Name ____

Student Number ____

Lecture section (L01-Dr. Dimitrov, L02-Dr. Wang) ____________

University of Calgary
Schulich School of Engineering
Fall 2011

ENGG 407 - Numerical Methods

Midterm

Saturday, November 5, 2011
2:00-3:30pm

1. Examination is closed book.
2. The exam consists of 40 multiple choice questions where each question has equal value. Total marks for the exam is 100.
3. Write the answers on the separate bubble sheet as well as in the spaces provided in this exam booklet.
4. No wireless devices or earphones are allowed during exam.
5. No calculators are allowed.
6. An aid sheet is provided at the end of the exam paper.
7. You do not need to simplify the numerical expressions unless stated
8. All angles are in radians. For example sin(x), x is assumed to be in radians

Circle the correct answer below each question. Then, mark your answers on the standard answer sheet. Each question has equal value.

1. A numerical method is characterized by the following features *except*:
(**The Bold answer is expected**)

a. Human problem-solver oriented
b. Using approximation techniques
c. Using iterative and/or recursive algorithms
d. Designed for problems that cannot be or difficult to be solved analytically

2. The given iteration formula $x_{i+1} = x_i - \frac{f^{(1)}(x_i)}{f^{(2)}(x_i)}$ is suitable for:

a. Newton′s method for root finding
b. nothing as the expression is meaningless
c. the Newton-Raphson method for optima finding
d. expressing the second order Taylor series of $f(x)$

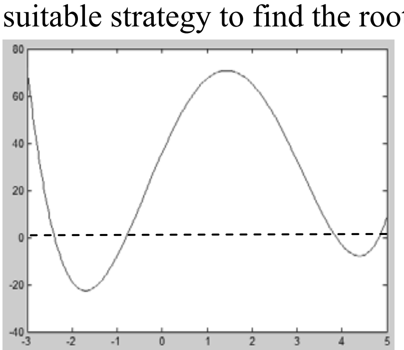
3. It is guaranteed that a root of a continuous nonlinear function $f(x)$ exists in [*a, b*] when
f(a) • *f(b)* < *0*. However, when *f(a)* • *f(b)* > *0*, there would be *N* roots existing in [*a, b*]. In this case, which of the following possibilities may be ruled out?

a. N = 0
b. N ≥ 2
c. N = 1
d. N = 2

4. The convergent con dition for s olving root(s) for a nonlinear functions *f(x)* in [*a, b*] is determined by the follows, *except*:

a. *(b-a)/2* < tolerance
b. $f(x_i) = 0$
d. $f(x_i)$ < tolerance
c. $\mathbf{f'(x_i) = 0}$

5. The plot of a function $f(x) = x^4 - 5.5x^3 - 7.2x^2 + 43x + 36 = 0$ is as follows. What is *NOT* a suitable strategy to find the roots of $f(x)$ in [-3, 5] using any single root finding method?



a. Using a spline or a section for screening
b. Using Newton`s method around the four points
c. Using the random method for root finding
d. Using the $f(a) \bullet f(b) < 0$ criterion

6. Which of the following methods is not suitable for curve fitting?

a. The polynomial regression method
b. The filtering method
c. The spline method
d. The power function regression method

7. In polynomial curve fitting, assume n is the number of data points, and m the order of the chosen polynomial. What is the condition between n and m?

a. $n \neq m$
b. $n > m$
c. $n < m$
d. $n >> m$

8. What is/are the eigenvalue(s) of the following matrix $\begin{bmatrix} 100 & 1 \\ 0 & 100 \end{bmatrix}$?

a. 0, 1
b. 100
c. 100, 100
d. $\infty, 0$

9. How to detect the trend of a nonlinear function *f(x)* as being descending at x_0 and $x_0+\Delta x$ where Δx is a positive real number close to machine epsilon?

***a.** $f(x_o) > f(x_o + \Delta x)$
b. $f(x_o) < f(x_o + \Delta x)$
c. $|f(x_o)| > |f(x_o + \Delta x)|$
d. $|f(x_o)| < |f(x_o + \Delta x)|$

10. Which of the following methods is not an iterative method for solving a system of linear equations?

a. The Jacobi method
b. The Newton-Seidel method
c. The Crout`s method
d. The explicit expression of linear equations

11. Given a system of linear equations $\mathbf{Ax} = \mathbf{b}$, according to the LU decom position method, where $\mathbf{A} = \mathbf{LU}$, what is the procedure for finding $\mathbf{x}$?

a. **x = U \ L**
***b. x = U \ (L \ b)**
c. **x = U \ (L / b)**
d. none of the above

12. Given **A = LU** $= \begin{bmatrix} 2 & 0 \\ 1 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}$, what is **A**?

