**MATH 1402A** **Term test #4 – solution** **21 November 2013**

Total marks: 35 marks

Non-programmable calculators are permitted. This test is closed book.
**Supply your answers on this sheet**, but TA's have extra paper if you need it.

**PLEASE PRINT** ________________________ ____________

First name Last name Student number

## Please show your work where appropriate!

**1.** [6] Evaluate the following integrals:

| a. $\int \frac{x}{x-1}\,dx$ | b. $\int_1^{10} \sqrt{3x+6}\,dx$ |
|---|---|

a. $\int \frac{x}{x-1}\,dx$   $u = x-1$, $du = dx$, $x = u+1$

$\Downarrow$

$\int \frac{u+1}{u}\,du = \int 1 + \frac{1}{u}\,du =$   $\frac{u}{u}+\frac{1}{u}$

$\ldots = u + \ln|u| + C$

$\ldots = x - 1 + \ln|x-1| + C$

or: $x + \ln|x-1| + C_1$

$(C_1 = C - 1)$

b. $\int_1^{10} \sqrt{3x+6}\,\underbrace{dx}_{\frac{1}{3}du}$

$u = 3x+6$
$\frac{1}{3}du = dx$
$u_1 = 3(1)+6 = 9$
$u_2 = 3(10)+6 = 36$

$\Downarrow$

$\frac{1}{3}\int_9^{36} \sqrt{u}\,du = \frac{1}{3}\int_9^{36} u^{1/2}\,du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_9^{36} =$

$\ldots = \frac{2}{9}\left[u^{3/2}\right]_9^{36} = \frac{2}{9}\left[36^{3/2} - 9^{3/2}\right] =$

$\ldots = \frac{2}{9}(216 - 27) = \frac{378}{9} = \boxed{42}$

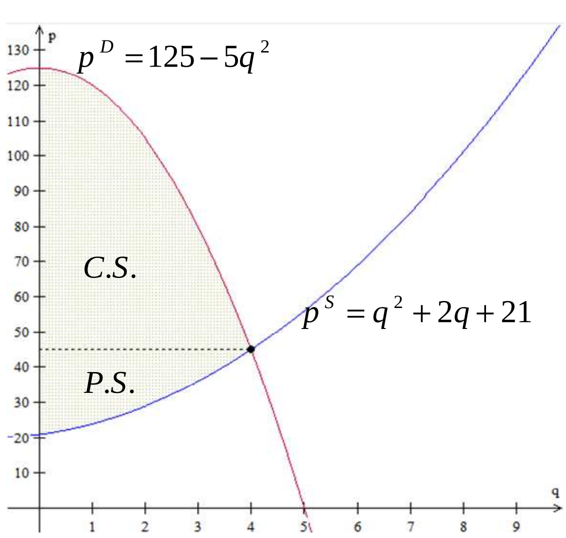
**2.** [4] Determine *TC*, the Total Cost, if *MC*, the Marginal Cost, is given by: $MC = 7q^2 + 9q - 10$ and *TC*=4000 for *q*=12.

$TC = \int MC\,dq = \int 7q^2 + 9q - 10\,dq = \frac{7}{3}q^3 + \frac{9}{2}q^2 - 10q + C$

$4000 = \frac{7}{3}(12)^3 + \frac{9}{2}(12)^2 - 10(12) + C \Rightarrow C = -560$

$\therefore\ TC = 7/3\,q^3 + 9/2\,q^2 - 10q - 560$

**3.** [6] Determine C.S. **and** P.S. for the following case. The **equilibrium point** is (4, 45).




$C.S. = \int_0^4 p^D - 45\,dq = \int_0^4 125 - 5q^2 - 45\,dq$

$\ldots = \int_0^4 80 - 5q^2\,dq = \left[80q - \frac{5}{3}q^3\right]_0^4 =$   $320 - 320/3$

$\ldots = 640/3 = 213\,1/3$

$P.S. = \int_0^4 45 - p^S\,dq = \int_0^4 45 - (q^2 + 2q + 21)\,dq =$

$\ldots = \int_0^4 24 - q^2 - 2q\,dq =$

$\ldots = \left[24q - \frac{1}{3}q^3 - q^2\right]_0^4 =$

$\ldots = 24(4) - \frac{1}{3}(4)^3 - (4)^2 =$   $96 - 64/3 - 16 = 80 - 64/3 = \frac{240}{3} - \frac{64}{3} = 176/3$

