

# .MATH 1300 D -MIDTERM # 1-2014.

NAME and I.D.# Version 1

**Instructions:** This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 40 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. **You may use the backs of pages.**

For long answer questions  **MUST SHOW YOUR WORK.**

**NO CALCULATORS. NO BOOKS. NO NOTES.**

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

Question	1	2	3	4
Answer	E	C	C	A

### Multiple Choice Questions (1-4)

Question 1 Solve the equation  $\log_9(x-1) - \log_9(x) = \frac{1}{2}$  for  $x$ .

- A)  $x = 2$    B)  $x = \frac{1}{2}$    C)  $x = 0$    D)  $x = -\frac{1}{2}$     E) For no value of  $x$ .

$$\log_9 \frac{(x-1)}{x} = \frac{1}{2}$$

$$(9)^{1/2} = \frac{x-1}{x}$$

$$3x = x - 1$$

$$2x = -1$$

$$x = -\frac{1}{2} \quad \times$$

Not in  
the  
domain

Question 2 Find the value or values of  $x$  for which the following function has slope 0.

$$f(x) = xe^{(-3x+2)}$$

- A)  $x = 3$    B)  $x = -3$    C)   $x = \frac{1}{3}$    D)  $x = 0$    E)  $x = 2$

$$f'(x) = e^{-3x+2} (1) - 3e^{-3x+2} x =$$

$$e^{-3x+2} (1 - 3x) = 0$$

$$x = \frac{1}{3}$$

Question 3 Calculate

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 - 2x - 3}$$

- A) 0    B) -1    C)  $\frac{1}{2}$     D) 1    E) The limit does not exist.

$$\lim_{x \rightarrow -1} \frac{(x-1)(x+1)}{(x-3)(x+1)} = \lim_{x \rightarrow -1} \frac{x-1}{x-3} = \frac{-2}{-4} = \frac{1}{2}$$

Question 4 Find the value of  $a$ , for which the following function is continuous everywhere:

$$f(x) = \begin{cases} |-x-2|+2 & \text{if } x \leq 1 \\ ax+2 & \text{if } x > 1 \end{cases}$$

- A) 3    B) -3    C)  $-\frac{2}{3}$     D)  $\frac{1}{2}$     E) 0

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} |-x-2|+2 = 5$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} ax+2 = a+2$$

$$a+2=5 \quad a=3$$

Long Answer Questions (5-7)

Question 5 (8 points)

Suppose a function  $y = f(x)$  is defined implicitly by the equation

$$x^2y + 2x = 2 + y^3$$

- 5 • Find  $\frac{dy}{dx}$ .
- 3 • Find the equation of a tangent line to the curve at the point (1, 1).

$$2xy + x^2y' + 2 = 3y^2y'$$

$$y'(x^2 - 3y^2) = -2xy - 2$$

$$y' = \frac{-2xy - 2}{x^2 - 3y^2}$$

$$y' = \frac{-2xy - 2}{x^2 - 3y^2} \Big|_{(1,1)} = \frac{-4}{1-3} = \frac{-4}{-2} = 2$$

$$y - 1 = 2(x - 1)$$

$$\underline{y = 2x - 1}$$

Question 6 (6 points)

Using only the definition of derivative as a limit, calculate  $f'(x)$  where

$$f(x) = \frac{3}{x-2}$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{3}{x+\Delta x-2} - \frac{3}{x-2}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\frac{3(x-2) - 3(x+\Delta x-2)}{(x+\Delta x-2)(x-2)}}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{3x - 6 - 3x - 3\Delta x + 6}{\Delta x(x-2)(x+\Delta x-2)}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{-3\Delta x}{\Delta x(x-2)(x+\Delta x-2)}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-3}{(x-2)(x+\Delta x-2)} = \frac{-3}{(x-2)^2}$$

Question 7 (6 points)

One thousand dollars is invested at a rate of 4%,

- 2 • How much will be in the account in 10 years if compounded 4 times per year? (You do not need to simplify your answers.)
- 4 • How long is required for the initial investment to double if the account compounds continuously?

