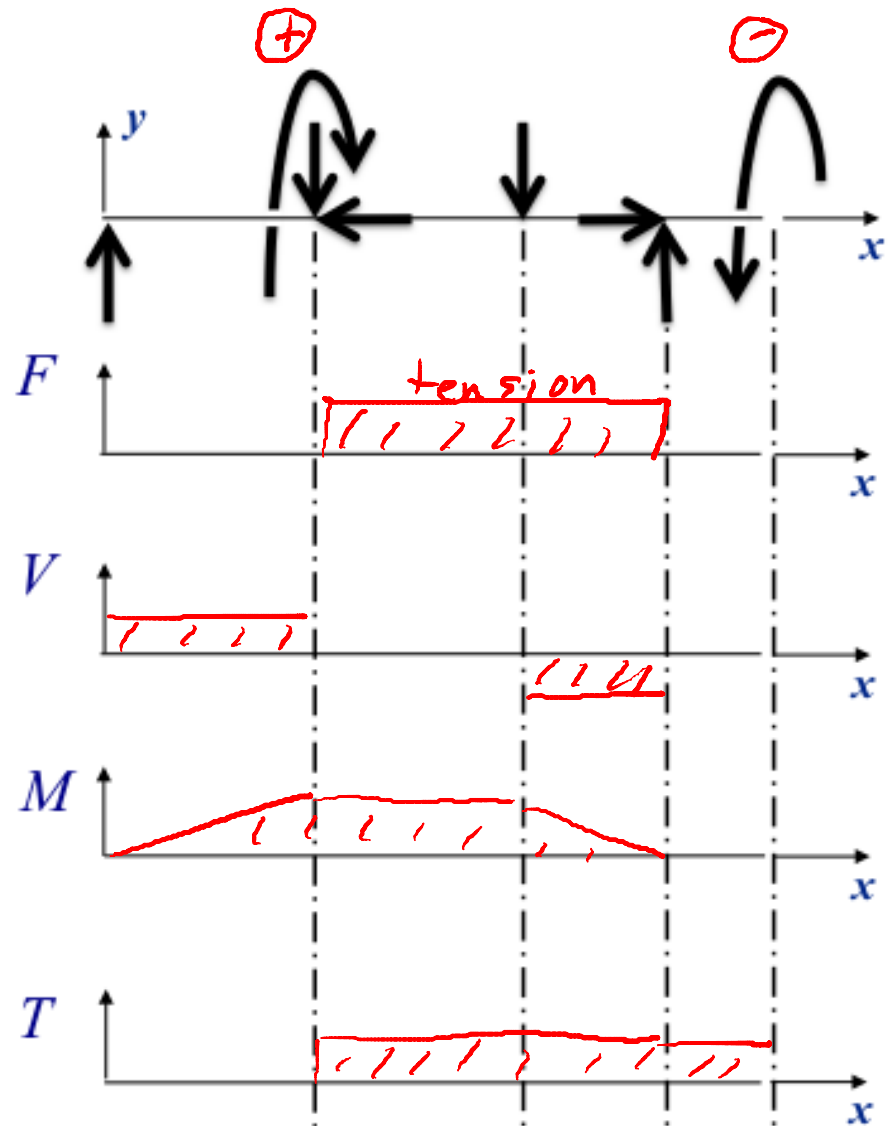
$$P_t = \frac{35}{5} = 7 \frac{\text{teeth}}{\text{inch}}$$

$$P_t = \frac{\pi}{P_t} = \frac{\pi}{7}$$

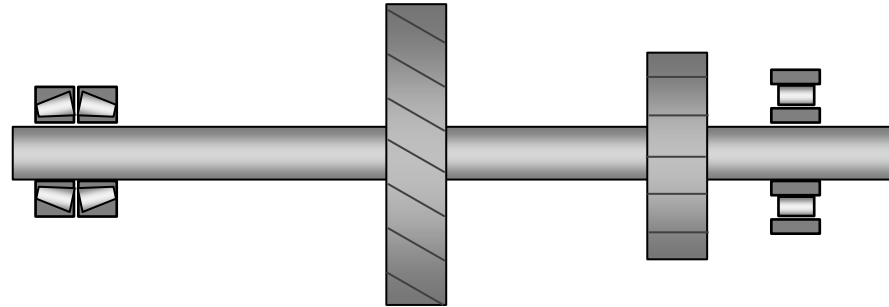
$$P_n = \frac{\pi}{7} \cos 30^\circ = \frac{\pi}{7} \cos 30^\circ = 0.388 \text{ inch}$$



