MATH 1402A **Term test #1 – solution** **26 September 2013**

Maximum score: **32** marks

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Non-programmable calculators are permitted. This test is closed book.

Supply your answers on this sheet, but TA's have extra paper if you need it.

PLEASE PRINT	First name	Last name	Student number

Please show your work where appropriate!

1. [2+3+3 marks] Determine the domain of the following functions, given the rule:

a. $z = f(x,y) = \dfrac{5}{x^2+3}$ $\Rightarrow \operatorname{dom}(f) = \mathbb{R}^2$

Indeed:

we require $\left.\begin{array}{l} x^2+3 \neq 0 \\ x^2 \neq -3 \end{array}\right\}$ ... which is true for all $x \in \mathbb{R}$

b. $y = f(x) = \dfrac{\sqrt{3x+12}}{x^2-1}$ $\Rightarrow \operatorname{dom}(f) = [-4,-1) \cup (-1,1) \cup (1,\infty)$

$= [-4,\infty) \setminus \{-1,1\}$

$= \{x \in \mathbb{R} \mid x \geq -4,\ x \neq -1, 1\}$

Indeed:

- Requirement 1: $3x+12 \geq 0$

  $\boxed{x \geq -4}$

AND

- Requirement 2: $x^2-1 \neq 0$

  $\boxed{\begin{array}{l} x \neq -1 \\ x \neq 1 \end{array}}$

c. $z = f(x,y) = \dfrac{\log_2(30x+5y-15)}{x}$ $\Rightarrow \operatorname{dom}(f) = \{(x,y) \in \mathbb{R}^2 \mid y > -6x+3,\ x \neq 0\}$

- Requirement 1: $30x+5y-15 > 0$

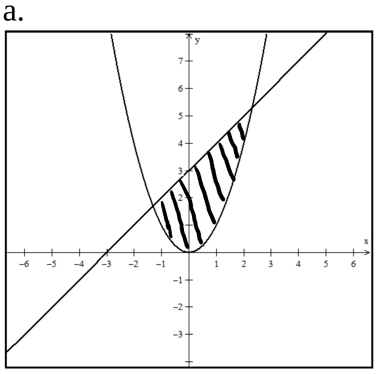
  $5y > -30x+15$

  $\boxed{y > -6x+3}$

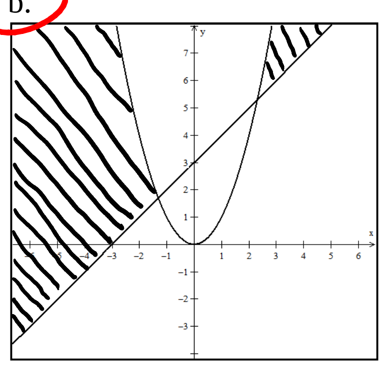
- Requirement 2: $\boxed{x \neq 0}$

2. [3] Let f be a function for which $dom(f) = \{(x,y) \in \Re^2 \mid y \leq x^2 \text{ and } y \geq x+3\}$. Select the graph which represents $dom(f)$. (Note: the curves are not labeled, but should be obvious!)

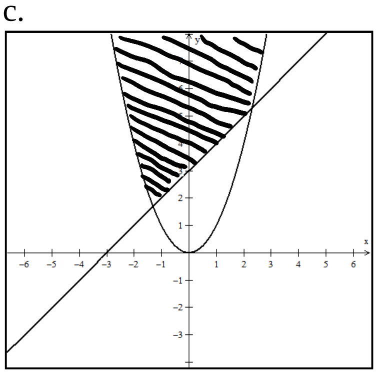
a.



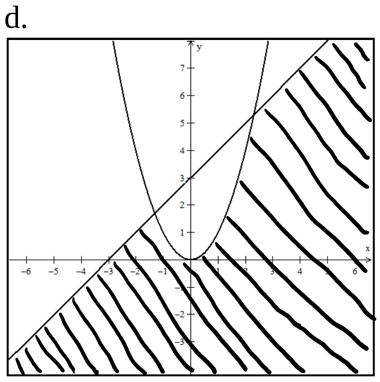
b.



c.



d.



