SYSC-3200 Fall 2012. Assignment 1 Solutions.

Marking notes: Formulations include marks for labelling constraints to explain what they mean, giving units for all variables, constraints, and the objective function, and explicitly listing the variable bounds (even when they are all nonnegativity bounds). Simplex solutions also include a mark for a concluding statement.

Question 1 [5 marks]

Variables:

NP: kg of nuts in PowerTrail
FP: kg of dried fruit in PowerTrail
CP: kg of chocolate chips in PowerTrail
NF: kg of nuts in FunTrail
FF: kg of dried fruit in FunTrail
CF: kg of choc. chips in FunTrail

Objective function:

maximize total revenue from sales ($)

Maximize Z = 26(NP+FP+CP) + 21(NF+FF+CF)

Constraints:

Availability of nuts (kg): $NP+NF \leq 250$

Availability of dried fruit (kg): $FP+FF \leq 200$

Availability of chocolate chips (kg): $CP+CF \leq 100$

PowerTrail nuts fraction: $NP/(NP+FP+CP) \geq 0.6$
$\rightarrow 0.4NP - 0.6FP - 0.6CP \geq 0$

PowerTrail dried fruit fraction: $FP/(NP+FP+CP) \geq 0.25$
$\rightarrow -0.25NP + 0.75FP - 0.25CP \geq 0$

PowerTrail chocolate chip fraction: $CP/(NP+FP+CP) \leq 0.1$
$\rightarrow -0.1NP - 0.1FP + 0.9CP \leq 0$

FunTrail chocolate chip fraction: $CF/(NF+FF+CF) \geq 0.25$
$\rightarrow -0.25NF - 0.25FF + 0.75CF \geq 0$

FunTrail dried fruit fraction: $FF/(NF+FF+CF) \geq 0.25$
$\rightarrow -0.25NF + 0.75FF - 0.25CF \geq 0$

Variable bounds:

$NP, FP, CP, NF, FF, CF \geq 0$

Question 2 [5 marks]

Variables:

x_{1g}: barrels of oil 1 used in making gasoline
x_{1h}: barrels of oil 1 used in making heating oil
x_{2g}: barrels of oil 2 used in making gasoline
x_{2h}: barrels of oil 2 used in making heating oil
a_g: dollars used to advertise gasoline
a_h: dollars used to advertise heating oil

Objective function:

Maximize net profit ($)

Maximize $Z = 25(x_{1g} + x_{2g}) + 20(x_{1h} + x_{2h}) - a_g - a_h$

Constraints:

Oil 1 availability (barrels): $x_{1g} + x_{1h} \leq 5000$

Oil 2 availability (barrels): $x_{2g} + x_{2h} \leq 10,000$

Gasoline quality (quality units): $(10x_{1g} + 5x_{2g})/(x_{1g} + x_{2g}) \geq 8 \rightarrow 2x_{1g} - 3x_{2g} \geq 0$

Heating oil quality (quality units): $(10x_{1h} + 5x_{2h})/(x_{1h} + x_{2h}) \geq 6 \rightarrow 4x_{1h} - x_{2h} \geq 0$

Gasoline demand generated by advertising (barrels): $5a_g - x_{1g} - x_{2g} = 0$

Heating oil demand generated by advertising (barrels): $10a_h - x_{1h} - x_{2h} = 0$

Variable bounds:

$x_{1g}, x_{1h}, x_{2g}, x_{2h}, a_g, a_h \geq 0$

Question 3 [10 marks]

(i) Formulation [3 marks]

Variables: M: rate at which to manufacture plastic mugs (mugs/day)
B: rate at which to manufacture plastic boxes (boxes/day)

