CHAPTER 5

CAPITAL ALLOCATION TO RISKY ASSETS

1. A. THE EXPECTED CASH FLOW IS (.5 × $70,000) + (.5 × 200,000) = $135,000.

With a risk premium of 8 percent over the risk-free rate of 6 percent, the required rate of return is 14 percent. Therefore, the present value of the portfolio is

$135,000/1.14 = $118,421

b. *If the portfolio is purchased for $118,421, and provides an expected cash inflow of $135,000, then the expected rate of return [E(r)] is derived as follows:*

$118,421 × [1 + E(r)] = $135,000

Therefore, E(r) = 14 percent. The portfolio price is set to equate the expected rate of return with the required rate of return.

c. *If the risk premium over T-bills is now 12 percent, then the required return is*

6% + 12% = 18%

The present value of the portfolio is now

$135,000/1.18 = $114,407

d. *For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount from expected value is a penalty for risk.*

2. *When we specify utility by $U = E(r) - .005A\sigma^2$, the utility from bills is 7%, while that from the risky portfolio is $U = 12 - .005A \times 18^2 = 12 - 1.62A$. For the portfolio to be preferred to bills, the following inequality must hold: $12 - 1.62A > 7$, or $A < 5/1.62 = 3.09$. A must be less than 3.09 for the risky portfolio to be preferred to bills.*

3. *Points on the curve are derived as follows:*

$$U = 5 = E(r) - .005A\sigma^2 = E(r) - .015\sigma^2$$

The necessary value of E(r), given the value of σ^2, is therefore:

σ	σ^2	**E(r)**
0%	0	5.0%
5	25	5.375
10	100	6.5
15	225	8.375
20	400	11.0
25	625	14.375

The indifference curve is depicted by the bold line in the following graph (labelled Q3, for Question 3).

4. Repeating the analysis in problem 3, utility is:

$$U = E(r) - .005A\sigma^2 = E(r) - .02\sigma^2 = 4$$

leading to the equal-utility combinations of expected return and standard deviation presented in the table below. The indifference curve is the upward-sloping line appearing in the graph of problem 3, labelled Q4 (for Question 4).

σ	σ^2	**E(r)**
0%	0	4.00%
5	25	4.50
10	100	6.00
15	225	8.50
20	400	12.00
25	625	16.50

The indifference curve in problem 4 differs from that in problem 3 in both slope and intercept. When A increases from 3 to 4, the higher risk aversion results in a greater slope for the indifference curve since more expected return is needed to compensate for additional σ. The lower level of utility assumed for problem 4 (4 percent rather than 5 percent), shifts the vertical intercept down by 1 percent.

5. The coefficient of risk aversion of a risk-neutral investor is zero. The corresponding utility is simply equal to the portfolio's expected return. The corresponding indifference curve in the expected return-standard deviation plane is a horizontal line, drawn in the graph of problem 3, and labelled Q5.

6. A risk lover, rather than penalizing portfolio utility to account for risk, derives greater utility as variance increases. This amounts to a negative coefficient of risk aversion. The corresponding indifference curve is downward-sloping, as drawn in the graph of problem 3, and labelled Q6.

7. The portfolio expected return can be computed as follows:

W_{bills}	×	**Return on bills**	+	**W_{market}**	×	**Exp. return on market**	=	**Portfolio expected return**	**Portfolio standard deviation (= w_{market} × 17.12)**
.0		5%		1.0		9.24%		9.24%	17.42%
.2		5		.8		9.24		8.39	13.94

.4	5	.6	9.24	7.54	10.45
.6	5	.4	9.24	6.70	6.97
.8	5	.2	9.24	5.85	3.48
1.0	5	.0	9.24	5.00	0

8. *Computing the utility from U = E(r) – .005 × A⌠^{2} = E(r) – .015⌠^{2} (because A = 3), we arrive at the following table.*