16. Based on the shaft loading shown in the top diagram, complete the diagrams for axial force, shear force, bending moment and torsion. Notes: The moments shown are oriented along the x-axis. You do not need to indicate values in your diagrams, but they must follow the usual sign convention. (2 marks)



18. The shaft shown has a pair of tapered roller bearings, a helical gear, a spur gear, and a cylindrical roller bearing. Without making any changes to these 4 elements, what are the TWO most significant changes you would suggest to this design and why? (2 marks)



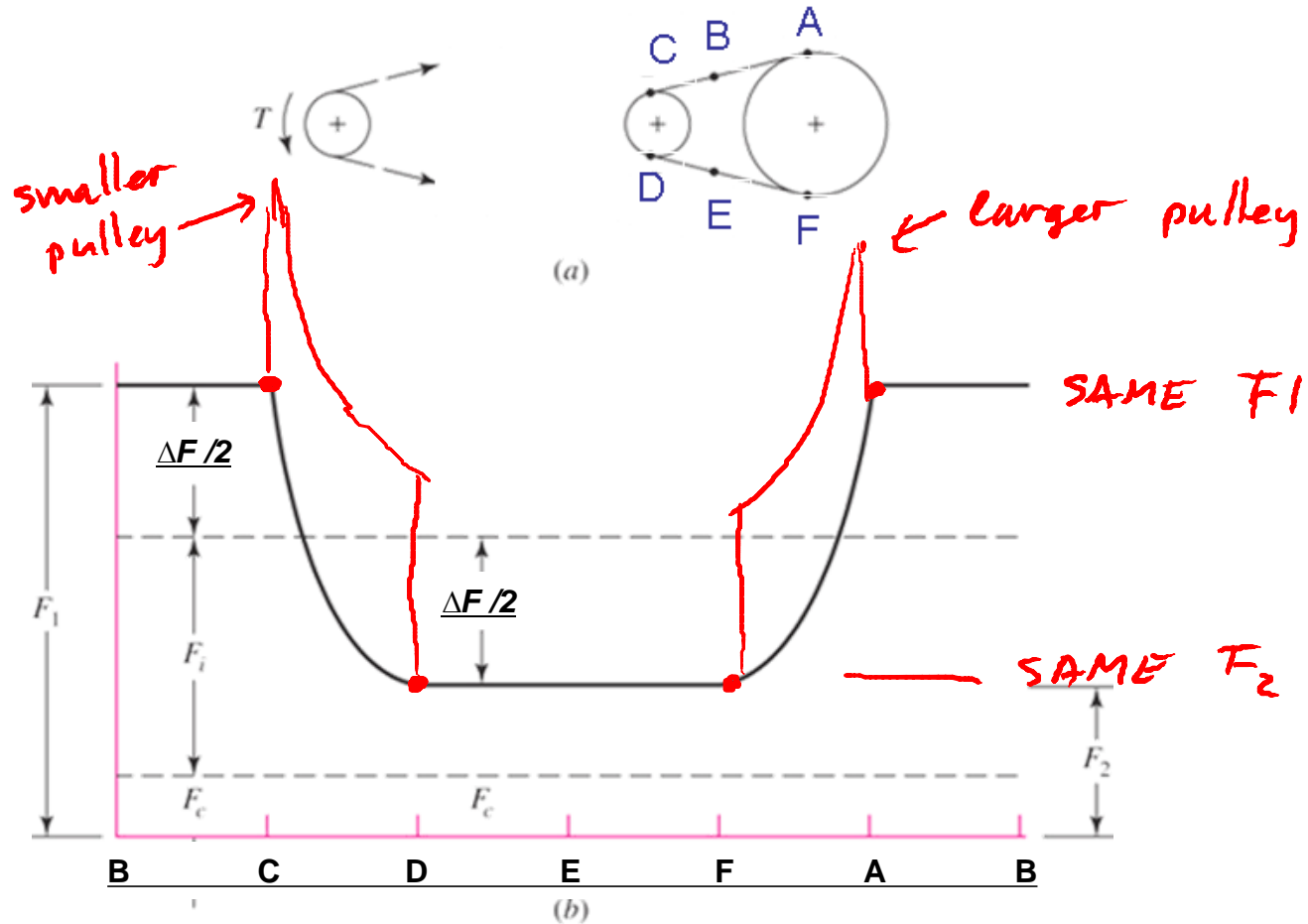
Change #1:

- Make shaft shorter
- or - Put components closer to bearings

Change #2:

- Add axial constraint (on left bearing)

19. The diagram shown below is the tension profile for a flat belt. On this diagram, sketch the tension profile for a V-belt belt that is transmitting the same torque, but otherwise has the same linear speed, initial tension, pulley diameters, and belt weight per unit length. (2 marks)



Question 20:

- A) For synchronous power transmission with a flexible element, what is the principal *operating* difference between roller chains and timing belts? Exclude materials and secondary effects such as lubrication, dirt, and maximum capacity. (1 mark)

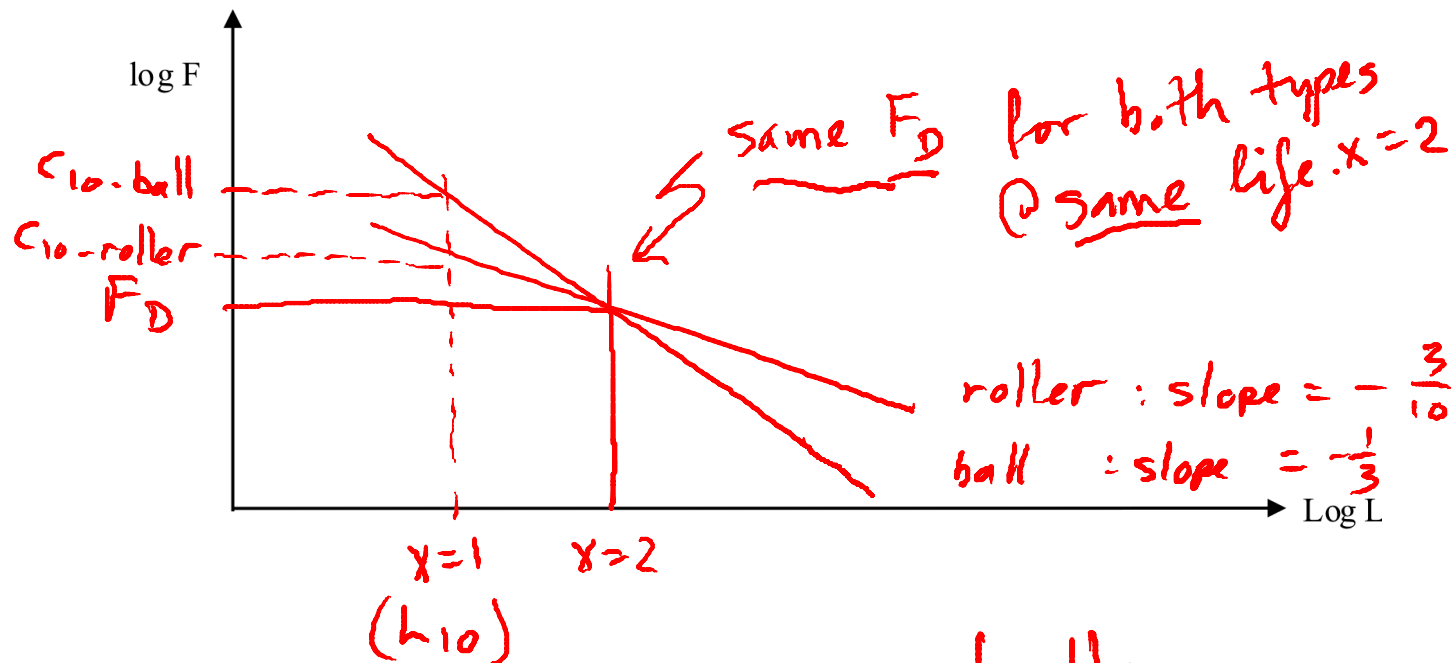
Roller chains have chordal speed variation



- B) What is the simplest way to minimize this difference? (1 mark)

