STAT 2507 MIDTERM TEST SUMMER 2014

Total Marks = 100

Last Name: _____First Name: _____

Student Number: _____

INSTRUCTIONS: This test consists of 20 multiple-choice questions. Please use scantron sheet to answer the questions. All questions are equally weighted. This is a 90-minute closed-book test. You are allowed to use non-programmable calculator. **Formula sheet, probability tables and a blank page for rough calculations are attached to the test paper. This test contains 8 pages.**

1) You asked ten of you classmates about their weight. On the basis of this information, you estimated that the average weight of all students at your school is 71.8kg. What is this an example of?
   (a) Descriptive statistics
   (b) Statistical inference*
   (c) A sample
   (d) A population

2) A market share of 78.5% can be represented in a pie chart by a slice having a central angle measure in degrees. In this case, what would be the size of the central angle of the market share?
   (a) 39.3 degrees
   (b) 78.5 degrees
   (c) 141.3 degrees
   (d) 282.6 degrees *

3) A stem-and-leaf plot is used to display the distribution of which of the following kinds of data?
   (a) Qualitative data
   (b) Quantitative data *
   (c) Two quantitative variables
   (d) Two qualitative variables

4) Which of the following is a property of a right-skewed distribution?
   (a) The mean is greater than the median *
   (b) The mean and median are equal
   (c) The mean is less than the median
   (d) The mean is less than the mode

5) Which of the following summary measures is most affected by outliers?
   (a) The first quartile
   (b) The second quartile
   (c) The third quartile
   (d) Variance *

6) The following data represent a sample of 10 scores on a 20-point statistics quiz: 16, 16, 16, 16, 16, 18, 18, 20, 20, and 20. After the mean, median, range and variance were calculated for the scores, it was discovered that one of the scores of 20 should have been an 18. Which of the following pairs of measures will change when the calculations are redone using the correct scores?
   (a) Mean and range
   (b) Median and range
   (c) Mean and variance *
   (d) Median and variance

7) If P(A) = 0.8, P(B) = 0.6 and P(AUB) = 0.9, then what is the value of P(A∩B) ?
   (a) 0.72
   (b) 0.60
   (c) 0.63
   (d) 0.50*

8) A sample of 50 values produced the following summary statistics: Q_1 = 10, Q_2 = 14.6, Q_3 = 16.7 and $\bar{x}$ =1.53. Based on this information, what are the left and right ends, respectively, of the box of the box plots using whiskers?
   (a) 5.3 and 32.0
   (b) 10.0 and 14.6
   (c) 10.0 and 16.7 *
   (d) 14.6 and 16.7

9) According to Tchebysheff's Theorem, what is the percentage of measurements in a data set that will fall within three standard deviations of the mean?
   (a) 16%
   (b) At least 68%
   (c) 75%
   (d) At least 89% *

10) Given that S_x^2 = 400, S_y^2 = 625, S_{xy} = 350 and n = 10, what is the correlation coefficient?
   (a) 0.875
   (b) 0.700 *
   (c) 0.56
   (d) 0.156

11) Which of the following values would be the correlation coefficient produced by a perfectly straight line sloping downward?
   (a) +1
   (b) -1 *
   (c) +2
   (d) -2

12) Suppose P(A) = 0.4, P(B) = 0.3 and P(A∩B) = 0, Which one of the following statements correctly defines the relationship between events A and B?
   (a) Events A and B are independent, but not mutually exclusive
   (b) Events A and B are mutually exclusive, but not independent *
   (c) Events A and B are neither mutually exclusive nor independent
   (d) Events A and B are both mutually exclusive and independent

13) Steve takes either a bus or subway to go to work, with probabilities 0.25 and 0.75 respectively. When he takes the bus, he is late 40% of the time. When takes the subway, he is late 30% of the time. If Steve is late for work on a particular day, what is the probability that he took the bus?
   (a) 0.4000
   (b) 0.1000
   (c) 0.4444
   (d) 0.3077 *

14) How many ways can one choose a combination of three items out of eight distinct items?
   (a) 28
   (b) 56 *
   (c) 112
   (d) 224

15) The probability distribution of the number of accidents in North York, Ontario, each day is given by

X	0	1	2	3	4	5
P(x)	0.20	0.10	0.30	0.15	0.20	0.05

Based on this distribution, what would be the expected number of accidents on a given day?

