MIDTERM (2)

Problem 1 (25 marks)

The Big B Company has hired a C290 student to model their firm's operations. Big B assembles and sells 2 products, the X and the Y. Big B imports all the small plastic parts needed to make an X and a Y and needs only to add labour and special fasteners to assemble the 2 products so they are complete and ready for sale. Assembly of each X requires 20 minutes of labour and each Y requires 30 minutes of labour. There are 190 hours of labour available during the next month. In addition, 2 fasteners and 1 fastener are needed to assemble one X and one Y respectively. For the next month, 540 fasteners are available.

In addition to these constraints, there are minimum monthly production levels of 30 units for X and 50 units for Y. Due to storage limitations, a maximum of 200 units of X can be stored in a month. A careful analysis of revenues and costs for labour and fasteners, has provided profit contributions of $5 00 for each X and $4.00 fo each Y. Big B is interested in maximizing profit.

Recognizing that the management at Big B are computer illiterate and are unlikely to understand an Excel based model, our C290 students has decided to use an algebraic/geometric approach to model the firm's operations. The correct algebraic formulation and partially completed graph are shown on the next page.

Constraints

Labour = .33 X .5Y ≤ 190h

Fasteners = 2X + Y ≤ 540 fast

$X \geq 30$ $X \leq 200$

$Y \geq 50$ $30 \leq X \leq 200$

P = 5X + 4Y

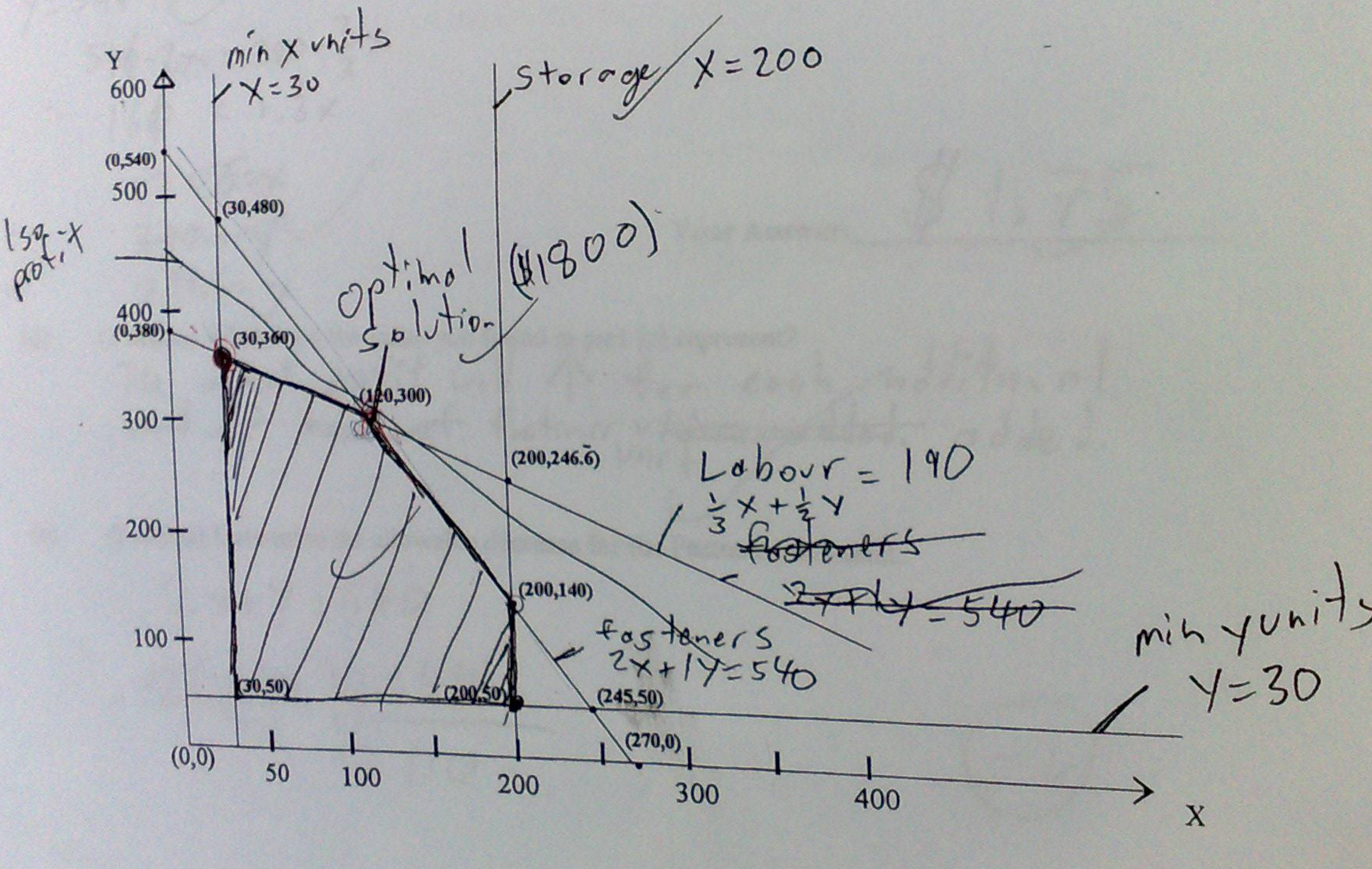
objective = max profit.