***a.** $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.
b. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
c. $A = \begin{bmatrix} 2 & 1.5 \\ 3 & 0.5 \end{bmatrix}$.
d. None of the above

13. Given **A = LU** $= \begin{bmatrix} 2 & 0 \\ 1 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 1.5 \\ 0 & 1 \end{bmatrix}$, what is det(**A**)?

a. 2
b. 3
c. 4
d. 1

14. The *Jocobi iterative method* for systems of linear equations is a spe cial case of the *Gauss-Siedel iterative method*. Given the general iterati on formula of the latter is $x_i = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j)$, what is the iterative expression of the former?

a. $x_i^{(k+1)} = \frac{1}{a_{ii}} J(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{(k)})$
b. $x_i^{(k)} = \frac{1}{a_{ii}} (b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{(k)})$
***c.** $x_i^{(k+1)} = \frac{1}{a_{ii}} (b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{(k)})$
d. none of the above

15. Given the general iterative formula of the 1-D *Random Search* method as $x^{(i+1)} = x_l^{(i)} + (x_u^{(i)} - x_l^{(i)})r^{(i)}$ for a function $f(x)$ in $[x_l,\ x_u]$, where r is a random number, $0 \le r \le 1$, how may it be extended to a 2-D problem $f(x, y)$ in $[x_l,\ x_u]$ and $[y_l,\ y_u]$?

a. $\begin{cases} x^{(i+1)} = x_l^{(i)} + (x_u^{(i)} - x_l^{(i)})r_x^{(i)} \\ y^{(i+1)} = y_l^{(i)} + (y_u^{(i)} - y_l^{(i)})r_y^{(i)} \end{cases}$

***b.** $\begin{cases} x^{(i+1)} = x_l^{(i)} + (x_u^{(i)} - x_l^{(i)})r^{(i)} \\ y^{(i+1)} = y_l^{(i)} + (y_u^{(i)} - y_l^{(i)})r^{(i)} \end{cases}$

c. $\begin{cases} x^{(i+1)} = (x_u^{(i)} - x_l^{(i)})r^{(i)} \\ y^{(i+1)} = (y_u^{(i)} - y_l^{(i)})r^{(i)} \end{cases}$

d. none of the above

16. Given a function $f(x) = 2\sin(x) + 1$, what is the first order Taylor expansion of the function at the point $x_0 = \pi$?

a. $f(x) = f(x_0) + 2\cos(x_0)(x - x_0)$
***b.** $f(x) = 1 - 2(x - \pi)$
c. $f(x) = f(x_0) + 2\cos(x)(x - x_0)$
d. $f(x) = 1 + 2(x - \pi)$

17. What is/are the eigenvalue(s) of the following matrix $\begin{bmatrix} 1 & 4 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$?

a. 1, 2, 3
b. 1, 2, 3, 4
c. 0, 1, 2
d. none of the above

18. Given two bytes for machine representation of float point numbers. Assume each byte (8 bits) is allocated for the mantissa and exponent, respectively, where the most significant bits are reserved for representing their signs. What is the range of number representation using this scheme?

a. $\pm 2^8 \bullet 10^{\pm 2^8}$
b. $\pm 2^7 \bullet 10^{\pm 2^7}$
***c.** $\pm 2^7 \bullet 2^{\pm 2^7}$
d. $2^{\pm 7} \bullet 2^{\pm 2^{15}}$

19. Which of the following methods is not suitable for solving the root(s) of a nonlinear function $f(x)$?

a. The bisection method

b. Newton`s method
c. The Regula-Falsi method
d. The Gaussian method

20. Which of the following methods is not suitable for directly solving a system of linear equations?

a. The Gaussian elimination method
b. The Gauss-Jordan elimination method
c. The Gauss-Seidel method
d. The LU decomposition method

