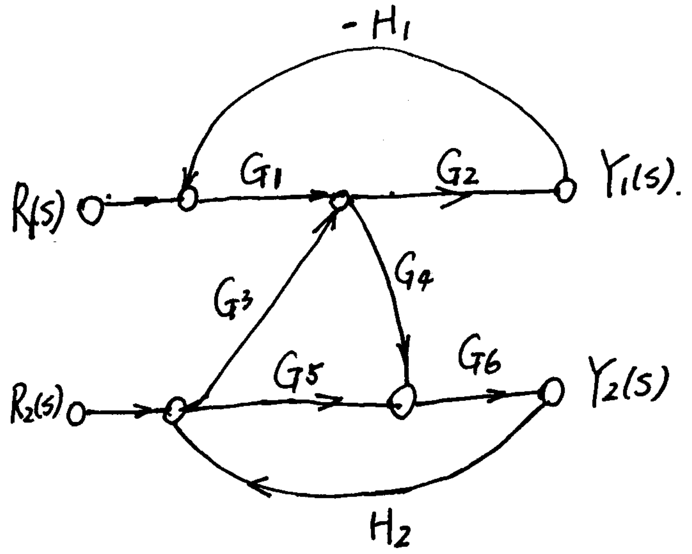
Solutions to Midterm   Section T.

①.



(2). $T_1(s) = \frac{Y_1(S)}{R_1(S)} = \frac{\sum_k \Delta_{ijk}\cdot P_{ijk}}{\Delta}$

$L_1 = -G_1H_1G_2 \qquad L_2 = G_3G_4G_6H_2 \quad L_3 = G_5G_6H_2$

$\Delta = 1-(L_1+L_2+L_3) + L_1\cdot L_3 = 1+G_1H_1G_2 - G_3G_4G_6H_2 - G_5G_6H_2 + (-G_1H_1G_2)(G_5G_6H$

$\Delta_{111} = 1 - L_3 = 1 - G_5G_6H_2$ Set $L_1 = L_2 = 0$

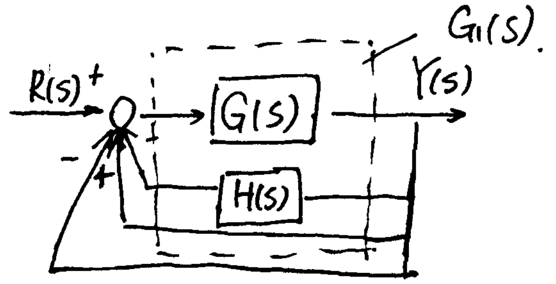
$P_{111} = G_1G_2.$

$$T_1(s) = \frac{(1-G_5G_6H_2)\cdot G_1G_2}{\Delta}$$

$$T_2(s) = \frac{Y_2(s)}{R_1(s)} = \frac{\sum_k \Delta_{12k}\cdot P_{12\cdot k}}{\Delta}$$

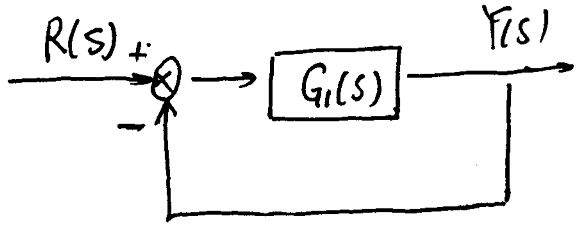
$\Delta_{121} = 1 \quad P_{121} = G_1G_4G_6.$

$$T_2(s) = \frac{G_1G_4G_6}{\Delta}$$

(2). 

$$G(s)=\frac{K}{s+10},\quad H(s)=\frac{14}{s^2+5s+6}. \quad (2)$$

(a) System becomes the unity negative feedback system.



Thus:

$$G_1(s)=\frac{G(s)}{1+GH-G(s)}=\frac{\frac{K}{s+10}}{1+\frac{K}{s+10}\cdot\frac{14}{s^2+5s+6}-\frac{K}{s+10}}$$

$$=\frac{K(s^2+5s+6)}{(s+10)(s^2+5s+6)+14K-(s^2+5s+6)K}$$

By checking feedforward transfer function $G_1(s)$, we know that the system is type 0 system.

(b). For type 0 system, the steady state error due to step input $R(s)=\frac{1}{s}$ is obtained by the formula:

$$e_{ss}=\frac{1}{1+K_p}=\frac{1}{1+G_1(0)}=\frac{1}{1+\frac{6K}{60+14K-6K}}=\frac{60+8K}{60+14K}$$

(C). steady-state error due to $T_d(s)=\frac{1}{s}$.

$$E_d(s)=0-\frac{Y(s)}{T_d(s)}\cdot\frac{1}{s}=-\frac{1}{1+GH}\cdot\frac{1}{s}=-\frac{1}{s\left(1+\frac{14K}{(s+10)(s^2+5s+6)}\right)}$$

$$e_{dss}=\lim_{s\to 0} sE_d(s)=\lim_{s\to 0}\left(-\frac{1}{1+\frac{14K}{(s+10)(s^2+5s+6)}}\right)=-\frac{60}{60+14K}$$

(3).