- increase # teeth of sprocket, or
- increase diameter of sprocket

21. An engineer has calculated a design bearing load, F_D . The desired reliability is $R = 0.90$, and the desired life is $x = 2.0$. However, it is not yet clear whether a deep-groove ball bearing or a cylindrical roller bearing should be implemented. If the two options are to have the same design load F_D , help the engineer determine which bearing type will have a higher C_{10} rating. Sketch the 90% reliability load-life curves for a ball bearing and a cylindrical roller bearing. Label pertinent ordinate values (F_D , $C_{10\text{-ball}}$ and $C_{10\text{-roller}}$) and abscissa values ($x = 1$, $x = 2$). Indicate below which type will have the higher C_{10} rating. (2 marks)

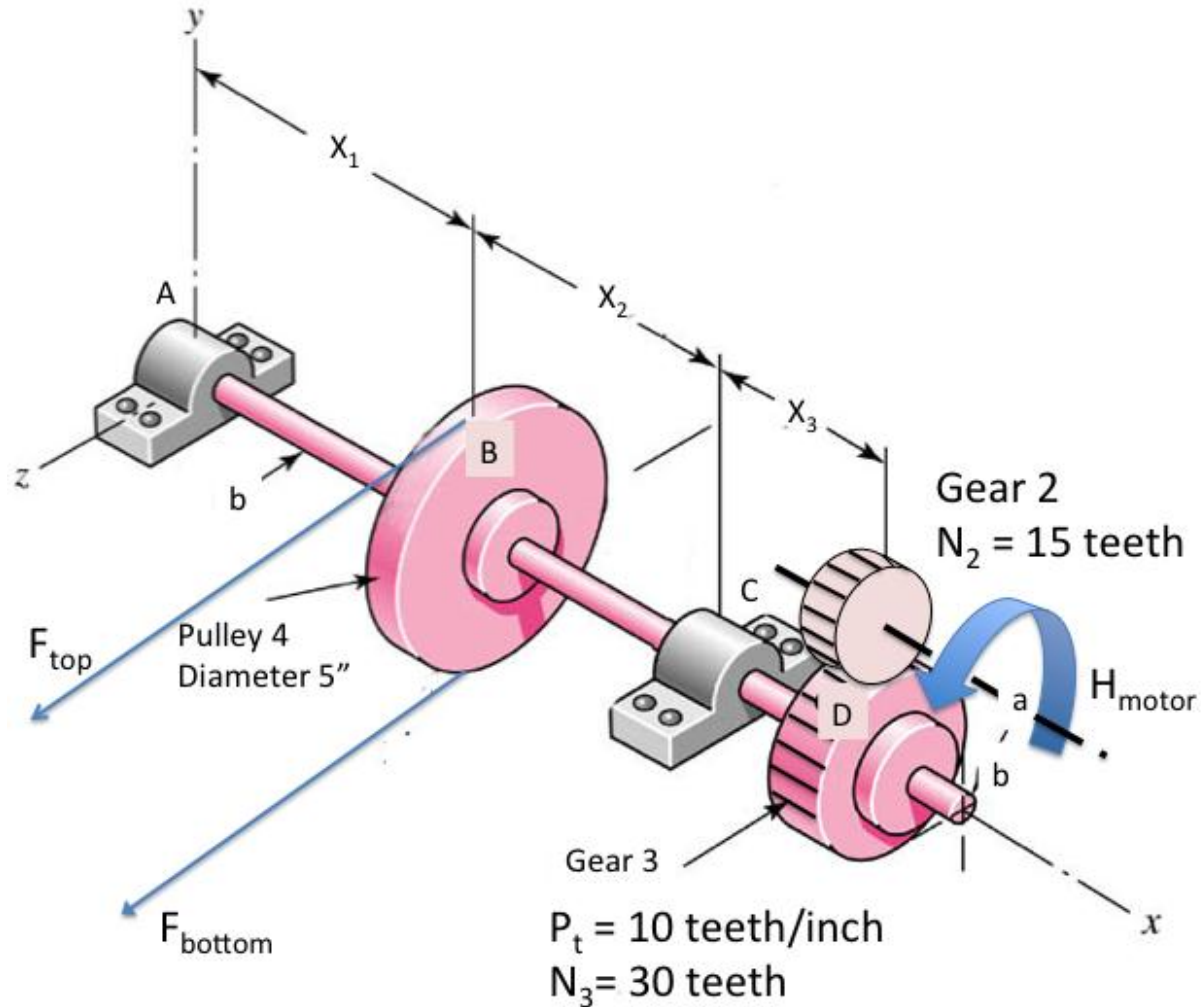


Bearing type with higher C_{10} catalogue rating: ball:

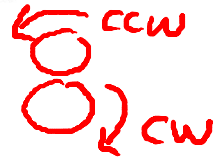
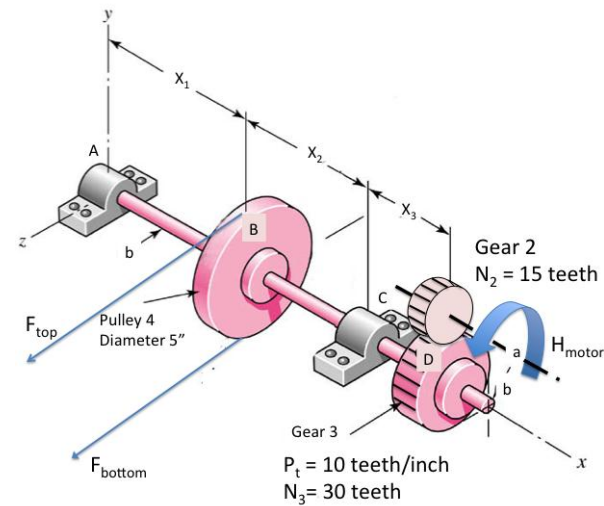
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Part 2 (open-book)



22. The motor connected to shaft (a) delivers a power of $H_{nom} = 2.00$ kW at a speed of 3000 revolutions per minute in a counter-clockwise direction, as shown in the figure. Spur gears 2 and 3 deliver the power with an efficiency of 95% to shaft (b) at location D. Shafts (a) and (b) are aligned on the y-axis. The V-belt on Pulley 4 at location B has a wrap angle of 180° . Forces F_{top} and F_{bottom} are parallel to the z-axis.



Determine the direction of rotation of shaft b, the magnitude of the torque on shaft (b) delivered to the V-belt sheave (4) and the linear speed of the V-belt in ft/min. (3 marks)

shaft b rotates CW

$$H_b = \text{efficiency} \cdot H_a = 0.95 \cdot 2000 = 1900 \text{ W}$$

$$n_b = e \cdot n_a = 0.5 (3000) = 1500 \text{ rev/min}$$

$$\omega_b = \frac{2\pi}{60} n_b = \frac{2\pi}{60} \cdot 1500 = 157.08 \text{ rad/sec}$$

$$T_b = \frac{H_b}{\omega_b} = \frac{1900}{157.08} = 12.10 \text{ Nm}$$

$T_b = 12.10 \text{ Nm}$

$$V = n_b d_4 \pi = 1500 \frac{\text{rev}}{\text{min}} \cdot \frac{5}{12} \pi = \mathbf{1963 \text{ ft/min} = V_{\text{belt}}}$$

23. Regardless of the answers to the previous question, assume $T_b = 8 \text{ Nm}$ at a rotational speed of 155 rad/sec in a clockwise direction. Assume a "V V" configuration (V-belt sheaves on driving and driven ends), so $K_1 = 1$. Assume a length factor of $K_2 = 1$, a service factor $K_s = 1.3$ and a design factor of 1.0.

Determine the smallest type of belt that will suffice from Table 17-12 (from types A through E), and justify your answer. (3 marks)

