Concordia University
Department of Mathematics & Statistics

Course: MATH 206/4 2013-2014

Examination: Midterm Test **Date & Time: March 2nd, 2PM**

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Special Instructions

- **Solve all problems and, in each case, clearly indicate which problem you are solving.**
- **Clearly identify your answers; if you don't show or explain your work, you cannot get full marks.**
- **Only approved calculators are permitted.**

Marks

[3] 1. The expression $(\sqrt{5}-2)^2$ can be written as $a-b\sqrt{c}$ where a, b and c are whole numbers. Find the values of a, b and c.

[3] 2. Simplify the expression $\sqrt{72} \times 3\sqrt{2}$.

[3] 3. Rationalize the denominator of the expression and simplify common factors:

$$\frac{2}{1-\sqrt{5}}$$

[3] 4. Remove the parentheses and combine like terms:

$$2(5x-2)+x^2-x(x+4)$$

[3] 5. Rewrite as a single fraction; combine like terms in the numerator and denominator.

$$\frac{2}{x}-\frac{x+1}{x^2-6}$$

[6] 6. Factor the polynomials completely:

a. $2x^2-5x+3$ b. $16x^4-81$

[3] 7. Use synthetic division to determine whether $x + 2$ is a factor of $x^3 - 3x + 2$.

[6] 8. Solve the following equations or show that there's no solution:

a. $x^2 = x + 2$
b. $2x - (x + 1) = 3(x + 5)$
c. $(x - 1)^2 = -2$

[6] 9. Solve the inequalities:

a. $4x + 7 > 4 - 2x$
b. $|3x - 1| \leq 2$

[3] 10. Write an equation of a line passing through the point (2, 1) and parallel to the line $y = 2x$.

[3] 11. Write an equation of the circle with diameter 4 and centre $(1, -1)$.

[6] 12. Find the domain of the functions:

a. $f(x) = \sqrt{x^2 - 1}$ b. $f(x) = \frac{-1}{\sqrt{x+1}}$

[2] 13. Clearly show and explain how the graph of $f(x) = (x + 2)^2$ can be obtained from the graph of $g(x) = x^2$.

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3 1. $(\sqrt{5}-2)^2 = (\sqrt{5}-2)(\sqrt{5}-2) = \sqrt{5}\sqrt{5} - 2\sqrt{5} - 2\sqrt{5} + 4 = 9 - 4\sqrt{5}$

$\Rightarrow a = 9, b = 4, c = 5$

3 2. $\sqrt{72} \times 3\sqrt{2} = 3\sqrt{72 \times 2} = 3\sqrt{144} = 3(12) = 36$

3 3. $\frac{2}{1-\sqrt{5}} = \frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})} = \frac{2(1+\sqrt{5})}{1+\sqrt{5}-\sqrt{5}-5} = \frac{2(1+\sqrt{5})}{-4} = \frac{(1+\sqrt{5})}{-2}$

3 4. $2(5x-2) + x^2 - x(x+4)$

$= 10x - 4 + x^2 - x^2 - 4x$

$= 6x - 4$

3 5. $\frac{2}{x} - \frac{x+1}{x^2-6} = \frac{2}{x} - \frac{x+1}{(x-\sqrt{6})(x+\sqrt{6})} = \frac{2(x-\sqrt{6})(x+\sqrt{6}) - (x+1)x}{(x-\sqrt{6})(x+\sqrt{6})\,x}$

$= \frac{2(x^2-6) - x^2 - x}{x(x^2-6)} = \frac{2x^2 - 12 - x^2 - x}{x(x^2-6)} = \frac{x^2 - x - 12}{x(x^2-6)}$

2@3 6. a) $2x^2 - 5x + 3 = (2x-3)(x-1)$

b) $16x^4 - 81 = (4x^2-9)(4x^2+9) = (2x-3)(2x+3)(4x^2+9)$

3 7.