(a) $P.O. = 100 \cdot e^{-\xi\pi/\sqrt{1-\xi^2}}$

$5\% < P.O < 10\% \Rightarrow \xi_{5\%} > \xi > \xi_{10\%} \Rightarrow \cos^{-1}(\xi_{5\%}) < \theta < \cos^{-1}(\xi_{10\%})$

where

$\theta_{5\%} < \theta < \theta_{10\%}$

$$\frac{-\xi_{5\%} \cdot \pi}{\sqrt{1-\xi_{5\%}^2}} = \ln(0.05) = -2.995$$

solve the equation: $\xi_{5\%} = 0.6901 \Rightarrow \theta_{5\%} = \cos^{-1}(\xi_{5\%}) = 46.4^\circ$

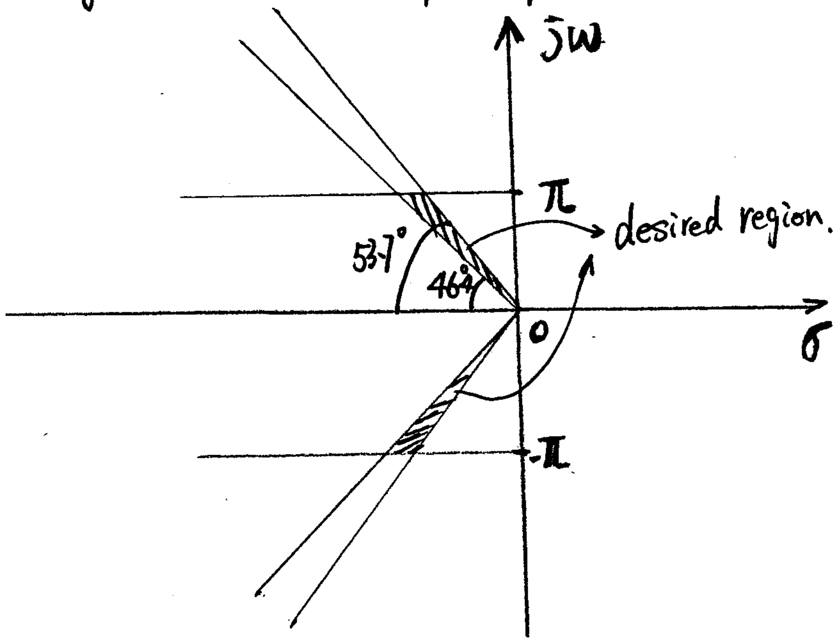
$$\frac{-\xi_{10\%} \cdot \pi}{\sqrt{1-\xi_{10\%}^2}} = \ln(0.1) = -2.3026$$

$\xi_{10\%} = 0.5912 \Rightarrow \theta_{10\%} = \cos^{-1}(\xi_{10\%}) = 53.7^\circ$

$46.4^\circ < \theta < 53.7^\circ$

(b) Peak time $T_p = \frac{\pi}{W_n\sqrt{1-\xi^2}} > 1$ second $\Rightarrow W_n\sqrt{1-\xi^2} < \pi \Rightarrow \mathrm{Im}(s) < \pi$

The desired region of the complex poles is shown as:



(4). (a) The closed-loop characteristic equation

$$1+K\cdot\frac{s+1}{s^2(s+2)(s+20)}=0$$

$$\Rightarrow \quad s^4+22s^3+40s^2+Ks+K=0$$

s^4	1	40	K
s^3	22	K	0
s^2	$\frac{880-K}{22}$	K	
s	a	0	
1	K		

$$a=\left(\frac{880-K}{22}\cdot K-22K\right)\Big/\frac{(880-K)}{22}$$

$$=\frac{880K-K^2-484K}{880-K}$$

$$=\frac{396K-K^2}{880-K}$$

In order to keep closed Loop stable, the first column should be designed with no sign change.

$$\frac{880-K}{22}>0 \qquad \frac{396K-K^2}{880-K}>0 \qquad K>0$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$K<880 \qquad K>0 \text{ or } K<396 \qquad K>0$$

In summary, the range for K should satisfy all above inequalities:

$$0<K<396$$

(b). The marginally stable roots exists only when elements in s row are equal to zeros.

$$\frac{396K-K^2}{880-K}=0 \Rightarrow K=0 \quad K=396$$

The roots are obtained from Auxiliary polynomial

$$\frac{484}{22}\cdot s^2+396=0 \Rightarrow s_{1,2}=\pm 4.26j$$

K shouldn't be zero since it makes both s and s^0 rows are equal to zeros.