MAT1339 A
Fall 2014
Assignment 1
Due: September 23: 6:00pm.

Please submit your assignment on or before 6:00pm. You need to put your assignment into the box in the Hall of Math Building, 585 King Edward.

Instructor: Dr. Hua

Instructions:
You should show your work for multiple choice questions.
For long questions you should write all the steps, similar to what we did in class.
You can work in a group but you **should** write your own assignments. You are not allowed to copy another student's work. Note that plagiarism is taken very seriously at the University of Ottawa.
This assignment is worth 5% of your final grade.

Question 1 (2 marks) Find the average rate of change for $f(x) = 2x^3 + x^2 + 4x$ on $-1 \leq x \leq 3$.

1. 40
2. $\frac{35}{2}$
3. 20
4. 15

$$\frac{f(3)-f(-1)}{3-(-1)} = \frac{80}{4} = 20$$

Question 2 (2 marks) Find the instantaneous rate of change of the function in question 1 at $x = -1$.

1. 8
2. 12
3. 4
4. 0

$$f'(x) = 6x^2 + 2x + 4$$

$$f'(-1) = 8$$

Question 3 (5 marks) Use the definition of derivative (first principle) to find the derivative of

$$f(x) = \sqrt{x^2 + 3}.$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{(x+h)^2+3} - \sqrt{x^2+3}}{h}$$

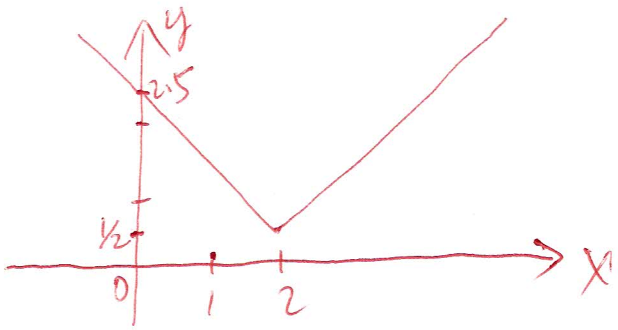
$$= \lim_{h \to 0} \frac{\left(\sqrt{(x+h)^2+3} - \sqrt{x^2+3}\right)\left(\sqrt{(x+h)^2+3} + \sqrt{x^2+3}\right)}{h\left(\sqrt{(x+h)^2+3} + \sqrt{x^2+3}\right)}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h\left(\sqrt{(x+h)^2+3} + \sqrt{x^2+3}\right)} = \lim_{h \to 0} \frac{2x+h}{\sqrt{(x+h)^2+3} + \sqrt{x^2+3}}$$

$$= \frac{x}{\sqrt{x^2+3}}$$

Question 4 (14 marks) Let $f(x) = |x-2| + \frac{1}{2}$.

1. (2 marks) Sketch the graph of $f(x)$.



2. (3 marks) Is $f(x)$ continuous? Justify your answer. (Hint: write $f(x)$ as a piecewise defined function.)

$$f(x) = \begin{cases} x - 1.5, & \text{if } x \geq 2 \\ 2.5 - x, & \text{if } x < 2 \end{cases}$$

① $f(x)$ is continuous when $x>2$ or $x<2$,
since

② At $x=2$: $f(2) = 0.5$

$\lim_{x \to 2^-} f(x) = 2.5 - 2 = 0.5$, $\lim_{x \to 2^+} f(x) = 2 - 1.5 = 0.5$

$\therefore \lim_{x \to 2} f(x) = 0.5 = f(2)$

$\therefore f(x)$ is continuous at $x=2$

$\therefore f(x)$ is continuous for all $x \in \mathbb{R}$.

3. (4 marks) Find $f'(1)$ and $f'(3)$.