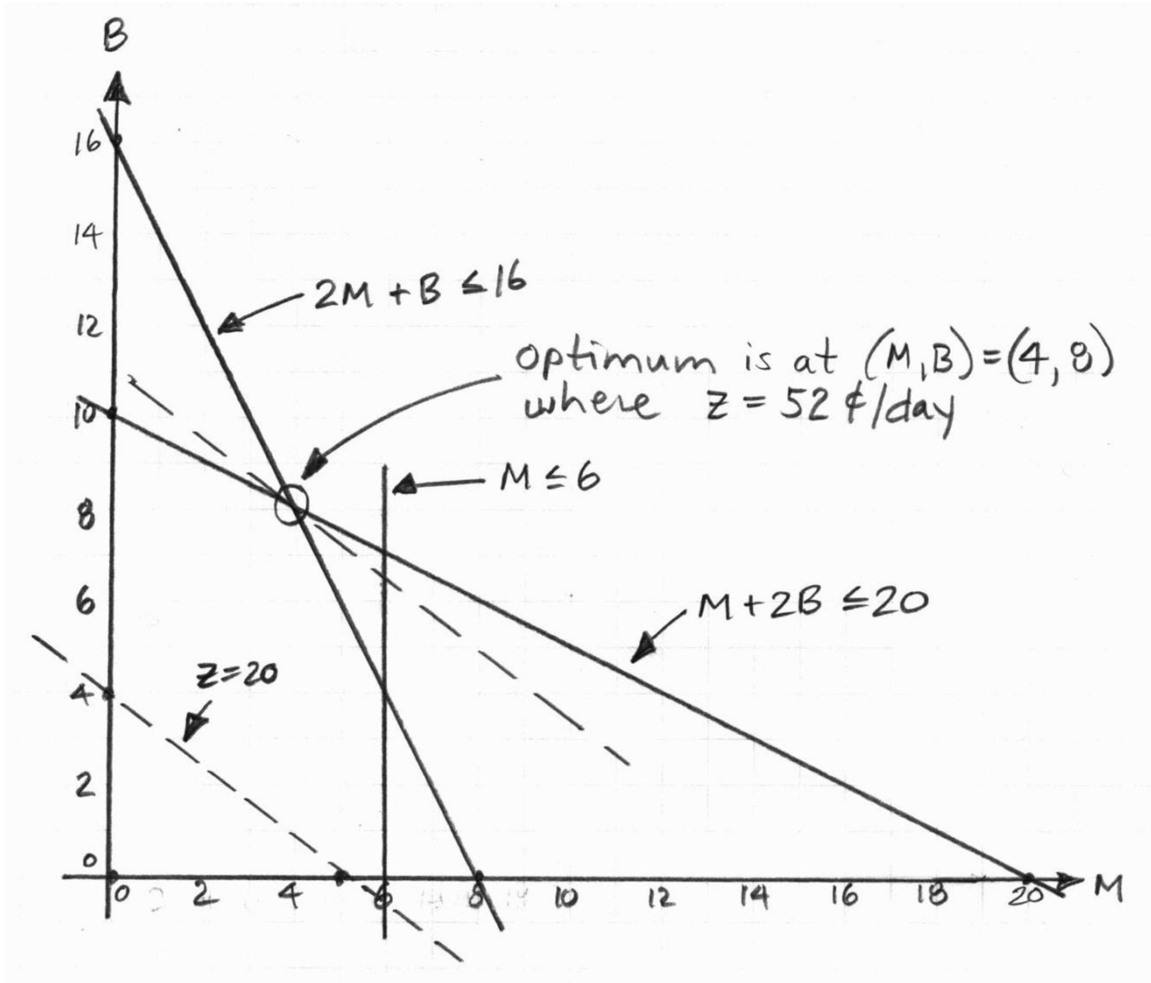
Objective function: maximize profit rate (¢/day)
$\text{Max } Z = 5M + 4B$

Constraints:

- Plastic supply (kg/day): $M + 2B \leq 20$
- Labour availability (hours/day): $2M + B \leq 16$
- Market limit on mugs (mugs/day): $M \leq 6$

Variable bounds: $M \geq 0, B \geq 0$

(ii) Graphical Solution [3 marks]



So mugs should be made at a rate of 4 per day and boxes at a rate of 8 per day to achieve a maximum profit rate of 52¢ per day.

(iii) Solution via simplex tableau [4 marks]

Convert to equality format:

Max $Z - 5M - 4B = 0$

$M + 2B + s_1 = 20$

$2M + B + s_2 = 16$

$M + s_3 = 6$

basic	Z	M	B	s_1	s_2	s_3	RHS	MRT
Z	1	-5	-4	0	0	0	0	-
s_1	0	1	2	1	0	0	20	20/1=20
s_2	0	2	1	0	1	0	16	16/2=8
s_3	0	1	0	0	0	1	6	6/1=6
Z	1	0	-4	0	0	5	30	-
s_1	0	0	2	1	0	-1	14	14/2=7
s_2	0	0	1	0	1	-2	4	4/1=4
M	0	1	0	0	0	1	6	No limit
Z	1	0	0	0	4	-3	46	-
s_1	0	0	0	1	-2	3	6	6/3=2
B	0	0	1	0	1	-2	4	No limit
M	0	1	0	0	0	1	6	6/1=6
Z	1	0	0	1	2	0	52	
s_3	0	0	0	0.333	-0.667	1	2	
B	0	0	1	0.667	-0.333	0	8	
M	0	1	0	-0.333	0.667	0	4	

The simplex method produces the same solution as the graphical method: mugs should be made at a rate of 4 per day and boxes at a rate of 8 per day to achieve a maximum profit rate of 52¢ per day.

Question 4 [10 marks]

[4 marks formulation, 4 marks simplex solution, 2 marks recognizing and explaining unboundedness]

Variables: A: production of apple juice (litres/min)
OJ: production of orange juice (litres/min)

Objective function: maximize profit rate ($/min)
Max Z = 1.2A + 1.8OJ

Constraints:

- Apple juice production (litres/min): $A \le OJ + 5 \rightarrow A - OJ \le 5$
- Orange juice production (litres/min): $OJ \le A + 2 \rightarrow -A + OJ \le 2$

Variable bounds: $A \ge 0$, $OJ \ge 0$

Convert to equality format:

Max $Z - 1.2A - 1.8OJ = 0$

$A - OJ + s_1 = 5$

$-A + OJ + s_2 = 2$

Basic	**Z**	**A**	**OJ**	**s_1**	**s_2**	**RHS**	**MRT**
Z	1	-1.2	-1.8	0	0	0	-
s_1	0	1	-1	1	0	5	No limit
s_2	0	-1	1	0	1	2	2/1=2
Z	1	-3	0	0	1.8	3.6	-
s_1	0	0	0	1	1	7	No limit
OJ	0	-1	1	0	1	2	No limit

The possible leaving basic variables are all tied at "no limit". This shows that the model is unbounded, which usually indicates that a constraint has been omitted, in this case likely a constraint on the amount of juice that can actually be sold, or on the capacity limits at the production facility.