Concordia University

Math 251	Class Test	October 2013
Instructor: C Cummins	Time: 75 mins	30 marks (30% of final grade)

All solutions must include a reasoned explanation.

Q1 (6 marks)

Let $M_{2\times 2}(I\!R)$ be the vector space of 2×2 matrices with real entries.

a) Let $W_1 = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times 2}(I\!R) \mid a = b\}$. Show that W_1 is a subspace of $M_{2\times 2}(I\!R)$.

b) Let $W_2 = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times 2}(I\!R) \mid a = b^2\}$. Is W_2 a subspace of $M_{2\times 2}(I\!R)$? Explain your answer carefully.

Q2 (6 marks)

Let P_2 be the vector space of polynomials of degree at most 2 with real coefficients. Let $\beta = \{1, 1 + x, 1 + x + x^2\}$ be an (ordered) basis of P_2 (you do not have to check this is a basis). Let $T : P_2 \to P_2$ be the transformation defined by $T(p(x)) = p(-x)$. For example $T(x + x^2) = -x + x^2$. Show that T is a linear transformation and find the matrix $[T]_\beta$ of T with respect to the basis β.

Q3 (6 marks)

Let $S = \{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}\}$

a) Prove that S is a basis of $M_{2\times 2}(I\!R)$.

b) Let $T : M_{2\times 2}(I\!R) \to M_{2\times 2}(I\!R)$ be the transformation given by $T(A) = A^t$, i.e. $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$. Show that T is a linear transformation and find $[T]_S$, the matrix of T with respect to the basis S.

Q4 (6 marks) Let $T : I\!R^3 \to I\!R^3$ be the linear transformation $T(x, y, z) = (x + y + z, x + y + z, x + y + z)$. Find a basis of the nullspace $N(T)$ and find a basis of the range space $R(T)$. What are the dimensions of these subspaces? Show that $I\!R^3 = N(T) \oplus R(T)$.

Q5 (6 marks)

Let V and W be vector spaces and let $T : V \to W$ be a linear transformation.

a) Show that the null space $N(T)$ is a subspace of V.

b) Show that the range space $R(T)$ is a subspace of W.