(a) 4.62
(b) 2.15
(c) 1.81
(d) 2.20 *

16) If the random variable x is binomially distributed with n=10 and p=0.05, what is P(x=1)?

(a) 0.914
(b) 0.315 *
(c) 0.550
(d) 0.074

17) Which of the following is NOT a characteristic of a binomial problem?

(a) There are n identical trials, and all trials are independent
(b) Each trial has two possible outcomes, which are traditionally labelled “failure” and “success”, and the probability of success p is the same on each trial
(c) We are interested in x, the number of successes observed during the n trials.
(d) The probability of failure may differ from trial to trial *

18) In order to determine the quality of a shipment of 20 parts, a sample of 3 items is randomly selected without replacement from the shipment. Four of 20 items in the shipment are actually defective. Let Y be a random variable representing the total number of defective items in the sample. Then P(Y=1) is

(a) 0.48
(b) 0.60
(c) 0.08
(d) 0.42 *

19) Your professor erases the board in class according to a Poisson process with an average of 2 times every 30 minutes. What is the probability that your professor will erase the board exactly 4 times during a 90 minute class?

(a) 0.542
(b) 0.161
(c) 0.089
(d) 0.134 *

20) When sampling without replacement, which of the following is the appropriate probability distribution to use?

(a) The binomial distribution
(b) The hypergeometric distribution *
(c) The Poisson distribution
(d) The normal approximation to the binomial distribution

Cumulative Binomial Probabilities: P(X ≤ x)