W_{bills}	W_{market}	**E(r)**	⌠	⌠ ⌠ ⌠ ⌠ ⌠ ⌠ ^{2}	**U(A = 3)**	**U(A = 5)**
.0	1.0	9.24%	17.42	303.46	4.69	1.65
.2	.8	8.39	13.94	194.32	5.48	3.53
.4	.6	7.54	10.45	109.20	5.90	4.81
.6	.4	6.70	6.97	48.58	5.97	5.49
.8	.2	5.85	3.48	12.11	5.69	5.55
1.0	5.0	5.0	0	0	5.0	5.0

The utility column implies that investors with A = 3 will prefer a position of 40 percent in the market and 60 percent in bills over any of the other positions in the table; those with A = 5 will prefer 20 percent in the market and 80 percent in bills.

9. *The column labelled U(A = 5) in the table above is computed from U = E(r) – .005 A⌠^{2} = E(r) – .025⌠^{2} (since A = 5). It shows that the more risk-averse investors will prefer the position with 20 percent in the market index portfolio, rather than the 40 percent market weight preferred by investors with A = 3.*

10. *Expected return = .3 ⌠ 8% + .7 ⌠ 18% = 15% per year*

    Standard deviation = .7 ⌠ 28% = 19.6%

11. *Investment proportions:*

    30.0% in T-bills

    .7 ⌠ 27% = 18.9% in stock A

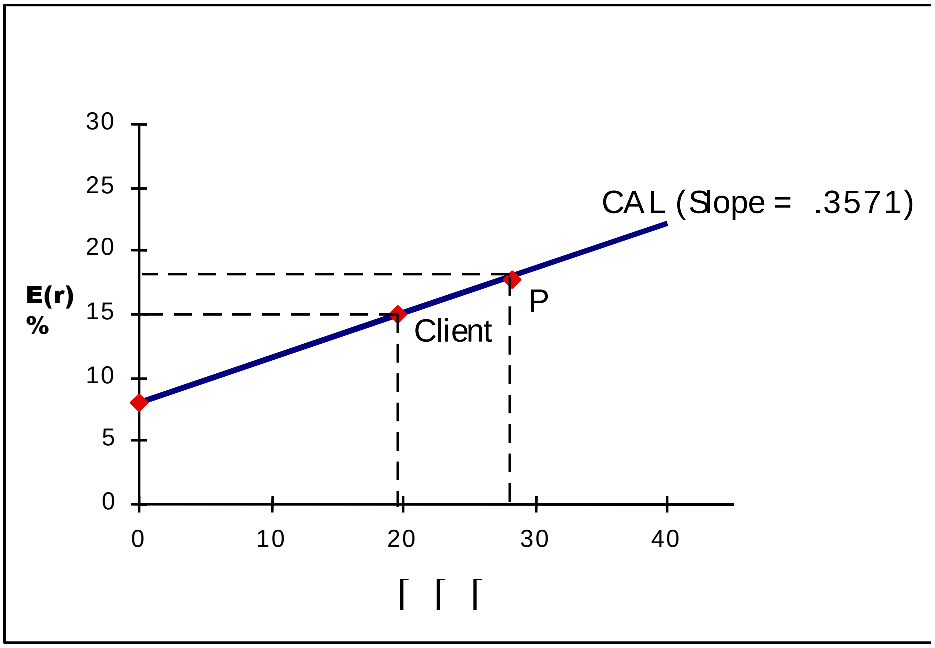
    .7 ⌠ 33% = 23.1% in stock B

    .7 ⌠ 40% = 28.0% in stock C

12. *Your reward-to-variability ratio = = .3571*

    Client's reward-to-variability ratio = = .3571

13.