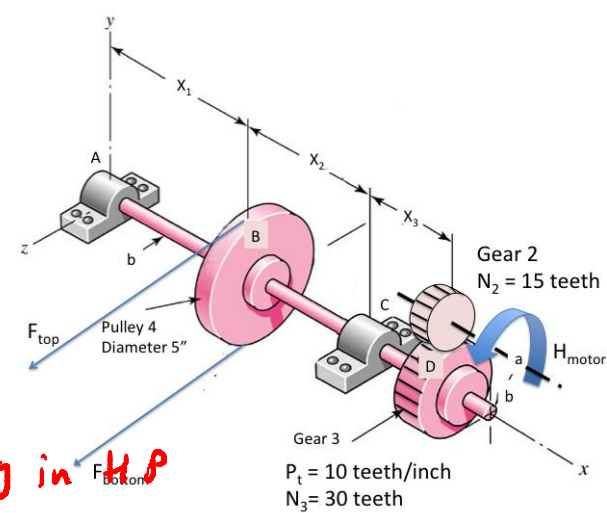


Table needs V-limit & power rating in HP

$$H_{nom} = T \omega = 8 \text{ Nm} \cdot 155 \frac{\text{rad}}{\text{sec}} = 1240 \text{ W} = 1.66 \text{ HP}$$

$$H_d = K_s H_{nom} = 1.3 \cdot (1.66) = 2.16 \text{ HP}$$

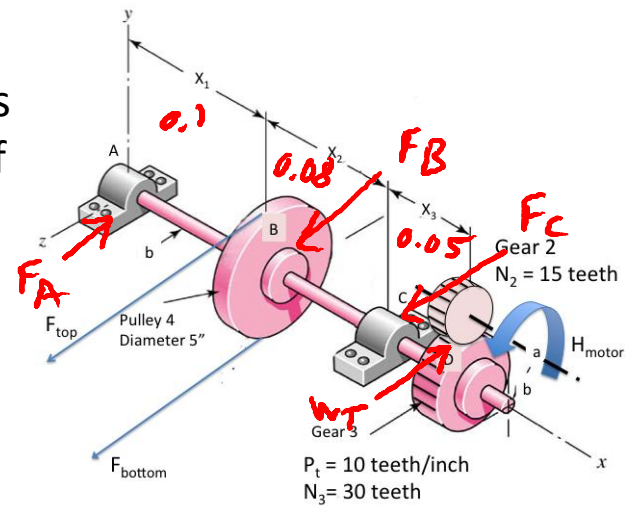
$$n_b = \frac{60}{2\pi} \omega_b = \frac{60}{2\pi} \cdot 155 = \underline{1480 \text{ rev/min}}$$

$$V = n_b \frac{\pi d}{12} = 1480 \frac{\pi \cdot 5}{12} = 1937 \text{ ft/min}$$

⇒ Belt 'B' @ 5" sheave (from drawing)

$$\text{HP rating} = 2.33 \text{ HP}$$

25. Regardless of the answers to the previous question, assume $F_{top} = 300\text{ N}$ and $F_{bottom} = 1000\text{ N}$. The geometry of the shaft is as follows: $x_1 = 10\text{ cm}$; $x_2 = 8\text{ cm}$; $x_3 = 5\text{ cm}$. The pressure angle of the gears is 20° and $W_t = 1167\text{ N}$. The bearings at A and C are self-aligning. **Determine the reaction forces on the shaft from bearing A and bearing C in the X-Z plane ONLY. (6 marks)**



I meant the opposite. If you used W_t in other direction, that's ok.

$$F_B = F_1 + F_2 = 1000 + 300 = 1300\text{ N}$$

$$F_D = W_t = 1167\text{ N}$$

$$\sum M_A = 0 : 0.1 F_B + 0.18 F_C - 0.23 F_D = 0$$

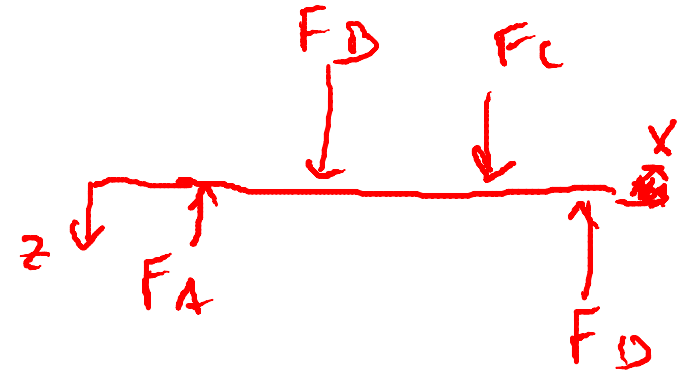
$$\text{Solve for } F_C : F_C = \frac{0.23 F_D - 0.1 F_B}{0.18} = \frac{0.23(1167) - 0.1(1300)}{0.18}$$