Let X = number of X's to assemble in the next month
Y = number of Y's to assemble in the next month

Max 5X + 4Y
st

Labour)	1/3 X + ½ Y <= 190	hours
Fasteners)	2X + 1 Y <= 540	units
Minimum X)	X >= 30	units
Minimum Y)	Y >= 50	units
Storage)	X <= 200	units

(a) (3 marks) The partially completed graph below correctly identifies all the constraints listed above and correctly identifies the values for the points of intersection. **Label** all the constraints on this graph and clearly (i) shade the feasible region and (ii) clearly identify at which intersection the optimal solution exists.



1560

unused resources

(b) (2 marks) Explain why the point (100,200) is not optimal.

it is inside the feasible region which means ~~the~~ it has "twins." In other words there are other points which will have the same output therefor this is not optimal. Also to be optimal, point must be on a binding constraint intersection.

(−1)

(c) (4 marks) Determine the shadow price of the Fastener constraint.

$5(120.75) + 4(299.5) = 1801.75$

$5(120) + 4(300) = 1800$

1.75

fasteners: $2x + y = 540$

labour: $\frac{1}{3}x + \frac{1}{2}y = 190$

$y = 540 - 2x$

$y = 380 - \frac{2}{3}x$

$540 - 2x = 380 - \frac{2}{3}x$

$160 = 1.\dot{3}x$

$120.75 = x$

~~300 = y~~

$299.5 = y$

Your Answer: $1.75

(d) (2 marks) What does the value you found in part (c) represent?

The amount profit will ↑ for each additional ~~unit of hour of~~ fastener unit ~~time added.~~ added.

(e) (2 marks) Determine the allowable decrease for the Fastener constraint.

$2x + y = 540$

$2(200) + 30 = 430$

110

(−2)

Your Answer: allowable decrease = 110 ~~units~~ fasteners units

(f) (2 marks) Determine the shadow price of the Min X constraint.

non binding

Your Answer: $0

(g) (3 marks) Suppose the shadow price for the Labour constraint is 4.5. Consider the following statement: This means that if we increase the Right Hand Side (RHS) by 1 hour, profit would go up by $4.50. This statement is true if:

(A) the cost of labour is relevant and the constraint is non-binding
(B) the cost of labour is sunk and the constraint is non-binding
(C) the cost of labour is relevant and the constraint is binding
(D) the cost of labour is relevant and the constraint is non-binding
(E) the cost of labour is relevant and the constraint is redundant
(F) the cost of labour is relevant and the constraint is redundant
(G) none of the above is correct

Your Answer: C

(h) (4 marks) Determine the allowable increase and allowable decrease on the profit contribution for Product Y.

