3-D FREE-BODY DIAGRAMS, EQUILIBRIUM EQUATIONS, CONSTRAINTS AND STATICAL DETERMINACY

Today's Objective:

Students will be able to:

a) Identify support reactions in 3-D and draw a free body diagram, and,

b) apply the equations of equilibrium.

In-Class Activities:

- Reading Quiz
- Applications
- Support Reactions in 3-D
- Equations of Equilibrium
- Concept Quiz
- Group Problem Solving
- Attention quiz

READING QUIZ

1. If a support prevents rotation of a body about an axis, then the support exerts a ________ on the body about that axis.

   A) Couple moment B) Force

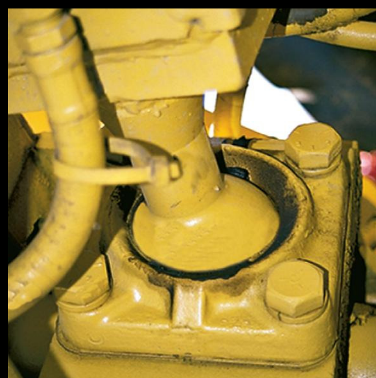
   C) Both A and B. D) None of the above.

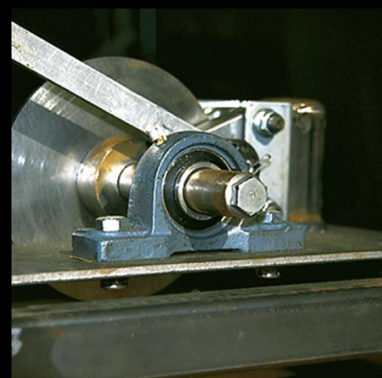
2. When doing a 3-D problem analysis, you have ________ scalar equations of equilibrium.

   A) 3 B) 4

   C) 5 D) 6

APPLICATIONS





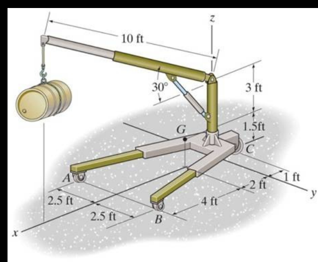
Ball-and-socket joints and journal bearings are often used in mechanical systems. To design the joints or bearings, the support reactions at these joints and the loads must be determined.

APPLICATIONS (continued)

The tie rod from point A is used to support the overhang at the entrance of a building. It is pin connected to the wall at A and to the center of the overhang B.

If A is moved to a lower position D, will the force in the rod change or remain the same? By making such a change without understanding if there is a change in forces, failure might occur.

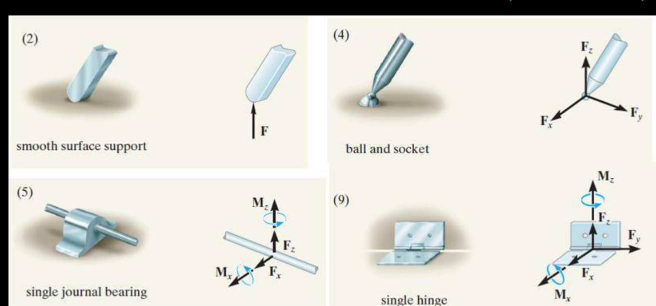
APPLICATIONS (continued)



The crane, which weighs 350 lb, is supporting a oil drum.

How do you determine the largest oil drum weight that the crane can support without overturning ?

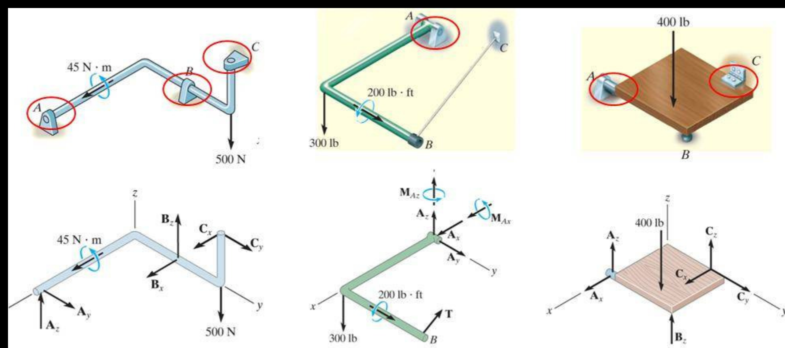
SUPPORT REACTIONS IN 3-D (Table 5-2)



A few examples are shown above. Other support reactions are given in your text book (Table 5-2).