When $x<2$: $f'(x) = (2.5-x)' = -1$

$\Rightarrow f'(1) = -1$

When $x>2$: $f'(x) = (x-1.5)' = 1$

$\Rightarrow f'(3) = 1$

4. (5 marks) Show that $f'(2)$ does not exist. (Hint: use the definition of derivatives.)

$$\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0^+} \frac{(2+h)-1.5-0.5}{h} = 1$$

$$\lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0^-} \frac{[2.5-(2+h)]-0.5}{h} = -1$$

$$\lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} \neq \lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h}$$

$\therefore f'(2) \nexists$

Question 5 (6 marks) Use the derivative rules to find $f'(x)$ when $f(x)$ is

1. $(2x^5 + 3x^2)(x^3 + 1)(\frac{1}{\sqrt{x}})$ $= (2x^5+3x^2)(x^3+1)(x^{-\frac{1}{2}})$

$$f'(x) = (2x^5+3x^2)'(x^3+1)(x^{-1/2}) + (2x^5+3x^2)(x^3+1)'(x^{-1/2}) + (2x^5+3x^2)(x^3+1)(x^{-\frac{1}{2}})'$$
$$= (10x^4+6x)(x^3+1)(x^{-1/2}) + (2x^5+3x^2)(3x^2)(x^{-1/2}) + (2x^5+3x^2)(x^3+1)(-\tfrac{1}{2}x^{-3/2})$$
$$= 10x^{13/2} + 16x^{7/2} + 6x^{1/2} + 6x^{13/2} + 9x^{7/2} - x^{13/2} - \tfrac{5}{2}x^{7/2} - \tfrac{3}{2}x^{1/2}$$
$$= 15x^{13/2} + 22.5x^{7/2} + 4.5x^{1/2} \quad \text{or} \quad \sqrt{x}(15x^6 + 22.5x^3 + 4.5)$$

2. $\frac{\frac{2}{8}x^8+3x+5}{\sqrt{x^3}}$ (Hint: write this as product of two functions and use the product rule)

$$f(x) = (\tfrac{2}{8}x^8+3x+5)(x^{-3/2})$$
$$f'(x) = (\tfrac{2}{8}x^8+3x+5)'(x^{-3/2}) + (\tfrac{2}{8}x^8+3x+5)(x^{-3/2})'$$
$$= (2x^7+3)(x^{-3/2}) + (\tfrac{1}{4}x^8+3x+5)(-\tfrac{3}{2}x^{-5/2})$$
$$= 2x^{11/2} + 3x^{-3/2} - \tfrac{3}{8}x^{11/2} - \tfrac{9}{2}x^{-3/2} - \tfrac{15}{2}x^{-5/2}$$
$$= \tfrac{13}{8}x^{11/2} - \tfrac{3}{2}x^{-3/2} - \tfrac{15}{2}x^{-5/2}$$
$$\text{or} \quad \tfrac{1}{8}x^{-5/2}(13x^8 - 12x - 60)$$

Question 6 (11 marks) The position of a moving object is given by $s(t) = \frac{1}{3}t^3 + t^2 - 3t + 2$. We start observing the object at time $t = 0$.

1. (1 mark) Find the velocity function $v(t)$.

$V(t) = s'(t) = t^2 + 2t - 3$

2. (1 mark) Find the acceleration function $a(t)$.

$a(t) = V'(t) = 2t + 2$

3. (2 marks) Find the time at which we see the object stop.

$V(t) = 0$, $t^2 + 2t - 3 = 0$ $\quad (t-1)(t+3) > 0$

$t = 1, -3$ (not possible)

4. (2 marks) Find the position of the object when we see it stop.

$S(1) = \frac{1}{3} + 1 - 3 + 2 = \frac{1}{3}$

5. (2 marks) Determine the interval of t on which the object is speeding up.

Speeding Up:

$V(t)a(t) > 0 \Rightarrow (t-1)(t+3)(2t+2) > 0,$

$\Rightarrow t > 1.$ $\therefore$ Speeding up for all $t > 1$.

6. (3 marks) Sketch the graphs of $s(t), v(t)$ and $a(t)$.