21. In the *bisection* method, the root of $f(x)$ is to be determined given an initial bracket of $[a,b]=[0,10]$. After 3 iterations, what is the length of the bracket ($L_3 = b_3 - a_3$)?

a. 1.25
b. 2.50
c. 5.00
d. 1.75

22. When minimizing the function $f(x)=x^3-x+1$ in the interval [a, b] = [0, 1] by using the golden section optimization method, after the first iteration, the interval [a, b] will be reduced to:

a. [0, 0.618]
b. [0.618, 1]
c. [0, 0.382]
d. [0.382, 1]

23. Which of the following methods is not an interpolation method?

a. The Newton`s polynomial method
b. The Lagrange polynomial method
c. The extrapolation method
d. The general curve fitting and predication method

24. Applying Newton`s method for optimization on a function $f(x)=x^4-x+1$, what is the first approximation of the minimum of $f(x)$ assuming $x^{(0)}=1$?

***a)** $x^{(1)}=0.75$
b) $x^{(1)}=0.50$
c) $x^{(1)}=0.25$
d) $x^{(1)}=0.125$

25. Given a general row of expression in a system of linear equations as $L_i = (a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n) = b_i, 0 \le i, j \le n$, what are the valid operations that keep the system's solutions invariant, *except:*

a. $kL_i \rightarrow L_i$

b. $L_i \leftrightarrow L_j$

***c.** $L_i \times kL_j \rightarrow L_j$

d. $kL_i + k'L_j \rightarrow L_j$

(`Hint: k is a nonzero real number.)

26. Which of the following methods is not suitable for solving the 2-D optimization problems?

a. The random search method
b. The sliced reduction method
c. The deepest ascent search method
d. The fix point method

27. For a polynomial regression curve fit, the following is true:

a. Low order polynomials should be avoided as they are inaccurate

b. Selecting too high a polynomial order will result in erroneous fitting of the noise or distortion in the data

c. Only cubic polynomials should be used

d. Coefficients can be efficiently determined using low order Lagrange polynomials

28. For the system of linear equations $\begin{cases} 3x_1 + x_2 + x_3 = 1 \\ 2x_1 + 5x_2 - 2x_3 = 0 \\ -x_1 + 4x_2 - 6x_3 = 1 \end{cases}$, the Jacobi method will:

a) definitely converge
b) definitely diverge
c) undeterminable
d) depend on the initial values

29. Which of the following methods is not suitable for solving the 1-D optimization problems?

a. The binary section search method
b. The golden section search method
c. Taylor`s method
d. Newton`s method

30. The inverse of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, A^{-1}, is:

a. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

d. $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

31. How can we tell that x_o maximizes $f(x)$?

a. $f(x_o) = 0$ and $f^{(1)}(x_o) = 0$

b. $f^{(1)}(x_o) = 0$ and $f^{(2)}(x_o) = 0$

c. $f^{(2)}(x_o) > 0$ and $f^{(1)}(x_o) = 0$

***d.** $f^{(2)}(x_o) < 0$ **and** $f^{(1)}(x_o) = 0$

32. In a binary floating point representation, what determines the machine epsilon?

a. the exponent value
b. number of bits in exponent
c. number of bits in mantissa
d. machine dependent

33. What is the first order Taylor expansion of the function $f(x, y) = \sin(xy)$ at the expansion point (x_0, y_0)?

a. $f_1(x, y) = \sin(x_0 y_0) + \cos(x_0 y_0)(x - x_0) + \cos(x_0 y_0)(y - y_0)$

b. $f_1(x, y) = \sin(x_0 y_0) + y_0 \cos(x_0 y_0)(x - x_0) + x_0 \cos(x_0 y_0)(y - y_0)$

c. $f_1(x, y) = \sin(x_0 y_0) + y_0 \cos(xy)(x - x_0) + x_0 \cos(xy)(y - y_0)$

d. $f_1(x, y) = \sin(xy) + y \cos(xy)(x - x_0) + x \cos(xy)(y - y_0)$

34. Given *f(x)* as a continuous function for which x_o satisfies $f^{(1)}(x_o) = 0$. How can we know that $f(x_o)$ is a possible minimum?