As a general rule, if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body. Similarly, if rotation is prevented, a couple moment is exerted on the body by the support.

IMPORTANT NOTE



A single bearing or hinge can prevent rotation by providing a resistive couple moment. However, it is usually preferred to use two or more properly aligned bearings or hinges. Thus, in these cases, only force reactions are generated and there are no moment reactions created.

EQUATIONS OF EQUILIBRIUM

(Section 5.6)

As stated earlier, when a body is in equilibrium, the net force and the net moment equal zero, i.e., $\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$.

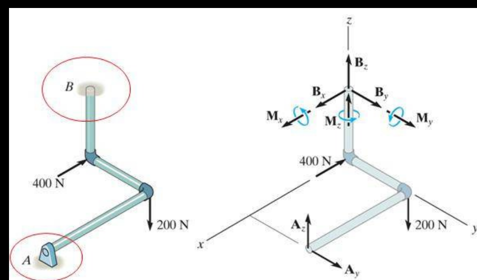
These two vector equations can be written as six scalar equations of equilibrium (EofE). These are

$$\sum F_X = \sum F_Y = \sum F_Z = 0$$

$$\sum M_X = \sum M_Y = \sum M_Z = 0$$

The moment equations can be determined about any point. Usually, choosing the point where the maximum number of unknown forces are present simplifies the solution. Any forces occurring at the point where moments are taken do not appear in the moment equation since they pass through the point.

CONSTRAINTS AND STATICAL DETERMINACY
(Section 5.7)

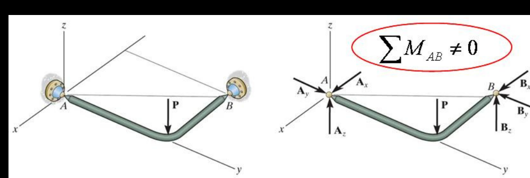


Redundant Constraints: When a body has more supports than necessary to hold it in equilibrium, it becomes statically indeterminate.

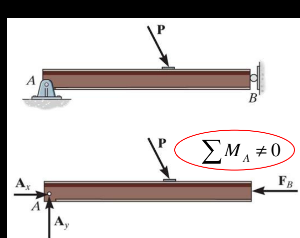
A problem that is statically indeterminate has more unknowns than equations of equilibrium.

Are statically indeterminate structures used in practice? Why or why not?

IMPROPER CONSTRAINTS



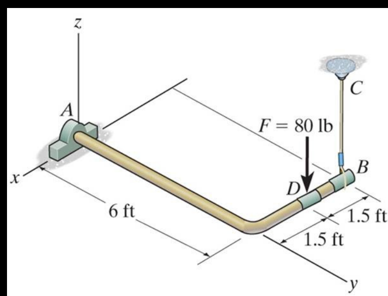
Here, we have 6 unknowns but there is nothing restricting rotation about the AB axis.



In some cases, there may be as many unknown reactions as there are equations of equilibrium.

However, if the supports are not properly constrained, the body may become unstable for some loading cases.

EXAMPLE



Given: The rod, supported by thrust bearing at A and cable BC, is subjected to an 80 lb force.

Find: Reactions at the thrust bearing A and cable BC.

Plan:

a) Establish the x, y and z axes.

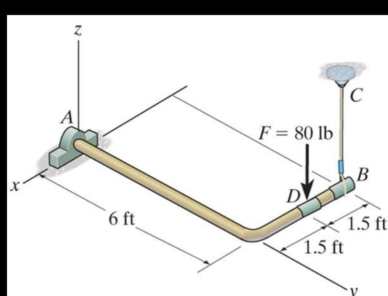
b) Draw a FBD of the rod.