14. a. $E(r_C) = r_f + [E(r_P) - r_f]\ y = 8 + 10y$

If the expected return of the portfolio is equal to 16 percent, then solving for y we get

$16 = 8 + 10y$, and $y = = .8$

Therefore, to get an expected return of 16 percent the client must invest 80 percent of total funds in the risky portfolio and 20 percent in T-bills.

b. Investment proportions of the client's funds:

20% in T-bills

.8 ⌈ 27% = 21.6% in stock A

.8 ⌈ 33% = 26.4% in stock B

.8 ⌈ 40% = 32.0% in stock C

c. ⌈ $_C$ = .8 ⌈ ⌈ $_P$ = .8 ⌈ 28% = 22.4% per year

15. a. ⌈ $_C$ = y ⌈ 28%. If your client wants a standard deviation of 18 percent at most, then

$y = 18/28 = .6429 = 64.29\%$ in the risky portfolio.

b. $E(r_C) = 8 + 10y = 8 + .6429$ ⌈ $10 = 8 + 6.429 = 14.429\%$

16. a.

y^* = = = **=** .3644

So the client's optimal proportions are 36.44 percent in the risky portfolio and 63.56 percent in T-bills.

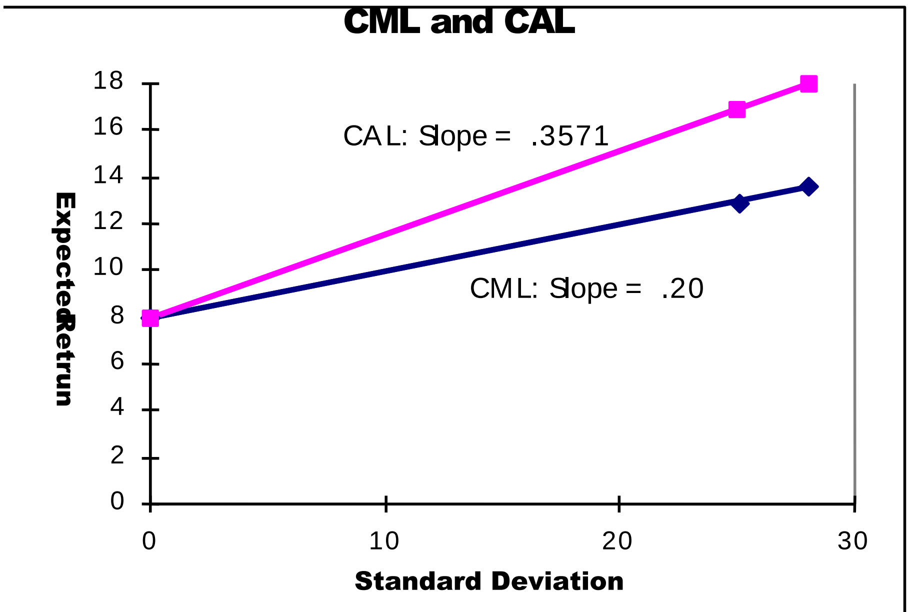
b. $E(r_C)$ = 8 + 10y^* = 8 + .3644 ⌈ 10 = 11.644%

≥ $_C$ = .3644 ⌈ 28 = 10.20%

17. a. *Slope of the CML = = .20*

The diagram is below.

b. *My fund allows an investor to achieve a higher mean for any given standard deviation than would a passive strategy, that is, a higher expected return for any given level of risk.*



18. a. *With 70 percent of his money in my fund's portfolio the client gets a mean return of 15 percent per year and a standard deviation of 19.6 percent per year. If he shifts that money to the passive portfolio (which has an expected return of 13 percent and standard deviation of 25 percent), his overall expected return and standard deviation become*

$E(r_C)$ = r_f + .7[$E(r_M)$ ⌈ r_f]

In this case, r_f = 8% *and* $E(r_M)$ = 13%. *Therefore,*

$E(r_C)$ = 8 + .7 ⌈ (13 – 8) = 11.5%

The standard deviation of the complete portfolio using the passive portfolio would be