n	x	p									x
		0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	
10	0	0.3487	0.1074	0.0282	0.0060	0.0010	0.0001	0.0000	0.0000	0.0000	0
	1	0.7361	0.3758	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	0.0000	1
	2	0.9298	0.6778	0.3828	0.1673	0.0547	0.0123	0.0016	0.0001	0.0000	2
	3	0.9872	0.8791	0.6496	0.3823	0.1719	0.0548	0.0106	0.0009	0.0000	3
	4	0.9984	0.9672	0.8497	0.6331	0.3770	0.1662	0.0473	0.0064	0.0001	4
	5	0.9999	0.9936	0.9527	0.8338	0.6230	0.3669	0.1503	0.0328	0.0016	5
	6	1.0000	0.9991	0.9894	0.9452	0.8281	0.6177	0.3504	0.1209	0.0128	6
	7	1.0000	0.9999	0.9984	0.9877	0.9453	0.8327	0.6172	0.3222	0.0702	7
	8	1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.6242	0.2639	8
	9	1.0000	1.0000	1.0000	0.9999	0.9990	0.9940	0.9718	0.8926	0.6513	9
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	10
11	0	0.3138	0.0859	0.0198	0.0036	0.0005	0.0000	0.0000	0.0000	0.0000	0
	1	0.6974	0.3221	0.1130	0.0302	0.0059	0.0007	0.0000	0.0000	0.0000	1
	2	0.9104	0.6174	0.3127	0.1189	0.0327	0.0059	0.0006	0.0000	0.0000	2
	3	0.9815	0.8389	0.5696	0.2963	0.1133	0.0293	0.0043	0.0002	0.0000	3
	4	0.9972	0.9496	0.7897	0.5328	0.2744	0.0994	0.0216	0.0020	0.0000	4
	5	0.9997	0.9883	0.9218	0.7535	0.5000	0.2465	0.0782	0.0117	0.0003	5
	6	1.0000	0.9980	0.9784	0.9006	0.7256	0.4672	0.2103	0.0504	0.0028	6
	7	1.0000	0.9998	0.9957	0.9707	0.8867	0.7037	0.4304	0.1611	0.0185	7
	8	1.0000	1.0000	0.9994	0.9941	0.9673	0.8811	0.6873	0.3826	0.0896	8
	9	1.0000	1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.6779	0.3026	9
	10	1.0000	1.0000	1.0000	1.0000	0.9995	0.9964	0.9802	0.9141	0.6862	10
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	11
12	0	0.2824	0.0687	0.0138	0.0022	0.0002	0.0000	0.0000	0.0000	0.0000	0
	1	0.6590	0.2749	0.0850	0.0196	0.0032	0.0003	0.0000	0.0000	0.0000	1
	2	0.8891	0.5583	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	0.0000	2
	3	0.9744	0.7946	0.4925	0.2253	0.0730	0.0153	0.0017	0.0001	0.0000	3
	4	0.9957	0.9274	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000	4
	5	0.9995	0.9806	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001	5
	6	0.9999	0.9961	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005	6
	7	1.0000	0.9994	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043	7
	8	1.0000	0.9999	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256	8
	9	1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109	9
	10	1.0000	1.0000	1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410	10
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9862	0.9313	0.7176	11
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	12
13	0	0.2542	0.0550	0.0097	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0
	1	0.6213	0.2336	0.0637	0.0126	0.0017	0.0001	0.0000	0.0000	0.0000	1
	2	0.8661	0.5017	0.2025	0.0579	0.0112	0.0013	0.0001	0.0000	0.0000	2
	3	0.9658	0.7473	0.4206	0.1686	0.0461	0.0078	0.0007	0.0000	0.0000	3
	4	0.9935	0.9009	0.6543	0.3530	0.1334	0.0321	0.0040	0.0002	0.0000	4
	5	0.9991	0.9700	0.8346	0.5744	0.2905	0.0977	0.0182	0.0012	0.0000	5
	6	0.9999	0.9930	0.9376	0.7712	0.5000	0.2288	0.0624	0.0070	0.0001	6
	7	1.0000	0.9988	0.9818	0.9023	0.7095	0.4256	0.1654	0.0300	0.0009	7
	8	1.0000	0.9998	0.9960	0.9679	0.8666	0.6470	0.3457	0.0991	0.0065	8
	9	1.0000	1.0000	0.9993	0.9922	0.9539	0.8314	0.5794	0.2527	0.0342	9
	10	1.0000	1.0000	0.9999	0.9987	0.9888	0.9421	0.7975	0.4983	0.1339	10
	11	1.0000	1.0000	1.0000	0.9999	0.9983	0.9874	0.9363	0.7664	0.3787	11
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987	0.9903	0.9450	0.7458	12
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	13

Cumulative Poisson Probabilities: p(X ≤ x)

x	Value of μ										x
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	1
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197	2
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810	3
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	4
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	5
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	6
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	7
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	8