$\ldots = \boxed{58\,2/3}$

# MATH 1402A — Term test #4 – solution — 21 November 2013

Total marks: 35 marks

**4.** [4] Determine *TR*, the Total Revenue, if *MR*, the Marginal Revenue, is given by $MR = 12 - 4q$. Assume that *TR*=0 when *q*=0. Use the result obtained for *TR* to determine the inverse demand function (i.e. $p^D$=*p*).

$$TR = \int MR\,dq = \int 12 - 4q \, dq = 12q - 2q^2 + C$$

$$TR = 0 \Rightarrow q = 0 \Rightarrow C = 0$$

$$\therefore \quad TR = 12q - 2q^2 = \underbrace{(12-2q)}_{p} \cdot \underbrace{q}_{q}$$

$$\therefore \quad p = p^D = 12 - 2q$$

**5.** [3] Given the following matrix, rewrite an equivalent matrix obtained after performing the 2 specified E.R.O.'s

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 6 \\ -4 & 0 & 1 & 3 \\ 3 & 1 & 3 & 11 \end{array}\right] \xrightarrow[R_3' = R_3 - 3R_1]{R_2' = R_2 + 4R_1} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 6 \\ 0 & 12 & -3 & 27 \\ 0 & -8 & 6 & -7 \end{array}\right]$$

**6.** Consider the following system of 3 equations and 3 unknowns: $\boxed{\begin{aligned} 2x_1 + x_2 &= -3x_3 \\ x_1 + x_2 &= -1 - 2x_3 \\ x_2 - x_3 &= -2 \end{aligned}}$

a. [1] Write down the augmented matrix for the system

b. [4] Carry the augmented matrix to a R.E.F. (row-echelon form).

$$A.M. = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & -2 \end{array}\right] \quad \cdots \text{b.c.} \quad \left\{\begin{aligned} 2x_1 + x_2 &= -3x_3 \\ x_1 + x_2 &= -1 - 2x_3 \\ x_2 - x_3 &= -2 \end{aligned}\right\} \sim \left\{\begin{aligned} 2x_1 + x_2 + 3x_3 &= 0 \\ x_1 + x_2 + 2x_3 &= -1 \\ x_2 - x_3 &= -2 \end{aligned}\right.$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & -2 \end{array}\right] \xrightarrow{R_1' = R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & -2 \end{array}\right] \xrightarrow{R_2' = R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$$\underbrace{\xrightarrow{R_3' = R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -2 & 0 \end{array}\right]}_{\text{R.E.F.}} \quad \underbrace{\xrightarrow{R_3' = -\frac{1}{2}R_3} \left[\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 1 \\ 0 & \boxed{1} & 1 & -2 \\ 0 & 0 & \boxed{1} & 0 \end{array}\right]}_{\text{OPTIONAL STEP}}$$

c. [3] Solve the system using back-substitution (i.e. start with row 3, then 2, then 1)

$$\begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -2 & 0 \end{array}\right] \\ \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \end{array} \Rightarrow \begin{array}{c|c|c} -2x_3 = 0 & x_2 + x_3 = -2 & x_1 + x_3 = 1 \\ x_3 = 0 & x_2 = -2 & x_1 = 1 \end{array}$$

$$\therefore \quad (x_1, x_2, x_3) = (1, -2, 0)$$

**7.** [4] Express the solution of this system in **parametric form**:

$$\begin{array}{c} \left[\begin{array}{ccc|c} \boxed{1} & 1 & -2 & 12 \\ 0 & \boxed{3} & -1 & 9 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \end{array}$$

$$\left\{\begin{aligned} x_1 + x_2 - 2x_3 &= 12 \\ 3x_2 - x_3 &= 9 \end{aligned}\right.$$

$x_3$ is free; set $x_3 = t$

$$\therefore \quad x_2 = 3 + \tfrac{1}{3}t$$

and $x_1 = 12 - (3 + \tfrac{1}{3}t) + 2t$

$$x_1 = 9 + \tfrac{5}{3}t$$

$$\boxed{\begin{aligned} \therefore \quad x_1 &= 9 + \tfrac{5}{3}t \\ x_2 &= 3 + \tfrac{1}{3}t \\ x_3 &= t \end{aligned}}$$