CHAPTER 4

RETURN AND RISK: ANALYZING THE HISTORICAL RECORD

1. Your holding period return for the next year on the money market fund depends on what 30-day interest rates will be each month when it is time to roll over maturing securities. The one-year savings deposit will offer a 7.5 percent holding period return for the year. If you forecast the rate on money market instruments to rise significantly above the current yield of 6 percent, then the money market fund might result in a higher HPR for the year. While the 20-year Government of Canada bond is offering a yield to maturity of 9 percent per year (150 basis points higher than the rate on the one-year savings deposit at the bank), you could wind up with a one-year HPR of much less than 7.5 percent on the bond if long-term interest rates rise during the year. If the Government of Canada bond's yields rise above 9 percent during the year, then the price of the bond will fall, and the capital loss will wipe out some or all of the 9 percent return you would have received if bond yields had remained unchanged over the course of the year.

2. The true statements are (c) and (e). The explanations follow.

   Statement (c): Let σ = the annual standard deviation of the risky investments and σ_1 = the standard deviation of the first investment alternative over the two-year period. Then

   $$\sigma_1 = \sqrt{2}\,\sigma$$

   Therefore, the annualized standard deviation for the first investment alternative is equal to

   $$\frac{\sigma_1}{2} = \frac{\sigma}{\sqrt{2}}$$

   Statement (e): The first investment alternative is more attractive to investors with lower degrees of risk aversion. The first alternative (entailing a sequence of two identically distributed and uncorrelated risky investments) is riskier than the second alternative (the risky investment followed by a risk-free investment). Therefore, the first alternative is more attractive to investors with lower degrees of risk aversion. Notice, however, that if you mistakenly believed that "time diversification" can reduce the total risk of a sequence of risky investments, you would have been tempted to conclude that the first alternative is less risky and therefore more attractive to more risk-averse investors. This is clearly not the case; the two-year standard deviation of the first alternative is greater than the two-year standard deviation of the second alternative.

3. a. If businesses increase their capital spending they are likely to increase their demand for funds. This will shift the demand curve in Figure 5.1 to the right and increase the equilibrium real rate of interest.

   b. Increased household saving will shift the supply of funds curve to the right and cause real interest rates to fall.

   c. An open market sale of Treasury securities by the Bank of Canada is equivalent to a reduction in the supply of funds (a shift of the supply curve to the left). The equilibrium real rate of interest will rise.

4. a. The inflation-plus GIC is safer because it guarantees the purchasing power of the investment. Using the approximation that the real rate equals the nominal rate minus the inflation rate, the GIC provides a real rate of 1.5 percent regardless of the inflation rate.

   b. The expected return depends on the expected rate of inflation over the next year. If the rate of inflation is less than 3.5 percent then the conventional GIC will offer a higher real return than the inflation-plus GIC; if inflation is more than 3.5 percent, the opposite will be true.

   c. If you expect the rate of inflation to be 3 percent over the next year, then the conventional GIC offers you an expected real rate of return of 2 percent, which is .5 percent higher than the real rate on the inflation-protected GIC. But unless you know that inflation will be 3 percent with certainty, the conventional GIC is also riskier. The question of which is the better investment then depends on your attitude toward risk versus return. You might choose to diversify and invest part of your funds in each.

d. No. We cannot assume that the entire difference between the nominal risk-free rate (on conventional GICs) of 5 percent and the real risk-free rate (on inflation-protected GICs) of 1.5 percent is the expected rate of inflation. Part of the difference is probably a risk premium associated with the uncertainty surrounding the real rate of return on the conventional GICs. This implies that the expected rate of inflation is less than 3.5 percent per year.

5. $E(r) = = .35 \lceil \quad 44\% + .30 \lceil \quad 14\% + .35 \lceil \quad (-16\%) = 14\%$

Variance = $.35 \lceil \quad (44 - 14)^2 + .30 \lceil \quad (14 - 14)^2 + .35 \lceil \quad (-16 - 14)^2 = 630$

Standard deviation = 25.10%

The mean is unchanged, but the standard deviation has increased, as the probabilities of higher and lower returns have increased.

6. Probability distribution of price and 1-year holding period return on 30-year Canada bonds (which will have 29 years to maturity at year's end):

Economy	**Probability**	**YTM**	**Price**	**Capital Gain**	**Coupon**	**HPR**
Boom	.20	11.0%	$ 74.05	–$25.95	$8.00	–17.95%
Normal growth	.50	8.0	100.00	0.00	8.00	8.00%
Recession	.30	7.0	112.28	12.28	8.00	20.28%

7. From Table 4.3 the average risk premium on S&P/TSX composite stocks for the period 1957–2012 was 4.24 percent per year. Adding this to a risk-free rate of 3 percent gives an expected return of 7.24 percent per year for the S&P/TSX Index portfolio.

8. The average rate of return and standard deviation are quite different in the sub periods:

	Stocks		**Bonds**	
	Mean	***Std. Dev.***	***Mean***	***Std. Dev.***
1957–2012	10.43%	16.88%	8.92%	9.71%
1957–1984	10.82	17.72	6.82	10.39
1985–2012	10.05	16.82	11.02	8.65

I would prefer to use the risk premiums and standard deviations estimated over the more recent period, because it reflects better the current lower average returns than the earlier period.

9. a. Real holding period return $= - 1 =$

$= = .0588 = 5.88\%$

b. The approximation gives a real HPR of 80% – 70% = 10%, which is clearly too high.

10. $E(q) = 0 \times .25 + 1 \times .25 + 2 \times .5 = 1.25$; $E(q^2) = (1 \times 0.25) + (4 \times 0.75) = 2.25$, implying $\text{Var}(q) = 2.25 - 1.25^2 = 0.6875$

11. a. Corresponding to plus or minus two standard deviations.