$5x + 4y$

$$-\frac{2}{1} \le -\frac{5}{4} \le -\frac{\frac{1}{3}}{\frac{1}{2}} \qquad \frac{5}{2} \le c_1 \le \frac{15}{2}$$

$$-\frac{2}{1} \le -\frac{5}{c_1} \le -\frac{2}{3}$$

$$-\frac{2}{1} \le -\frac{5}{c_1} \qquad -\frac{5}{c_1} \le -\frac{2}{3} \qquad c_1 \le \frac{15}{6}$$

$$-c_1 \le -\frac{5}{2} \qquad -15 \le -2c_1$$

$$c_1 \ge \frac{5}{2} \qquad -\frac{15}{2} \le c_i \qquad \frac{15}{2} \ge c_1$$

Your Answer: $\frac{5}{2} \le c_1 \le \frac{15}{2}$

allowable decrease = 1.5
allowable increase = 3.5

(i) (3 marks) Suppose that management at Big B have decided to add a Packaging constraint to their model where each X requires 3 minutes and each Y requires 2 minutes of time. In all, 25 hours (1,500 minutes) are available for this task. Explain the effect this would have on the optimal solution.

This would have no effect as it would be a non-binding constraint. Th total minutes needed would be $3(120) + 2(300) = 960$ minutes which is less than th 1500 minutes available.

Please do not write below this line.

MIDTERM

Problem 3 (12 marks)

The Ferris Company produces precision medical diagnostic equipment at two factories. Three Medical Centres (MC's) have placed confirmed orders for this month's production output. The following table shows what the cost would be for shipping 1 unit from each Factory to each Medical Centre. The production capacity at Factory 1 is 500 units and the capacity at Factory 2 is 600 units. Confirmed orders have been received from each of the 3 Medical Centres and are listed in the table below. An additional consideration is that due to a union contract requirement, a minimum of 250 units must be shipped from Factory 2 to Medical Centre 3.

A decision needs to be made about the shipping plan for how many units to ship from each Factory to each Medical Centre to exactly meet the confirmed orders at the lowest possible shipping cost.

Unit Costs	MC 1	MC2	MC3
Factory 1	$60	$80	$70
Factory 2	$40	$90	$60
Confirmed Order size	200 units	300 units	400 units

demand

(a) (2 marks) Clearly define the decision variables in this problem.

Let $F1_{mci}$ = # of units shipped from factory 1 to medical centre i in the production period (month) $i = 1, 2, 3$

Let $F2_{mci}$ = # of units shipped from factory 2 to medical centre i in the production period (month) $i = 1, 2, 3$

✓

(b) (2 marks) Write the objective function?

minimize ✓

$$60(F1_{mc1}) + 80(F1_{mc2}) + 70(F1_{mc3}) + 40(F2_{mc1}) + 90(F2_{mc2}) + 60(F2_{mc3})$$

✓

(c) (8 marks) List the constraints for this problem. Be sure to identify each constraint by name and indicate the correct units for each constraint

S.T.

Production Capacity factory 1 $F1_{mc1} + F1_{mc2} + F1_{mc3} \leq 500$ units of medical diagnostic equipment ✓

Production Capacity factory 2 $F2_{mc1} + F2_{mc2} + F2_{mc3} \leq 600$ units " ✓

Union contract $F2_{mc3} \geq 250$ units " ✓

MC1 order $F1_{mc1} + F2_{mc1} = 200$ units " ✓

MC2 order $F1_{mc2} + F2_{mc2} = 300$ units " ✓

MC3 order $F1_{mc3} + F2_{mc3} = 400$ units " ✓

non-neg constraint?

(-1)

4