⌈ $_C$ = .7 ⌈ ⌈ $_M$ = .7 ⌈ 25% = 17.5%

Therefore, the shift entails a decline in the mean from 14 percent to 11.5 percent and a decline in the standard deviation from 19.6 percent to 17.5 percent. Since both mean return and standard deviation fall, it is not yet clear whether the move is beneficial or harmful. The disadvantage of the shift is that if my client is willing to accept a mean return on his total portfolio of 11.5 percent, he can achieve it with a lower standard deviation using my fund portfolio, rather than the passive portfolio. To achieve a target mean of 11.5 percent, we first write the mean of the complete portfolio as a function of the proportions invested in my fund portfolio, y:

$E(r_C)$ = 8 + y(18 ⌈ 8) = 8 + 10y

Because our target is: $E(r_C)$ = 11.5%, the proportion that must be invested in my fund is determined as follows:

11.5 = 8 + 10y, y = = .35

The standard deviation of the portfolio would be: $⌈_C$ = y ⌈ 28% = .35 ⌈ 28% = 9.8%.

Thus, by using my portfolio, the same 11.5 percent expected return can be achieved with a standard deviation of only 9.8 percent as opposed to the standard deviation of 17.5 percent using the passive portfolio.

b. The fee would reduce the reward-to-variability ratio, that is, the slope of the CAL. Clients will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee.

Slope of CAL with fee = =

Slope of CML (which requires no fee) = = .20. Setting these slopes equal we get

= .20

10 ⌈ f = 28 ⌈ .20 = 5.6

f = 10 ⌈ 5.6 = 4.4% per year

19. a. The formula for the optimal proportion to invest in the passive portfolio is

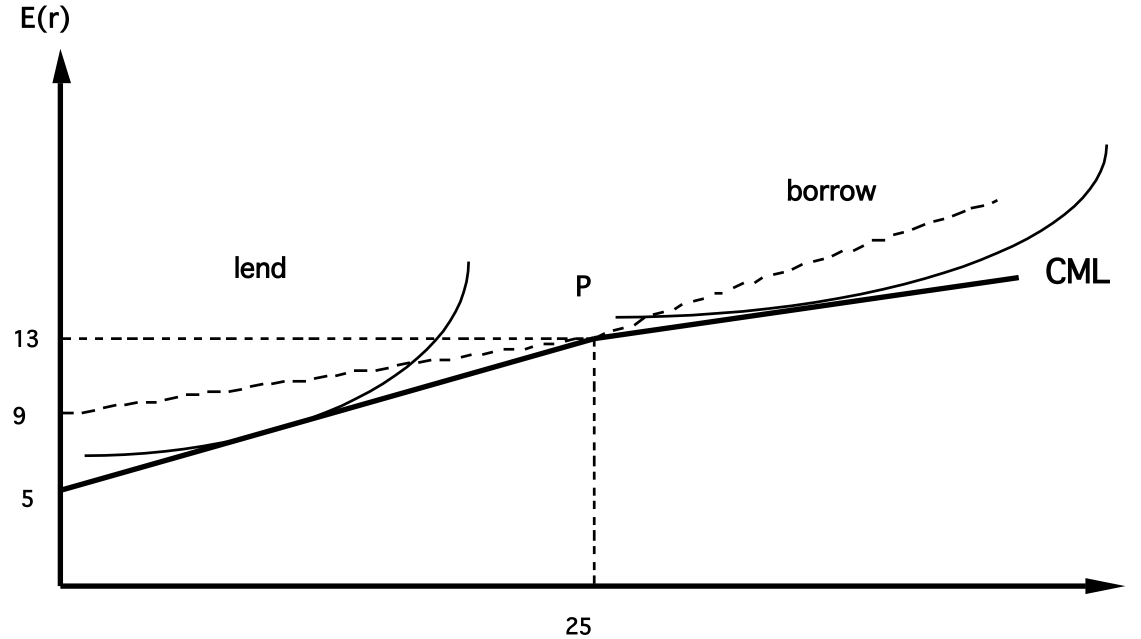
y* =

With $E(r_M)$ = 13%; r_f = 8%; $⌈_M$ = 25%; A = 3.5, we get

y* = = .2286

b. The answer here is the same as in 9(b). The fee that you can charge a client is the same regardless of the asset allocation mix of your client's portfolio. You can charge a fee that will equalize the reward-to-variability ratio of your portfolio with that of your competition.