$$\textcircled{\bullet} \quad A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$
$$A(10) = 1000 \left(1 + \frac{0.04}{4}\right)^{40}$$

$$\textcircled{\bullet} \quad A(t) = P_0 e^{rt}$$

$$2P_0 = P_0 e^{0.04t}$$

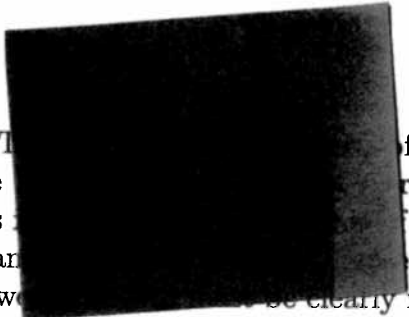
$$2 = e^{0.04t} \Rightarrow \ln 2 = 0.04t$$

$$t = \frac{\ln 2}{0.04}$$

**Space for additional work**

# MATH 1300 D -MIDTERM # 1-2014.

NAME and I.D.# Version 2



**Instructions:** There are 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as follows. The total score for the exam is 40 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. **You may use the backs of pages.**

**For long answer questions, YOU MUST SHOW YOUR WORK.**

**NO CALCULATORS. NO BOOKS. NO NOTES.**

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

Question	1	2	3	4
Answer	E	C	B	A

### Multiple Choice Questions (1-4)

Question 1 Solve the equation  $\log_4(x-1) - \log_4(x) = \frac{1}{2}$  for  $x$ .

A)  $x = -1$  B)  $x = 1$  C)  $x = 0$  D)  $x = -\frac{1}{2}$   E) For no value of  $x$ .

$$\log_4 \frac{x-1}{x} = \frac{1}{2} \quad 2 = \frac{x-1}{x}$$

$$2x = x - 1 \Rightarrow x = -1 \quad \text{NOT in the Domain}$$

Question 2 Find the value or values of  $x$  for which the following function has slope 0.

$$f(x) = xe^{(-2x+5)}$$

A)  $x = 2$  B)  $x = -3$   C)  $x = \frac{1}{2}$  D)  $x = 0$  E)  $x = -2$

$$f'(x) = e^{-2x+5} - 2e^{-2x+5}x$$

$$= e^{-2x+5}(1-2x) = 0 \quad \begin{array}{l} 1 = 2x \\ x = 1/2 \end{array}$$

Question 3 Calculate

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 2x - 3}$$

- A) 0     B)  $\frac{1}{2}$     C)  $\frac{1}{4}$     D) 2    E) The limit does not exist.

$$\lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x+3)(x-1)} = \lim_{x \rightarrow 1} \frac{x+1}{x+3} = \frac{2}{4}$$

Question 4 Find the value of  $a$ , for which the following function is continuous everywhere:

$$f(x) = \begin{cases} |-x-3|+1 & \text{if } x \leq 2 \\ ax+4 & \text{if } x > 2 \end{cases}$$

- A) 1    B) -1    C)  $-\frac{2}{3}$     D)  $\frac{1}{2}$     E) 0

$$\lim_{x \rightarrow 2^+} f(x) = 2a+4 = \lim_{f(x) \rightarrow 2^-} |-x-3|+1 = 6$$

$$2a+4=6$$

$$2a=2$$

$$a=1$$

Long Answer Questions (5-7)

Question 5 (8 points)

Suppose a function  $y = f(x)$  is defined implicitly by the equation

$$x^2y = 3 + y^3 - 3x$$

- ⑤ • Find  $\frac{dy}{dx}$ .
- ③ • Find the equation of a tangent line to the curve at the point  $(1, 1)$ .

$$2xy + x^2y' = 3y^2y' - 3$$

$$y'(x^2 - 3y^2) = -3 - 2xy$$

$$y' = \frac{-3 - 2xy}{x^2 - 3y^2} \Big|_{(1,1)} = \frac{-5}{1-3} = \frac{-5}{-2} = \frac{5}{2}$$

⊙

$$y - 1 = \frac{5}{2}(x - 1)$$

$$y = \frac{5}{2}x - \frac{3}{2}$$

Question 6 (6 points)

Using only the definition of derivative as a limit, calculate  $f'(x)$  where

$$f(x) = \frac{2}{x-3}$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\frac{2}{x+\Delta x-3} - \frac{2}{x-3}}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\frac{2(x-3) - 2(x+\Delta x-3)}{(x+\Delta x-3)(x-3)}}{\Delta x} =$$

$$\lim_{\Delta x \rightarrow 0} \frac{2x-6 - 2x - 2\Delta x + 6}{\Delta x (x+\Delta x-3)(x-3)} =$$

$$\lim_{\Delta x \rightarrow 0} \frac{-2\Delta x}{\Delta x (x+\Delta x-3)(x-3)} = \lim_{\Delta x \rightarrow 0} \frac{-2}{(x+\Delta x-3)(x-3)}$$

$$= \frac{-2}{(x+3)^2}$$

**Question 7 (6 points)**

One thousand dollars is invested at a rate of 3%,

- 2 • How much will be in the account in 8 years if compounded 5 times per year? (You do not need to simplify your answers.)
- 4 • How long is required for the initial investment to double if the account compounds continuously?