-2	1	0	-3	2
		-2	4	-2
	1	-2	1	0

Since Remainder $= 0$

$\Rightarrow x+2$ is a Factor of $x^3 - 3x + 2$

3@2 8. a) $x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x+1=0$	$x-2=0$
$x=-1$	$x=2$

b) $2x - (x+1) = 3(x+5)$

$2x - x - 1 = 3x + 15$

$2x - x - 3x = 15 + 1$

$-2x = 16$

$x = \frac{16}{-2}$

$x = -8$

c) $(x-1)^2 = -2$

Since $(x-1)^2$ is always ≥ 0

$\Rightarrow (x-1)^2$ cannot equal $-2 \Rightarrow$ No solution

9 a) $4x+7 > 4-2x$

$4x+2x > 4-7$

$6x > -3$

$x > -\frac{3}{6}$

$x > -\frac{1}{2}$

2@3

b) $|3x-1| \leq 2$

$\Rightarrow 3x-1 \leq 2$ AND $3x-1 \geq -2$

$3x \leq 2+1$ AND $3x \geq -2+1$

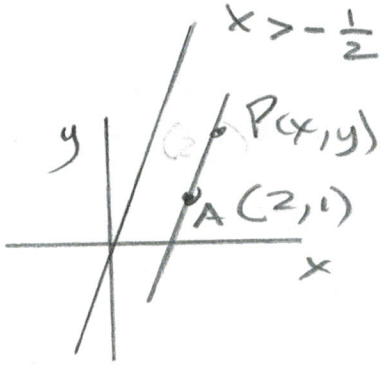
$3x \leq 3$ AND $3x \geq -1$

$x \leq 1$ AND $x \geq -\frac{1}{3}$

FINAL ANSWER

$-\frac{1}{3} \leq x \leq 1$

3 10.



Step 1 Slope of $y = 2x$ is $\Rightarrow$ required line has slope 2.

Step 2 Let $P(x,y)$ be ANY point on required line

$M_{AP} = \frac{y_2-y_1}{x_2-x_1}$

$\frac{2}{1} = \frac{y-1}{x-2}$

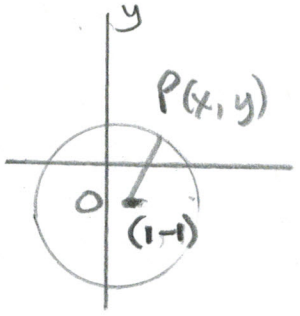
$1(y-1) = 2(x-2)$

$y-1 = 2x-4$

$y = 2x-4+1$

$y = 2x-3$

3 11.



Let $P(x,y)$ be ANY point on circle

distance $OP = 2$

$\sqrt{(x-1)^2+(y+1)^2} = 2 \Rightarrow x^2-2x+1+y^2+2y+1 = 4$

$\Rightarrow x^2+y^2-2x+2y-2 = 0$

12 a) $f(x) = \sqrt{x^2-1} \Rightarrow x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \in \text{Reals} \mid x \geq 1 \text{ or } x \leq -1$

2@3

b) $f(x) = -\frac{1}{\sqrt{x+1}}$

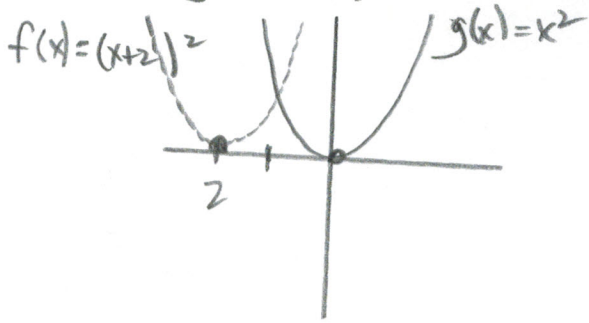
(i) $x+1 \neq 0 \Rightarrow x \neq -1$

(ii) $x+1 \geq 0 \Rightarrow x \geq -1$

$\Big\}\ x \in \text{Reals} \mid x > -1$

13. $y = (x+2)^2$

2



Step 1 Sketch $g(x)$ or $y = x^2$

Step 2 x is Replaced by $(x+2)$

$\Rightarrow$ graph of $f(x)$ or $y = (x+2)^2$ Shifts horizontally Left 2 units.