12. a. The T-bill rate projection is the average real rate plus the inflation of 3 percent. From Table 4.3 this is 2.43% + 3% = 5.43%.

b. Since the average excess return over the T-bill rate is 4.20 percent, the expected return on large stocks would be 5.43% + 4.20% = 9.63%.

c. The risk premium on the stock market is not expected to change.

13. In Figure 4.1 the increased business demand for funds is expected to shift the demand curve up. Unless the government expands the supply through monetary policy the interest rates will rise.

14. a. Probability distribution of the HPR on the stock market and put:

		Stock		Put	
State of the Economy	**Probability**	**Ending Price + Dividend**	**HPR**	**Ending Value**	**HPR**
Excellent	.25	$ 131.00	31.00%	$ .00	⌈ 100%
Good	.45	$ 114.00	14.00%	$ .00	⌈ 100%
Poor	.25	$ 93.25	−6.75%	$ 20.25	68.75%
Crash	.05	$ 48.00	⌈ 52.00 %	$ 64.00	433.33%

Remember that the cost of the index fund is $100 per share, and the cost of the put option is $12.

b. The cost of one share of the index fund plus a put option is $112. The probability distribution of the HPR on the portfolio is:

State of the Economy	**Probability**	**Ending Price + Put + Dividend**	**HPR**	
Excellent	.25	$ 131.00	17.0%	= (131 ⌈ 112)/112
Good	.45	$ 114.00	1.8%	= (114 ⌈ 112)/112
Poor	.25	$ 113.50	1.3%	= (113.50 ⌈ 112)/112
Crash	.05	$ 112.00	.0%	= (112 ⌈ 112)/112

c. Buying the put option guarantees the investor a minimum HPR of .0% regardless of what happens to the stock's price. Thus, it offers insurance against a price decline.

15. The probability distribution of the dollar return on CD plus call option is:

State of the Economy	**Probability**	**Ending Value of CD**	**Ending Value of Call**	**Combined Value**
Excellent	.25	$ 114.00	$16.50	$130.50
Good	.45	$ 114.00	$ .00	$114.00
Poor	.25	$ 114.00	$ .00	$114.00
Crash	.05	$ 114.00	$ .00	$114.00

16. a. The CC real rate r is found by setting $100 = 84.49e^{10r}$. Solving, we find $r = \ln(100.84.49)/10 = 1.69$ percent.

b. Since the real-estate investment average expected growth is 2 percent per quarter, its annual CC growth rate is $4 \times \ln(1.02) = 7.92$ percent. The annual risk premium, therefore, is $7.92 - 1.69 = 6.23$ percent.

c. From the previous question it follows that the quarterly CC average excess return is $6.23/4 = 1.56$ percent. The variance of the effective quarterly rate is $.1^2 = 0.01$ or 1 percent. If m and s represent the CC quarterly mean and variance of the

real estate investment, the formula that gives the variance of the effective quarterly yield of the real estate investment is $(e^s - 1)e^{2m+s}$, equal to .01 in this case. The Excel solver yields s = .00955, corresponding to an annual CC variance of 4 × .00955 = .0382 and a standard deviation of SQRT(.0382) = .1954, or 19.54 percent.

d. In ten years the CC mean and standard deviation excess return parameters are .623 and SQRT(.382) = .6181. The probability that the excess return will be negative is therefore equal that the probability that the standard normal variable will be less than –.623/.6181 = –1.008, which from Excel is equal to .1567. If we want the probability that the total real return will be negative we use –.792/.6181 = –1281, equal to .1001.

17. 19%

18. For X:
$\sigma^2 = 0.2 \times (-0.2)^2 + 0.5 \times (0.18)^2 + 0.3 \times (0.5)^2 - [E(X)]^2 = 0.0592$
Implying standard deviation = sqrt(0.0592) = 0.243

For Y:
$\sigma^2 = 0.2 \times (-0.15)^2 + 0.5 \times (0.20)^2 + 0.3 \times (0.10)^2 - [E(Y)]^2 = 0.0175$
Implying standard deviation =sqrt(0.0175) = 0.132

19. 10%

20. Expected return from investing in equities is 0.6 x 50,000 + 0.4 x (-30,000) = $6,000

Expected return from T-bills is #5,000, implying that the expected risk premium is $1,000.

21. .2 x (-25%) + .3 x 30% + .5 x 24% = 16%

22. a. Probability distribution of HPR on the stock market and put:

		Stock		**Put**	
State of the Economy	***Probability***	***Ending Price + $4 Dividend***	***HPR***	***Value***	***HPR***
Boom	.25	$144	44%	0	–100%
Normal growth	.50	114	14%	0	–100%
Recession	.25	84	–16%	$30	150%

Remember that the cost of the stock is $100 per share, and that of the put is $12.

b. The cost of one share of stock plus a put is $112. The probability distribution of HPR on the stock market plus put is:

State of the Economy	***Probability***	**Stock + Put + $4 Dividend** ***Ending Value***	***HPR***
Boom	.25	$144	28.6% (144 – 112)/112
Normal growth	.50	114	1.8 (114 – 112)/112
Recession	.25	114	1.8

c. Buying the put option guarantees you a minimum HPR of 1.8 percent regardless of what happens to the stock's price. Thus, it offers insurance against a price decline.

23. The probability distribution of the dollar return on CD plus call option is:

Economy	***Probability***	***Ending Value CD***	***Ending Value Call***	***Combined Value***
Boom	.25	114 (107.55x1.06)	$30	$144
Normal growth	.50	114	0	114
Recession	.25	114	0	114