20. If r_f = 5% but r= 9%, then the CML and indifference curves are as follows:



21. For y to be less than 1.0 (so that the investor is a lender), risk aversion must be large enough that

y = < 1

⌈ ⌈ ⌈ ⌈ ⌈ ⌈ = 1.28

For y to be greater than 1.0 (so that the investor is a borrower), risk aversion must be small enough that

⌈ y = > 1

⌈ ⌈ ⌈ ⌈ ⌈ = .64

For values of risk aversion within this range, the investor neither borrows nor lends, but instead holds a complete portfolio comprised only of the optimal risky portfolio:

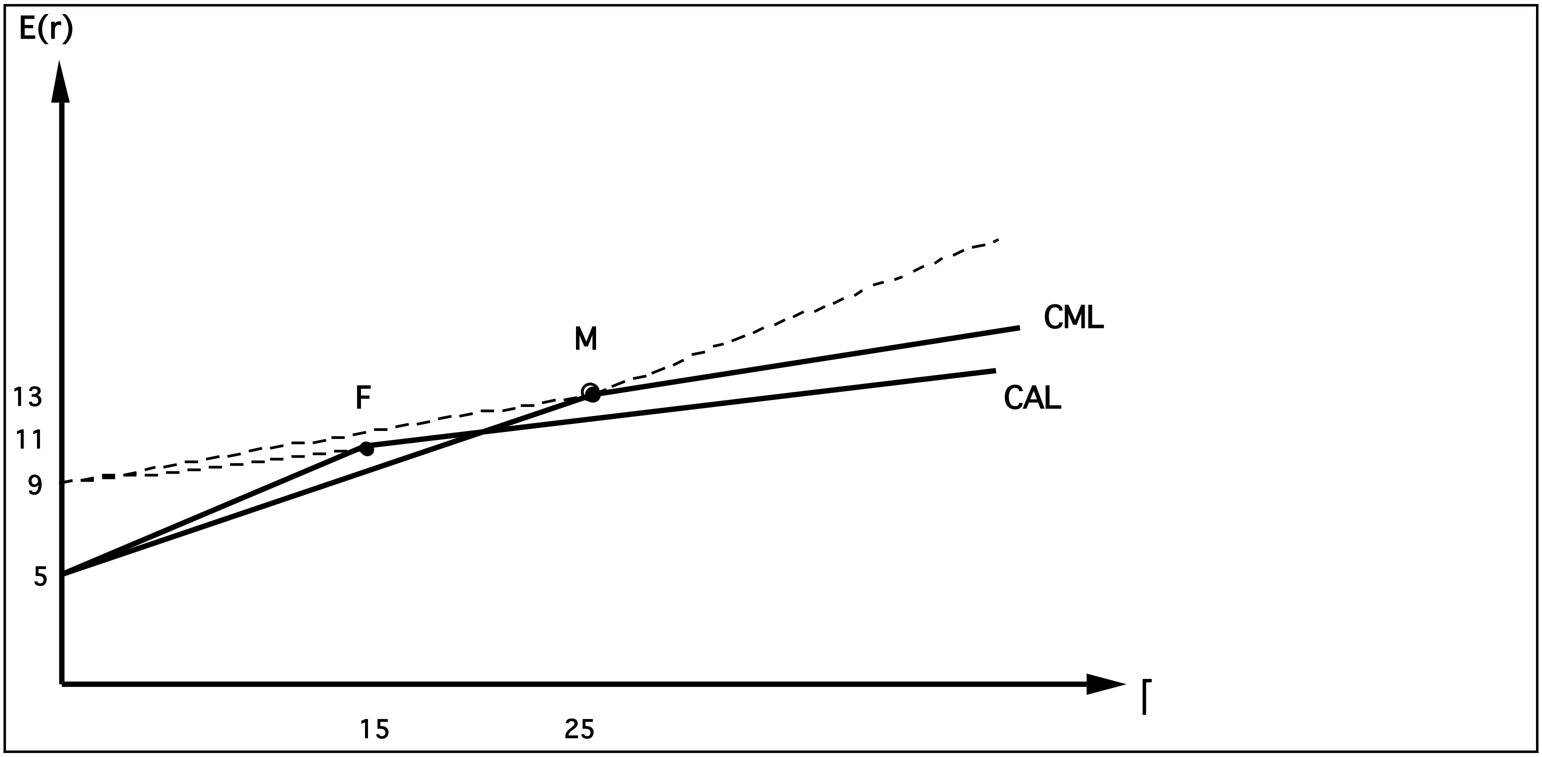
y = 1 for .64 ⌈ ⌈ ⌈ ⌈ ⌈ ⌈ ⌈ ⌈ ⌈ 1.28

22. a. The graph of problem 23 has to be redrawn here with E(r) = 11% and ⌈ = 15%.

b. For a lending position, ⌈ ⌈ ⌈ ⌈ ⌈ ⌈ = 2.67

For a borrowing position, ⌈ ⌈ ⌈ ⌈ ⌈ ⌈ = .89

In between, y = 1 for .89 ⌈ ⌈ ⌈ A⌈ ⌈ ⌈ ⌈ ⌈ 2.67



23. The maximum feasible fee, denoted f, depends on the reward-to-variability ratio.

For y < 1, the lending rate, 5 percent, is viewed as the relevant risk-free rate, and we solve for f from

=

f = 6Γ Γ = 1.2%

For y > 1 the borrowing rate, 9 percent, is the relevant risk-free rate. Then we notice that even without a fee, the active fund is inferior to the passive fund because

= .13 < = .16

More-risk-tolerant investors (who are more inclined to borrow) therefore will not be clients of the fund even without a fee. (If you solved for the fee that would make investors who borrow indifferent between the active and passive portfolio, as we did above for lending investors, you would find that f is negative: that is, you would need to pay them to choose your active fund.) The reason is that these investors desire higher risk–higher return complete portfolios and thus are in the borrowing range of the relevant CAL. In this range the reward to variability ratio of the index (the passive fund) is better than that of the managed fund.