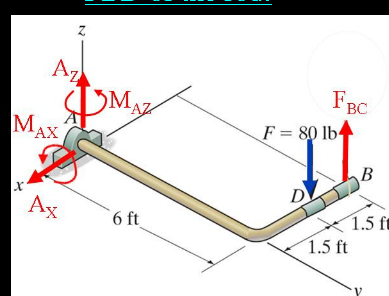
c) Write the forces using scalar equations.

d) Apply scalar equations of equilibrium to solve for the unknown forces.

EXAMPLE (continued)

FBD of the rod:





Applying scalar equations of equilibrium in appropriate order, we get

$\sum F_X = A_X = 0$; $A_X = 0$

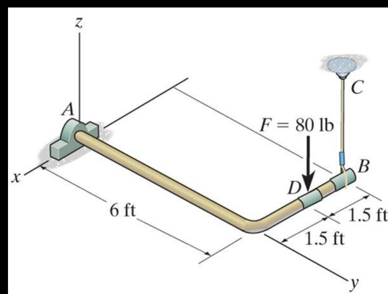
$\sum F_Z = A_Z + F_{BC} - 80 = 0$;

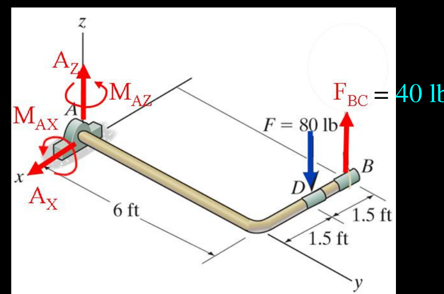
$\sum M_Y = -80\,(1.5) + F_{BC}\,(3.0) = 0$;

Solving these last two equations: $F_{BC} = 40$ lb, $A_Z = 40$ lb

EXAMPLE (continued)

FBD of the rod:





Now write scalar moment equations about what point? Point A!

$M_X = (M_A)_X + 40\,(6) - 80\,(6) = 0$; $(M_A)_X = 240$ lb ft

$\sum M_Z = (M_A)_Z = 0$; $(M_A)_Z = 0$

CONCEPT QUIZ

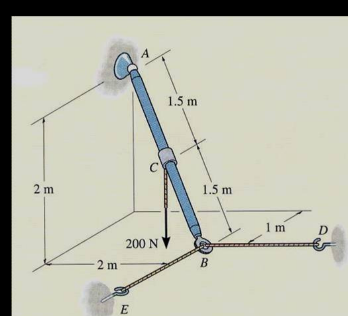
1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?

   A) 5 force and 1 moment reaction

   B) 5 force reactions

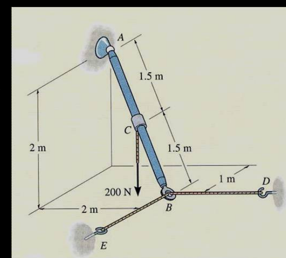
   C) 3 force and 3 moment reactions

   D) 4 force and 2 moment reactions



CONCEPT QUIZ (continued)

2. If an additional couple moment in the vertical direction is applied to rod AB at point C, then what will happen to the rod?



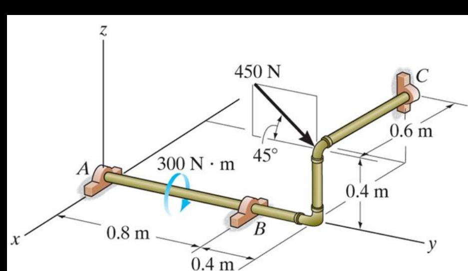
A) The rod remains in equilibrium as the cables provide the necessary support reactions.

B) The rod remains in equilibrium as the ball-and-socket joint will provide the necessary resistive reactions.

C) The rod becomes unstable as the cables cannot support compressive forces.

D) The rod becomes unstable since a moment about AB cannot be restricted.

GROUP PROBLEM SOLVING



Given: A rod is supported by smooth journal bearings at A, B, and C. Assume the rod is properly aligned.