$$F_C = 768\text{ N}$$

$$\sum F = 0 : F_A = F_B + F_C - F_D$$

$$F_A = 1300 + 768 - 1167$$

$$F_A = 901\text{ N}$$



26. Regardless of the answers to the previous question, assume the bearing reaction force magnitudes are $F_A = 1200 \text{ N}$ and $F_C = 800 \text{ N}$. Select a single-row, 02-series, deep-groove bearing from Table 11-2 that will be appropriate for both locations A and C. Use an application factor of 1.5, a life of $x=10$ and a reliability of $R = 0.995$. Use these parameters:

$$L_{10} = 10^6 \quad x_0 = 0.02 \quad \vartheta = 4.459 \quad b = 1.483$$

Determine the C_{10} value required and the bearing bore diameter at the more critical location of the two (A or C). (6 marks)

A is more critical : Force is larger

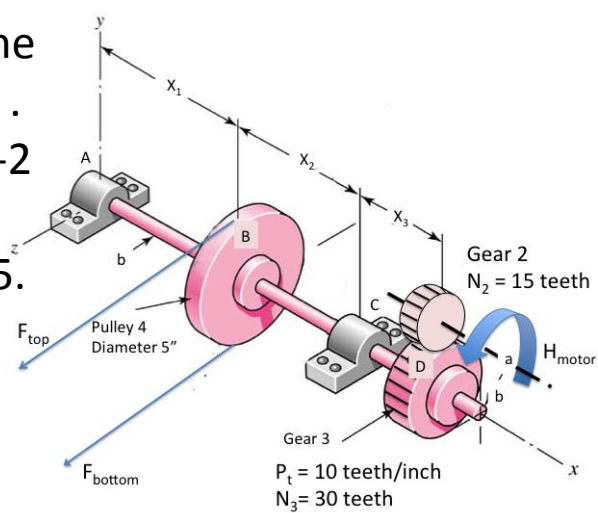
$$\Rightarrow C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\vartheta - x_0) \ln\left(\frac{1}{R_D}\right)^{1/b}} \right]^{\frac{1}{a}}$$

$$\Rightarrow C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\vartheta - x_0) (1 - R_D)^{1/b}} \right]^{\frac{1}{a}}$$

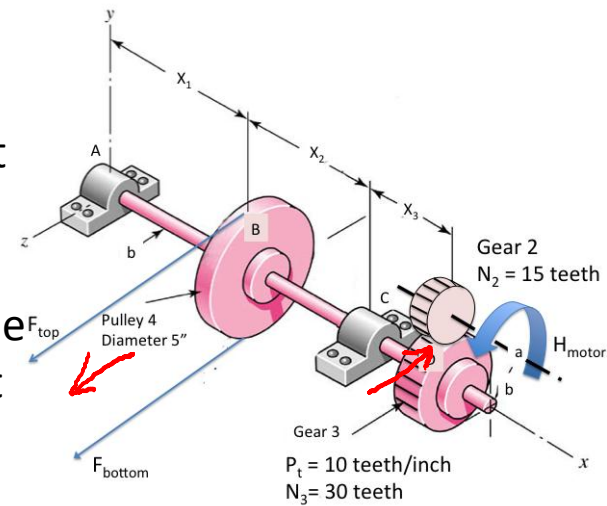
$$= 1.5(1200) \left[\frac{10}{0.02 + (4.459 - 0.02) (1 - 0.995)^{1/1.483}} \right]^{\frac{1}{3}}$$

$$C_{10} = 7384 \text{ N}$$

Bore of 15 mm will suffice.



27. Regardless of the answers to the previous question, assume the following for a calculation of stresses at B (the pulley): the shaft is made of steel with $S_{ut} = 800$ MPa and $S_y = 600$ MPa, and it has a fully-corrected endurance limit of $S_e = 300$ MPa at this location. The pulley is restrained to the shaft using a keyway (for the keyway, assume $K_t = 2.2$, $K_{ts} = 3.0$, $q = 0.75$, and $q_s = 0.85$). The following loading information is to be used: the bending moment magnitude at B due to the power transmission forces is 90 Nm. The torque T transmitted by the shaft from gear 3 to the pulley is 44.45 Nm. **To assure a factor of safety of 2.0, determine the diameter needed at location B (the pulley) to prevent fatigue failure, based on the ASME-elliptic criterion. (6 marks)**



