Lab Section 01 **SOLUTIONS** MATHEMATICS MATH*1080 Lab Quiz #1 Y

NO CALCULATORS ALLOWED

Surname:_____________________________________ Initials:___________ Lab Section #________________
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Student #:________________________________ Class Section #______________

1. Assume that for a set of $n = 10$ data points (x_i, y_i) that exhibits a linear pattern, $\sum_{i=1}^{n} x_i = 30$, $\sum_{i=1}^{n} y_i = 120$, $\sum_{i=1}^{n} x_i^2 = 120$, and $\sum_{i=1}^{n} x_i y_i = 450$. Use the formula

$$nb + \left(\sum_{i=1}^{n} x_i\right)m = \left(\sum_{i=1}^{n} y_i\right)$$
$$\left(\sum_{i=1}^{n} x_i\right)b + \left(\sum_{i=1}^{n} x_i^2\right)m = \sum_{i=1}^{n} x_i y_i$$

to determine the Least Squares Line $y = mx + b$ that fits these data points.

[3] **SOLUTION**

From the formula and the given values

$$nb + \left(\sum_{i=1}^{n} x_i\right)m = \left(\sum_{i=1}^{n} y_i\right) \quad \Rightarrow \quad 10b + 30m = 120 \quad \ldots \boxed{1}$$

$$\left(\sum_{i=1}^{n} x_i\right)b + \left(\sum_{i=1}^{n} x_i^2\right)m = \sum_{i=1}^{n} x_i y_i \quad \Rightarrow 30b + 120m = 450 \quad \ldots \boxed{2}$$ (1)

Multiple $3 \times \boxed{1}$ $\Rightarrow \quad 30b + 90m = 360 \quad \ldots \boxed{3}$

Compute $\boxed{2} - \boxed{3}$**:** $30m = 90 \quad \Rightarrow m = 3$ (1)

Substitute in $\boxed{1}$ **and solve for** b $10b + 30(3) = 120$: $\Rightarrow 10b = 30 \Rightarrow b = 3$

Least Squares Line: $y = 3x + 3$ (1)

2. On joining a real estate firm, an employee must choose one of two salary options based on the monthly net profit. Let x denote the monthly net profit in dollars.
Option A: The monthly salary $A(x)$ is $1,400 plus 5% of the monthly net profits.
Option B: The monthly salary $B(x)$ is $1,600 plus 6% of the amount of the monthly net profit that is greater than $20,000.
For what monthly net profits is the monthly salary in Option B equal to the monthly salary in Option A?

[4] **SOLUTION**

$$A(x) = 1400 + .05x \quad \text{and} \quad B(x) = \begin{cases} 1600 & \text{if } x \le 20,000 \\ 1600 + .06(x - 20,000) & \text{if } x > 20,000 \end{cases}$$ (1)

For if $x \le 20,000$ $\quad A(x) = B(x)$ (1)

$$1400 + .05x = 1600 \Rightarrow .05x = 200 \quad \Rightarrow \boxed{x = 4000}$$

For if $x > 20,000$ $\quad A(x) = B(x)$

$$1400 + .05x = 1600 + .06(x - 20,000)$$ (1)

$$\Rightarrow 1400 + .05x = 1600 + .06x - 1200$$

$$\Rightarrow \quad .01x = 1000$$

$$\Rightarrow \boxed{x = 100,000}$$ (1)

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3. In an epidemic, assume that the number of people infected is a quadratic function of the number of days the epidemic has been running. If the epidemic lasts 200 days and infects 1500 people at its peak then find the quadratic model. Assume that at time t = 0 the number of people infected is zero.

[3]

SOLUTION

Let $I(t)$ denote the number of people infected on day t.

$I(t) = A(t-B)^2 + C$

From graph: $\boxed{B = 100}$ **and** $\boxed{C = 1500}$

$\therefore\ I(t) = A(t-100)^2 + 1500$

For $t = 0,\ \ I(0) = A(0-100)^2 + 1500 = 0$

$\Rightarrow\ 10000A = -1500\ \Rightarrow A = \frac{-1500}{10000} = -\frac{3}{20}$

$\therefore\ \boxed{I(t) = -\frac{3}{20}(t-100)^2 + 1500}$

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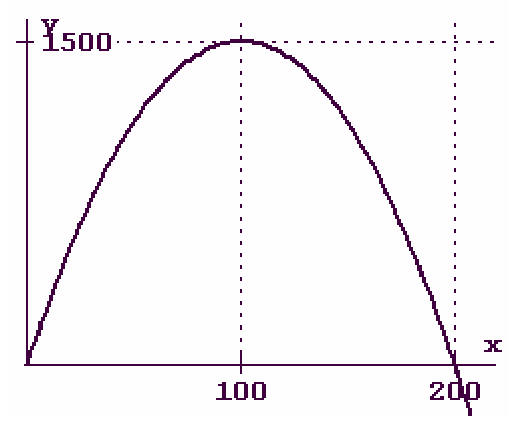
(1) (1) (1)



4. When discovered, the temperature of a body was 26 °C and the ambient temperature was a constant 22 °C. The body was quickly taken to the morgue where the ambient temperature was maintained at 5 °C. After 2 hours the body temperature was found to be 19 °C. Note that the normal body temperature is 37 °C and the formula for the cooling of a body is given by $T(t) = T_A + (T_0 - T_A)e^{-kt}$. Determine the decay rate k of the body.

[2] **SOLUTION**

Find the decay rate k in the morgue where: $T_0 = 26,\qquad T_A = 5$

At $t = 2:\quad T(2) = 19 = 5 + (26-5)e^{-k2} = 5 + 21e^{-k2}$ (1)

$\Rightarrow 21e^{-k2} = 14 \qquad \Rightarrow e^{-k2} = \frac{14}{21} = \frac{2}{3} \qquad \Rightarrow -k2 = ln(\frac{2}{3})$

$\therefore \boxed{k = -\frac{1}{2}ln(\frac{2}{3})}$ **(1/hours)** (1)

5. Assume that the weights of a radioactive substance on the 3rd and 7th days of an experiment were 50 grams and 40 grams respectively. Determine the decay rate of the substance?

[3] **SOLUTION**

General model is $A(t) = A_0e^{-kt}$ **where** $k > 0$.

We are given that $A(3) = 50$ **and** $A(7) = 40$.

To find the decay rate k

$A(3) = 50 = A_0e^{-3k}$ ....[1] (1)

$A(7) = 40 = A_0e^{-7k}$ ....[2]

Divide equation [1] by equation [2]

$\frac{A_0e^{-3k}}{A_0e^{-7k}} = \frac{50}{40} \qquad \Rightarrow \quad e^{-3k+7k} = \frac{5}{4}$ (1)

$\Rightarrow \quad e^{4k} = \frac{5}{4} \quad \Rightarrow 4k = ln(\frac{5}{4}) \quad \Rightarrow \quad k = \frac{1}{4}\,ln(\frac{5}{4})$ **hr**$^{-1}$ (1)

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