$$\textcircled{\bullet} \quad A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$
$$A(10) = 1000 \left(1 + \frac{0.03}{5}\right)^{40}$$

$$\textcircled{\bullet} \quad A(t) = P_0 e^{rt}$$
$$2P_0 = P_0 e^{0.03t}$$
$$2 = e^{0.03t}$$

$$\ln 2 = 0.03t \quad t = \frac{\ln 2}{0.03}$$

**Space for additional work**

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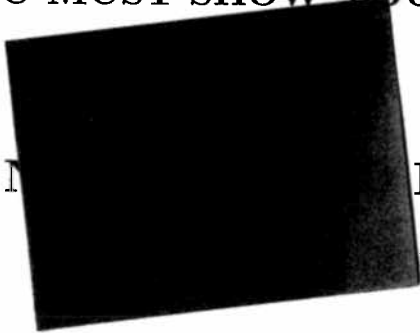
NAME and I.D.# \_\_\_\_\_ Version 3 \_\_\_\_\_

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Multiple Choice Answers:

Question	1	2	3	4
Answer	E	C	B	A

### Multiple Choice Questions (1-4)

Question 1 Solve the equation  $\log_4(x-1) - \log_4(x) = \frac{1}{2}$  for  $x$ .

- A)  $x = -1$    B)  $x = 1$    C)  $x = 0$    D)  $x = -\frac{1}{2}$     E) For no value of  $x$ .

$$\log_4 \frac{x-1}{x} = \frac{1}{2} \quad 2 = \frac{x-1}{x} \quad 2x = x-1$$
$$x = -1 \Rightarrow \text{Not in the Domain}$$

Question 2 Find the value or values of  $x$  for which the following function has slope 0.

$$f(x) = xe^{(-3x+2)}$$

- A)  $x = 3$    B)  $x = -3$     C)  $x = \frac{1}{3}$    D)  $x = 0$    E)  $x = 2$

$$f'(x) = e^{-3x+2} - 3e^{-3x+2}x$$
$$= e^{-3x+2}(1-3x) = 0$$
$$1 = 3x \quad x = \frac{1}{3}$$

Question 3 Calculate

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 2x - 3}$$

- A) 0    B)  $\frac{1}{2}$  ✓    C)  $\frac{1}{4}$     D) 2    E) The limit does not exist.

$$\lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x+3)(x-1)} \quad x \neq 1 = \lim_{x \rightarrow 1} \frac{x+1}{x+3} = \frac{1}{2}$$

Question 4 Find the value of  $a$ , for which the following function is continuous everywhere:

$$f(x) = \begin{cases} |-x-2|+2 & \text{if } x \leq 1 \\ ax+2 & \text{if } x > 1 \end{cases}$$

- A) 3 ✓    B) -3    C)  $-\frac{2}{3}$     D)  $\frac{1}{2}$     E) 0

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$|-1-2|+2 = a+2$$

$$5 = a+2 \quad a = 3$$

### Long Answer Questions (5-7)

#### Question 5 (8 points)

Suppose a function  $y = f(x)$  is defined implicitly by the equation

$$x^2y = 3 + y^3 - 3x$$

- Find  $\frac{dy}{dx}$ .
- Find the equation of a tangent line to the curve at the point  $(1, 1)$ .

$$2xy + x^2y' = 3y^2y' - 3$$

$$y'(x^2 - 3y^2) = -3 - 2xy$$

$$y' = \frac{-3 - 2xy}{x^2 - 3y^2}$$

$$y' \Big|_{(1,1)} = \frac{-5}{-2} = \frac{5}{2}$$

$$\textcircled{1} \quad y - 1 = \frac{5}{2}(x - 1)$$

$$y = \frac{5}{2}x - \frac{3}{2}$$

**Question 6 (6 points)**

Using only the definition of derivative as a limit, calculate  $f'(x)$  where

$$f(x) = \frac{3}{x-2}$$

look @ v 1 solutions

**Question 7 (6 points)**

*One thousand dollars is invested at a rate of 3%,*

- *How much will be in the account in 8 years if compounded 5 times per year? (You do not need to simplify your answers.)*
- *How long is required for the initial investment to double if the account compounds continuously?*

LOOK @ version 2  
solutions.

Space for additional work