24. a. If 1957–2012 is assumed to be representative of future expected performance, A = 2, $E(r_M)$ Γ r_f = 4.24%, and $Γ_M$ = 17.42% (we use the standard deviation of the risk premium from the last row of Table 5.7), then y* is given by

y* = = 4.24/(.01 × 2 × 17.42^2)= .6986

That is, 69.86 percent should be allocated to equity and 30.14 percent to bills.

b. If 1999–2012 is assumed to be representative of future expected performance, A = 2, $E(r_M)$ Γ r_f = 6.04%; and $Γ_M$ = 19.73%, then y* is given by

y* = 6.04/(.01 × 2 × 19.73^2)= .7758

Therefore, 77.58 percent of the complete portfolio is allocated to equity and 22.42 percent to bills.

c. In (a) the market risk premium is expected to be lower while the market risk is expected to be at a lower level than in (b). The fact that the reward-to-volatility ratio is expected to be lower in (a) (4.24/17.42 = .243 versus 6.04/19.73 = .31) explains the much smaller proportion invested in equity.

25. Assuming no change in tastes, an unchanged risk aversion coefficient, A, the denominator of the equation for the optimal investment in the risky portfolio will be higher. The proportion invested in the risky portfolio will depend on the relative change in the expected risk premium (the numerator) compared to the change in the perceived market risk. Investors perceiving higher risk will demand a higher risk premium to hold the same portfolio they held before. If we assume that the risk-free rate is unaffected, the increase in the risk premium would require a higher expected rate of return in the equity market.

