Ass 3

1\. a) X_i = # of heads $S = \frac{1}{n}\sum_1^n X_i$

$\Rightarrow$ Since it's tosses and we only have 2 results, head or tail. Let's define head as 1 and tail as 0

$\Rightarrow$ X_i — 0 $\Rightarrow (1-p) = 0.5$; 1 $\Rightarrow p = 0.5$

$X_i \sim B(0.5, 0.25)$

$\Rightarrow E(X) = 0.5$

$D(X) = \sigma^2 = 0.5^2 \Rightarrow \sigma = 0.5$

$\Rightarrow$ $P(40\% \le \frac{1}{n}\sum_1^n X_i \le 60\%)$

$= P(40\% - p \le \frac{1}{n}\sum_1^n X_i - p \le 60\% - p)$

$= P\left(\frac{40\% - p}{\sqrt{p(1-p)}/\sqrt{n}} \le \frac{\frac{1}{n}\sum_1^n X_i - p}{\sqrt{p(1-p)}/\sqrt{n}} \le \frac{60\% - p}{\sqrt{p(1-p)}/\sqrt{n}}\right)$

$= P\left(\frac{0.4 - 0.5}{0.5/\sqrt{120}} \le \frac{\frac{1}{120}\sum_1^{120} X_i - 0.5}{0.5/\sqrt{120}} \le \frac{0.6 - 0.5}{0.5/\sqrt{120}}\right)$

$= F\left(\frac{0.6-0.5}{0.5/\sqrt{120}}\right) - F\left(\frac{0.4-0.5}{0.5/\sqrt{120}}\right)$

$\approx F(2.19) - F(-2.19) \approx 0.978$

b) $P\left(\frac{1}{n}\sum_1^n X_i \ge \frac{5}{8}\right)$

$= P\left(\frac{1}{n}\sum_1^n X_i - p \ge \frac{5}{8} - p\right)$

$= P\left(\frac{\frac{1}{n}\sum_1^n X_i - p}{\sqrt{p(1-p)}/\sqrt{n}} \ge \frac{0.625 - p}{\sqrt{p(1-p)}/\sqrt{n}}\right)$

$= P\left(\frac{\frac{1}{120}\sum_1^{120} X_i - 0.5}{0.5/\sqrt{120}} \ge \frac{0.625 - 0.5}{0.5/\sqrt{120}}\right)$

$= 1 - F(2.74)$

$\approx 1 - 0.998 = 0.002$

$\bar{X} \ge 0.55 \Rightarrow \bar{X} - 0.46 \ge 0.55 - 0.46$

$\frac{\bar{X} - 0.55}{\sqrt{0.46 \times 0.54}/\sqrt{120}} \ge \cdots$

2\. a) Since the votes for the Candidate shows a binomial distribution $B(p, 1-p)$ where $p = 46\%$ and $1-p = 54\%$

The $E(X) = \mu = 0.46$, $V(X) = \sigma^2 = 0.46 \times 0.54 \Rightarrow \sigma = \sqrt{0.46 \times 0.54} \approx 0.498$

Then the probability than a majority of votes in favor of the candidate from the 200 voters is $P\left(\frac{1}{n}\sum_1^n X_i \ge \frac{101}{200}\right)$

$= P\left(\frac{1}{200}\sum_1^n X_i \ge 0.505\right)$