MATH 1402A **Term test #3 – solution** **7 November 2013**

Total marks: 35 marks

Non-programmable calculators are permitted. This test is closed book.
<u>Supply your answers on this sheet</u>, but TA's have extra paper if you need it.

PLEASE PRINT ______________________________ ____________

First name Last name Student number

Please show your work where appropriate!

1. [5] Determine $\int -3x^{-1} - \frac{4}{5\sqrt[5]{x}} + (x+1)^2 - e^x \; dx$

$$\ldots = \int -\frac{3}{x} - \frac{4}{5}x^{-1/5} + x^2 + 2x + 1 - e^x \, dx = -3\ln|x| - \frac{4}{5}\frac{x^{4/5}}{4/5} + \frac{x^3}{3} + x^2 + x - e^x + C_1$$

$$\ldots = -3\ln|x| - \sqrt[5]{x^4} + \frac{1}{3}x^3 + x^2 + x - e^x + C_1$$

2. [4] Determine $\int 2(2x+1)^3 \; dx$. There are 2 ways to do this; pick one.

$(2x+1)^3 = 8x^3 + 12x^2 + 6x + 1$

$u = 2x+1 \Rightarrow du = 2dx$

$\therefore \int 2(2x+1)^3 dx = \int u^3 du =$

$\ldots = \frac{1}{4}u^4 + C_1 = \frac{1}{4}(2x+1)^4 + C_1$

$2\int 8x^3 + 12x^2 + 6x + 1 \; dx =$

$\ldots = 2\left(\frac{8}{4}x^4 + \frac{12}{3}x^3 + \frac{6}{2}x^2 + x\right) + C_1$

$\ldots = 4x^4 + 8x^3 + 6x^2 + 2x + C_1$

3. [4] Determine $\int \frac{e^{\sqrt{x}} + 1}{2\sqrt{x}} dx$ (HINT: separate the integrand first, then integrate)

$$= \int \frac{e^{\sqrt{x}}}{2\sqrt{x}} + \frac{1}{2}x^{-1/2} dx = \int e^u du + \frac{1}{2}\frac{x^{1/2}}{1/2} + C_1$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$$\ldots = e^u + x^{1/2} + C_1 = e^{\sqrt{x}} + \sqrt{x} + C_1$$

4. Let $F(x) = \int x^{-1} - 2\sqrt[3]{x^2} + x^4 - 2\sqrt[5]{x^4} + 1 \; dx$. $= \int \frac{1}{x} - 2x^{2/3} + x^4 - 2x^{4/5} + 1 \; dx =$

a. [5] Determine the most general antiderivative $F(x)$.
b. [2] Determine the constant of integration if $F(-1) = 3$

$$\ldots \; F(x) = \ln|x| - 2\frac{x^{5/3}}{5/3} + \frac{1}{5}x^5 - 2\frac{x^{9/5}}{9/5} + x + C_1$$

$$\ldots = \ln|x| - \frac{6}{5}x^{5/3} + \frac{1}{5}x^5 - \frac{10}{9}x^{9/5} + x + C_1$$

$$F(-1) = 3 = \ln|-1| - \frac{6}{5}(-1)^{5/3} + \frac{1}{5}(-1)^5 - \frac{10}{9}(-1)^{9/5} + (-1) + C_1$$

$$3 = \frac{6}{5} - \frac{1}{5} + \frac{10}{9} - 1 + C_1 \Longrightarrow C_1 = 17/9$$

$$\boxed{\therefore F(x) = \ln|x| - \frac{6}{5}x^{5/3} + \frac{1}{5}x^5 - \frac{10}{9}x^{9/5} + x + \frac{17}{9}}$$

5. [4] Determine the area under the curve $y = \sqrt[4]{x^3}$ between the values $x = 1$ and $x = 16$. The curve lies above the x axis.

$$A = \int_1^{16} \sqrt[4]{x^3} \, dx = \int_1^{16} x^{3/4} dx = \left[\frac{x^{7/4}}{7/4}\right]_1^{16} = \frac{4}{7}\left[x^{7/4}\right]_1^{16}$$

$$\ldots = \frac{4}{7}(128 - 1) = \frac{508}{7} \simeq 72.57$$

6. **Profit maximization:** you are given the following information about a firm: the wage rate (***w***) is 200, the rental rate (***r***) is 12.5, the price (***p***) is 240 and production output is given by the model $Q = 2.5\,K^{0.25}\,L^{0.5}$. Use ***any method*** to determine:

a. [4] The values of K and L that maximize profits.	b. [1] The resulting production output Q.	c. [1] The profit Π.

$$\pi = pQ - rK - wL = 240(2.5K^{0.25}L^{0.5}) - 12.5K - 200L$$

$$\frac{\partial \pi}{\partial L} = 0 \Rightarrow \quad 300\,K^{0.25}L^{-0.5} - 200 = 0 \iff 3K^{0.25}L^{-0.5} = 2 \quad (1)$$

$$\frac{\partial \pi}{\partial K} = 0 \Rightarrow \quad 150\,K^{-0.75}L^{0.5} - 12.5 = 0 \iff 12K^{-0.75}L^{0.5} = 1 \quad (2)$$

$$(1) \div (2): \quad \frac{1}{4}KL^{-1} = 2 \Rightarrow K = 8L$$

substitute in (1): $3(8L)^{0.25}L^{-0.5} = 2$

$$3 \cdot 8^{0.25} \cdot L^{0.25} \cdot L^{-0.5} = 2$$

$$L^{-0.25} = \frac{2}{3 \cdot 8^{0.25}}$$

$$L = \left(\frac{2}{3 \cdot 8^{0.25}}\right)^{-4} = 40.5$$

$K = 8L \Rightarrow \boxed{K = 324} \rightarrow Q = 2.5(324)^{0.25}(40.5)^{0.5} = \boxed{67.5 = Q}$

$$\Pi = 240(2.5(324)^{0.25}(40.5)^{0.5}) - 12.5(324) - 200(40.5) = \boxed{4050 = \Pi}$$

7. [5] Determine the area of the region enclosed between the curves $y = x^2$ and $y = 3x + 4$. It is known that the 2 curves meet at the points (–1, 1) and (4, 16).

at $x = 0 \Rightarrow y = 0$ for $y = x^2$

$\Rightarrow y = 4$ for $y = 3x+4 \Rightarrow f_1(x) = 3x+4$

$$A = \int_{-1}^{4} f_1(x) - f_2(x)\,dx = \int_{-1}^{4} (3x+4) - x^2\,dx =$$

$$\ldots = \int_{-1}^{4} 3x + 4 - x^2\,dx = \left[\frac{3}{2}x^2 + 4x - \frac{x^3}{3}\right]_{-1}^{4} =$$

$$\ldots = \left(\frac{3}{2}(4)^2 + 4(4) - \frac{4^3}{3}\right) - \left(\frac{3}{2}(-1)^2 + 4(-1) - \frac{(-1)^3}{3}\right)$$

$$= \left(24 + 16 - 64/3\right) - \left(3/2 - 4 + 1/3\right)$$

$$\qquad\qquad\qquad 9/6 - \frac{24}{6} + \frac{2}{6}$$

$$= \frac{56}{3} + \frac{13}{6} = \frac{125}{6} = 20\tfrac{5}{6}$$