26. a. $E(r_C) = 8\% = 5\% + y \times (11\% - 5\%) \geq$ $y = \frac{.08 - .05}{.11 - .05} = .50$

b. $\sigma_C = y \times \sigma_P = .50 \times 15\% = 7.5\%$

c. The first client is more risk-averse, allowing a smaller standard deviation.

27. Johnson requests the portfolio standard deviation to equal one-half the market portfolio standard deviation. The market portfolio $\sigma_M = 20\%$, which implies $\sigma_P = 10\%$. The intercept of the CML equals $r_f = 0.05$ and the slope of the CML equals the Sharpe ratio for the market portfolio (35%). Therefore using the CML:

$$E(r_P) = r_f + \frac{E(r_M) - r_f}{\sigma_M}\sigma_P = 0.05 + 0.35 \times 0.10 = 0.085 = 8.5\%$$

28. b

29. 3. [Utility for each portfolio = E(r) – .005 × 4 × σ^2. We chose the portfolio with the highest utility value.]

30. 4. [When investors are risk-neutral, A = 0, and the portfolio with the highest utility is the one with the highest expected return.]

31. b [Investor's aversion to risk]

32. Indifference curve 2

33. Point E

34. (.6 × $50,000) + [.4 × (≥ $30,000)] ≥ $5,000 = $13,000

35. (b)

36.

Expected return for equity fund = T-bill rate + risk premium = 6% + 10% = 16%

Expected rate of return of the client's portfolio = (.6 × 16%) + (.4 × 6%) = 12%

Expected return of the client's portfolio = .12 × $100,000 = $12,000 (which implies expected total wealth at the end of the period = $112,000)

Standard deviation of client's overall portfolio = .6 × 14% = 8.4%

37. Reward-to-volatility ratio = $\frac{.10}{.14} = 0.$

APPENDIX 5A

1. Your $50,000 investment will grow to $50,000(1.06) = $53,000 by year-end. Without insurance your wealth will then be:

	Probability	Wealth
No fire:	.999	$253,000
Fire:	.001	$ 53,000

Which gives expected utility

$.001 \times \log_e(53{,}000) + .999 \times \log_e(253{,}000) = 12.439582$

and a certainty equivalent wealth of

exp(12.439582) = $252,604.85

With fire insurance at a cost of $P, your investment in the risk-free asset will be only

$(50,000 – P)

Your year-end wealth will be certain (since you are fully insured) and equal to

(50,000 – P) × 1.06 + 200,000

Setting this expression equal to $252,604.85 (the certainty equivalent of the uninsured house) results in P = $372.78. This is the most you will be willing to pay for insurance. Note that the expected loss is "only" $200, meaning that you are willing to pay quite a risk premium over the expected value of losses. The main reason is that the value of the house is a large proportion of your wealth.

2. a. With .5 coverage, your premium is $100, and your investment in the safe asset is $49,900 which grows by year-end to $52,894. If there is a fire, your insurance proceeds are only $100,000. Your outcome will be:

	Probability	Wealth
Fire	.001	$152,894
No fire	.999	$252,894

Expected utility is

$.001 \times \log_e(152{,}894) + .999 \times \log_e(252{,}894) = 12.440222$

and

$W_{CE} = \exp(12.440222) = \$252{,}767$

b. *With full coverage, your premium is $200, and your investment in the safe asset is $49,800 which grows by year-end to $52,788. If there is a fire, your insurance proceeds are $200,000. Your outcome will be:*

	Probability	Wealth
Fire	.001	$252,788
No fire	.999	$252,788

This option eliminates all risk, so $W_{CE} = \$252{,}788$.

c. *With 1.5 coverage, your premium is $300, and your investment in the safe asset is $49,700 which grows by year-end to $52,682. If there is a fire, your insurance proceeds are $300,000. Your outcome will be:*

	Probability	Wealth
Fire	.001	$352,682
No fire	.999	$252,682

Expected utility is

$.001 \times \log_e(352{,}682) + .999 \times \log_e(252{,}682) = 12.4402205$

and

$W_{CE} = \exp(12.440222) = \$252{,}766$