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3. [1+3] Let $z = f(x,y) = 12x^3 - x + 12y + 10$.

a. Determine $f(-1,3)$ $= 12(-1)^3 - (-1) + 12(3) + 10 = -12 + 1 + 36 + 10$

$= 35$

b. Determine the rule $y = g(x)$ of the function obtained when considering an iso–z section (i.e. constant z) where $z = z_0 = -6$.

$$z = z_0 = -6 = 12x^3 - x + 12y + 10$$

$$12y = -12x^3 + x - 16 \Rightarrow y = -x^3 + \tfrac{1}{12}x - \tfrac{4}{3} = g(x)$$

4. [4] $z = f(x,y) = \underbrace{x^2}_{u} \underbrace{\ln(1+x^2y^2)}_{v}$ Determine $\frac{\partial z}{\partial x}$.

$u = x^2 \quad v = \ln(1+x^2y^2)$

$u' = 2x \quad v' = \dfrac{2xy^2}{1+x^2y^2}$

$\therefore$ By the product rule:

$$\frac{\partial z}{\partial x} = uv' + vu' = x^2\left(\frac{2xy^2}{1+x^2y^2}\right) + 2x\ln(1+x^2y^2)$$

5. [4] Given $z = f(x,y) = x^3 + y^3 - x^2y - xy^2 - 3x + 5y$. Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

- $\frac{\partial z}{\partial x} = 3x^2 - 2xy - y^2 - 3 \Rightarrow \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(3x^2 - 2xy - y^2 - 3) = \boxed{-2x - 2y}$
- $\frac{\partial z}{\partial y} = 3y^2 - x^2 - 2xy + 5 \Rightarrow \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(3y^2 - x^2 - 2xy + 5) = \boxed{-2x - 2y}$

6. [3] Let $Q = f(K,L) = 6K^{\frac{2}{5}} L^{\frac{3}{5}}$ be the rule for a production function. Determine the rule $K = g(L)$ that represents an isoquant for which $Q = Q_0 = 54$. **Express the answer as a root**.

$$Q = Q_0 = 54 = 6K^{2/5}L^{3/5}$$

$$K^{2/5} = \frac{9}{L^{3/5}} \Rightarrow K = \frac{9^{5/2}}{L^{3/2}} \Rightarrow K = g(L) = \frac{243}{L^{3/2}} = \frac{243}{\sqrt{L^3}}$$

7. [3] Given $K = g(L) = \frac{64}{L^4}$, determine ***MRS***, the <u>marginal rate of substitution</u> $\left|\frac{dK}{dL}\right|$ for $L = 4$.

$$K = g(L) = 64L^{-4} \Rightarrow \frac{dK}{dL} = 64(-4)L^{-5} \Rightarrow \frac{dK}{dL} = -\frac{256}{L^5}$$

$$\therefore MRS = \frac{256}{L^5} \Rightarrow MRS\Big|_{L=4} = \frac{256}{4^5} = \boxed{\frac{1}{4}}$$

8. [3] Let $Q = f(K,L) = AK^{\alpha}L^{\beta}$ (with A, α, β subject to the **<u>usual constraints</u>**). Show that $APK > MPK$.

- $APK = \frac{Q}{K} = \frac{AK^{\alpha}L^{\beta}}{K} = AK^{\alpha-1}L^{\beta}$
- $MPK = \frac{\partial Q}{\partial K} = A\alpha K^{\alpha-1}L^{\beta}$

$$\frac{APK}{MPK} = \frac{AK^{\alpha-1}L^{\beta}}{A\alpha K^{\alpha-1}L^{\beta}}$$

$\ldots = \frac{1}{\alpha} > 1$

(b.c. $\alpha < 1$)