Find: The reactions at all the supports for the loading shown.

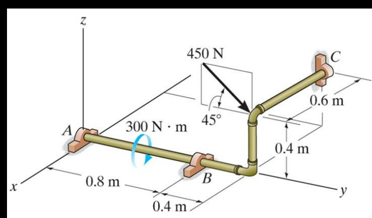
Plan:

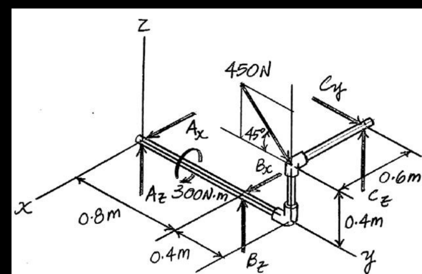
a) Draw a FBD of the rod.

b) Apply scalar equations of equilibrium to solve for the unknowns.

GROUP PROBLEM SOLVING (continued)

A FBD of the rod:





Applying scalar equations of equilibrium in appropriate order, we get

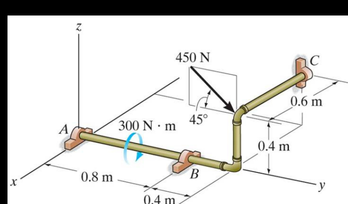
$\sum F_Y = 450 \cos 45° + C_Y = 0$; $C_Y = -318$ N

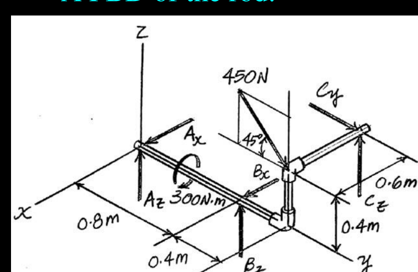
$\sum M_Y = C_Z (0.6) - 300 = 0$; $C_Z = 500$ N

$\sum M_Z = - B_X (0.8) - (- 318) (0.6) = 0$; $B_X = 239$ N

GROUP PROBLEM SOLVING (continued)

A FBD of the rod:





$\sum M_X = B_Z (0.8) - 450 \cos 45° (0.4) - 450 \sin 45° (0.8 + 0.4)$

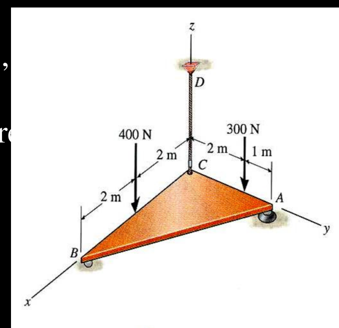
$+ 318 (0.4) + 500 (0.8 + 0.4) = 0$; $B_Z = - 273$ N

$\sum F_X = A_X + 239 = 0$; $A_X = - 239$ N

$\sum F_Z = A_Z - (- 273) + 500 - 450 \sin 45° = 0$; $A_Z = 90.9$ N

ATTENTION QUIZ

1. A plate is supported by a ball-and-socket joint at A, a roller joint at B, and a cable at C. How many unknown support reactions are there in this problem?



   A) 4 forces and 2 moments

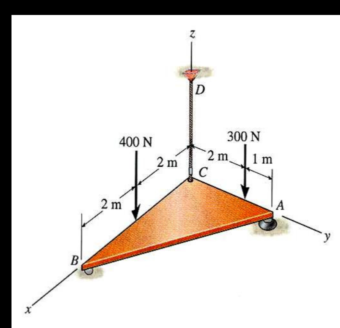
   B) 6 forces

   C) 5 forces

   D) 4 forces and 1 moment

ATTENTION QUIZ

2. What will be the easiest way to determine the force reaction B_Z ?



   A) Scalar equation $\sum F_Z = 0$

   B) Vector equation $\sum \boldsymbol{M_A} = 0$

   C) Scalar equation $\sum M_Z = 0$

   D) Scalar equation $\sum M_Y = 0$

End of the Lecture

Let Learning Continue