x	Value of μ										x
	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	15.00	
0	0.1353	0.0498	0.0183	0.0067	0.0025	0.0009	0.0003	0.0001	0.0000	0.0000	0
1	0.4060	0.1991	0.0916	0.0404	0.0174	0.0073	0.0030	0.0012	0.0005	0.0000	1
2	0.6767	0.4232	0.2381	0.1247	0.0620	0.0296	0.0138	0.0062	0.0028	0.0000	2
3	0.8571	0.6472	0.4335	0.2650	0.1512	0.0818	0.0424	0.0212	0.0103	0.0002	3
4	0.9473	0.8153	0.6288	0.4405	0.2851	0.1730	0.0996	0.0550	0.0293	0.0009	4
5	0.9834	0.9161	0.7851	0.6160	0.4457	0.3007	0.1912	0.1157	0.0671	0.0028	5
6	0.9955	0.9665	0.8893	0.7622	0.6063	0.4497	0.3134	0.2068	0.1301	0.0076	6
7	0.9989	0.9881	0.9489	0.8666	0.7440	0.5987	0.4530	0.3239	0.2202	0.0180	7
8	0.9998	0.9962	0.9786	0.9319	0.8472	0.7291	0.5925	0.4557	0.3328	0.0374	8
9	1.0000	0.9989	0.9919	0.9682	0.9161	0.8305	0.7166	0.5874	0.4579	0.0699	9
10	1.0000	0.9997	0.9972	0.9863	0.9574	0.9015	0.8159	0.7060	0.5830	0.1185	10
11	1.0000	0.9999	0.9991	0.9945	0.9799	0.9467	0.8881	0.8030	0.6968	0.1848	11
12	1.0000	1.0000	0.9997	0.9980	0.9912	0.9730	0.9362	0.8758	0.7916	0.2676	12
13	1.0000	1.0000	0.9999	0.9993	0.9964	0.9872	0.9658	0.9261	0.8645	0.3632	13
14	1.0000	1.0000	1.0000	0.9998	0.9986	0.9943	0.9827	0.9585	0.9165	0.4657	14
15	1.0000	1.0000	1.0000	0.9999	0.9995	0.9976	0.9918	0.9780	0.9513	0.5681	15
16	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9963	0.9889	0.9730	0.6641	16
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9947	0.9857	0.7489	17
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9976	0.9928	0.8195	18
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9965	0.8752	19
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9170	20
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9469	21
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9673	22
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9805	23
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9888	24
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9938	25
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9967	26
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9983	27
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	28
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	29
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	30
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	31
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	32
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	33

CHAPTER 2

- $S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}{(n-1)}$

CHAPTER 3

- $S_{xy} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{(n-1)}$
- Correlation Coefficient:

$$r = \frac{S_{xy}}{S_x S_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

$$= \frac{\sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{\sqrt{\left(\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right)\left(\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right)}}$$

- Slope: $b = r\left(\frac{S_y}{S_x}\right)$
- Intercept: $a = \bar{y} - b\bar{x}$

CHAPTER 4

- Permutation: $P_r^n = \frac{n!}{(n-r)!}$
- Combination: $C_r^n = \frac{n!}{r!(n-r)!}$
- Events A and B are independent if and only if $P(A \cap B) = P(A)\,P(B)$ or $P(A|B) = P(A)$ or $P(B|A) = P(B)$
- Conditional Probability:
  $P(B|A) = \frac{P(A \cap B)}{P(A)};\ P(A|B) = \frac{P(A \cap B)}{P(B)}$
- If events A and B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$, otherwise, in general $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A) = 1 - P(A^c)$ where A^c is the complement event.
- Total probability rule: If $A_1, A_2, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for an event B,

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \ldots + P(B|A_k)P(A_k) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

- Bayes' Rule:

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$$

- Expected value and variance of random variable X
  $E(X) = \mu = \sum_x xp(x), \quad \sigma^2 = \sum_x (x-\mu)^2 p(x) = (\sum_x x^2 p(x)) - \mu^2$

CHAPTER 5

- Binomial distribution with parameters n, p:
  $P(X = k) = C_k^n\, p^k (1-p)^{(n-k)}, \quad for\ k = 1, 2, 3, \cdots n$
  Where $C_k^n = \frac{n!}{k!(n-k)!}\ and\ n! = n(n-1)\ldots.(2)(1)$
  E(X) = μ = np,
  Var(X) = σ^2= np(1-p)
- Poisson distribution with parameter μ:
  $P(X = k) = \frac{\mu^k e^{-\mu}}{k!},\ for\ k = 1, 2, \cdots \propto$
  E(X) = μ
  Var(X) = σ^2 = μ

- Hyper geometric distribution with parameters N, M, n:

  $P(X = k) = \frac{C_k^M C_{n-k}^{N-M}}{C_n^N}$

  Where $C_n^N = \frac{N!}{n!(N-n)!}$

  E(X) = μ = $n\left(\frac{M}{N}\right)$

  Var(X) = σ^2 = $n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right)$