$$k_f = 1 + q(K_t - 1) = 1 + 0.75(2.2 - 1) = 1.9$$

$$k_{fs} = \dots = 2.7$$

$$T_m = 44.45 \text{ Nm} ; M_a = 90 \text{ Nm}$$

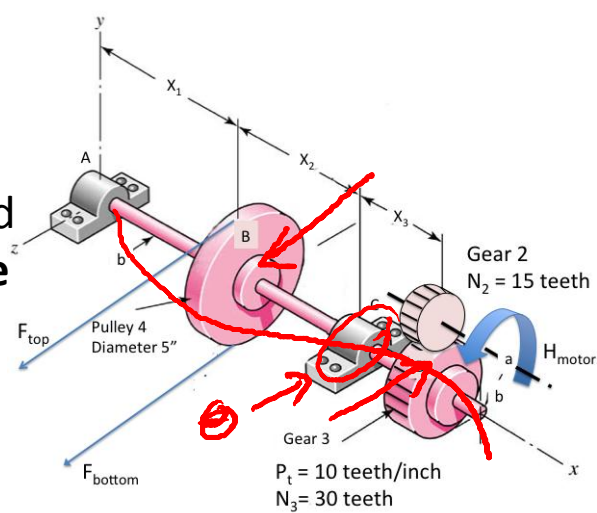
$$\text{ASME -elliptic : } A = \frac{k_f M_a}{S_a} = \frac{1.9(90)}{300 \cdot 10^6} = 5.7 \cdot 10^{-7}$$

$$B = \frac{k_{fs} T_m}{S_y} = \frac{2.7(44.45)}{600 \cdot 10^6} = 2.0 \cdot 10^{-7}$$

$$d = \left\{ \frac{16n}{\pi} [4A^2 + 3B^2] \right\}^{\frac{1}{2}} \Bigg|^{1/3} = \left\{ \frac{16 \cdot 2}{\pi} [4(5.7 \cdot 10^{-7})^2 + 3(2 \cdot 10^{-7})^2] \right\}^{\frac{1}{2}} \Bigg|^{1/3}$$

$$d = 22.98 \cdot 10^{-3} \text{ m} \Rightarrow \boxed{d = 22.98 \text{ mm}}$$

28. **BONUS:** Assume the diameter of the shaft is 30 mm for its entire length ($I = 74.8 \times 10^{-9} \text{ m}^4$). The radial force F_B can be found from: $F_{top} = 300 \text{ N}$ and $F_{bottom} = 1000 \text{ N}$, and is aligned with the $+z$ axis. Assume the total radial force at D is $F_D = -1200 \text{ N}$ also aligned along the z -axis. Assume E for steel = 80 GPa. **Calculate the slope of the shaft at bearing C. (4 marks)**



Assuming that this is the most critical location with respect to angular deflection, what can you conclude from the appropriateness of our choice of deep-groove bearings (see Table 7-2)? (1 mark) *Take derivative of deflection \Rightarrow slope ; use superposition*

① saddle force

$$y = \frac{F_B b_B x}{6EI l_B} (x^2 + b^2 - l^2)$$

$$\theta_B |_{x=l} = \frac{F_B b_B}{6EI l_B} (3l^2 + b^2 - l^2)$$

$$= \frac{1300 \cdot (0.08) \cdot 2 (0.18^2 + 0.08^2)}{6(80 \cdot 10^9)(74.8 \cdot 10^{-9})(0.18)}$$

$$\theta_B = 0.0011 \text{ rad}$$

② overhung force

$$y = \frac{F_D a_D x}{6EI l} (l^2 - x^2)$$

$$\theta_D |_{x=l} = \frac{F_D a_D}{6EI l} (l^2 - 3l^2) = \frac{F_D a_D l}{3EI}$$

$$\theta_D = \frac{1200 \cdot (0.05)(0.18)}{3(80 \cdot 10^9)(74.8 \cdot 10^{-9})} = 0.0006 \text{ rad}$$

$$\theta = \theta_B + \theta_D = 0.0011 + 0.0006 = \underline{\underline{0.0017 \text{ rad}}}$$

Deep-groove OK
but not conservative