a. $f^{(2)}(x_o) = 0$

b. $f^{(2)}(x_o) \neq 0$

c. $f^{(2)}(x_o) < 0$

***d.** $f^{(2)}(x_o) > 0$

35. Which of the following procedur es will typically result in the fastest conve rgence in root finding given $f(x)=0$?

a. The bisection method
b. The overlapping method based on the golden ratio
c. The regula falsi method
d. The update recursion of $x_{i+1}=x_i-f(x_i)/f^{(1)}(x_i)$

36. Given that we ha ve data points of (x_1,y_1), (x_2,y_2) and (x_3,y_3) what is a Lag range polynomial suitable for curve fitting?

a. ax^2+bx+1
b. $\dfrac{(x-x_1)(x-x_2)(x-x_3)}{x_1x_2x_3}$
c. $\dfrac{(x-x_1)(x-x_2)}{(x_1-x_3)(x_1-x_3)}$
d. $\dfrac{(x-x_1)(x-x_3)}{(x_2-x_3)(x_2-x_1)}$

37. A disadvantage of Newton's iterative method for root finding is that it:

a. Only applicable to force related problems
b. Requires the second order numerical derivative
c. Requires the algebraic form of the derivative
d. Only applicable when the Jacobian is used

38. In numerical optimization, the equivalence of the maximum and minimum at x*, $f_{max}(x^*)$ and $f_{min}(x^*)$, can be expressed as:

a. For all x*, $f_{max}(x^*) = - f_{min}(x^*)$
b. For all x*, $f_{max}(x^*) = f_{min}(-x^*)$
c. For all x*, $f_{max}(-x^*) = f_{min}(x^*)$
d. For some x*, $f_{max}(x^*) = - f_{min}(x^*)$

39. Given that a deterministic curve fit is requi red through 20 points what is the best and m ost practical approach?

a. Use Lagrange interpolation function
b. Use an interpolation polynomial of $f(x)=\sum_{n=0}^{20} a_n x^n$
c. Use multiple lower order splines
d. Use a least square regression curve fit

40. A least square regression curve fit is applied to a set of N points of (t_1, x_1) (t_2, x_2) $\cdots$ (t_N, x_N). The regression function is $f(t) = C$, C is a constant. Which of the following is true:

a. $C = 1$

b. $C = \frac{1}{N}\sum_{i=1}^{N} x_i^2$

***c.** $C = \frac{1}{N}\sum_{i=1}^{N} x_i$

d. Insufficient information to determine the regression curve fit

Aid Sheet

Derivative Notation

$$f^{(n)}(x) = \frac{d^n f(x)}{dx^n}$$

Taylor expansion Formulas

$$\widehat{f_N}\left(x\right) = \sum_{n=0}^{N} \frac{1}{n!} f^{(n)}(x_o)\left(x - x_o\right)^n$$

$$\hat{f}_1(\mathbf{x}) = f(\mathbf{x}_0) + J(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)$$

$$\hat{f}_2(\mathbf{x}) = f(\mathbf{x}_0) + J(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0) + \tfrac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \mathbf{H}(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)$$

Jacobian

$$J^T\left(x_{1,0}, x_{2,0}, \dots, x_{N,0}\right) = \begin{bmatrix} f^{(x_1)}\left(x_{1,0}, x_{2,0}, \dots, x_{N,0}\right) \\ f^{(x_2)}\left(x_{1,0}, x_{2,0}, \dots, x_{N,0}\right) \\ \vdots \\ f^{(x_N)}\left(x_{1,0}, x_{2,0}, \dots, x_{N,0}\right) \end{bmatrix}$$

Hessian

$$\mathbf{H} = \begin{bmatrix} f_1^{(x_1)} & f_1^{(x_2)} & \cdots & f_1^{(x_n)} \\ f_2^{(x_1)} & f_2^{(x_2)} & \cdots & f_2^{(x_n)} \\ \vdots & \vdots & \ddots & \vdots \\ f_n^{(x_1)} & f_n^{(x_2)} & \cdots & f_n^{(x_n)} \end{bmatrix}$$

Inverse of a 2x2 Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad - bc}$$

Quadratic $ax^2 + bx + c = 0$

roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ min/max $\dfrac{-b}{2a}$

Regula-Falsi method update $x_0 = \dfrac{af\left(b\right) - bf\left(a\right)}{f\left(b\right) - f\left(a\right)}$

Least squares regression

$$\mathbf{M} = \mathbf{AP} \qquad \mathbf{P} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{M}$$