Name (print): ___________________ **Student Number:** ___________________

Test #1 (____/30)

Part A: Multiple Choice (1 Mark Each, No Penalty For Incorrect Answers)

Question 1: Which of the following is a solution to the augmented matrix given below:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ -1 & 2 & 0 & -1 \\ 1 & 1 & 1 & 14 \\ 0 & 3 & 0 & 0 \end{array}\right]$$

a) $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ c) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ d) There is no solution

Question 2: Which of the following will be undefined:

a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e \\ f \end{bmatrix}$ b) $c\begin{bmatrix} a \\ b \end{bmatrix}$ c) $\begin{bmatrix} a \\ b \end{bmatrix}\begin{bmatrix} d & e & f \end{bmatrix}$ d) $\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ e) They are all defined

Part B: Fill in the Blank (1 Mark Each)

Question 1: Give an example of an augmented matrix that will not have a solution:

As long as the augmented matrix row reduces to a row with [0 0 0 … 0 | #] it will have no solution. The easiest is to have such a matrix written here.

Question 2: The formula for the inverse of a 2x2 matrix is:

$$\frac{1}{da - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Question 3: What do we call a matrix that row reduces to the identity matrix?

A non-singular matrix.

Question 4: Give an example of a matrix that is in RREF that has 2 free variables.

I will accept matrices that are augmented or not, provided that A has 2 columns with no pivot. For example:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Question 5: What are the elementary row matrices corresponding to the following for a 4x4 matrix:

a) Row swap 2 and 4. Call this matrix E_1

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

b) Row replace Row 3 with 2(R1)-R3. Call this matrix E_2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Let's say we had a matrix A. How would you matrix multiply E_1 and E_2 with A to apply the row operations E_1 first then E_2 second?

$$E_2 E_1 A$$

Part C: Short Answer (Show all work)

Question 1: a) Determine the Inverse of the matrix given: $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ [4 Marks]

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \text{R2→R2+R1} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \text{R3→R3-2R1} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 & 0 \\ 0 & -2 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\text{R3→R3+R2} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \text{R2→3R2-2R3} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 6 & 0 & 5 & 1 & -2 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \text{R1→3R1-R2-R3}$$

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 0 & -1 & 2 & 1 \\ 0 & 6 & 0 & 5 & 1 & -2 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \text{R1÷3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 6 & 0 & 5 & 1 & -2 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \text{R2÷6} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{3} & \frac{2-}{3} & \frac{1}{3} \\ 0 & 1 & 0 & \frac{5}{6} & \frac{1}{6} & -\frac{1}{3} \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \text{R3÷3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 0 & \frac{5}{6} & \frac{1}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right]$$

(Note that they may combine steps, this is fine and in fact I hope they would combine a few of these steps). Give them 4 marks if they get correct inverse, take off 0.5 marks per minor error. Give them at least 1 mark if they knew to start off with [A | I] to find the inverse of A)

a) Use the inverse to solve the following system: $Ax = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ [2 Marks]

We simply perform $x = A^{-1}b = \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{5}{6} & \frac{1}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{1}{2} \\ 0 \end{bmatrix}$

[1 Mark if they know $A^{-1}b$, 1 Mark for the correct answer (if they found the incorrect inverse, please follow their work through, they are still able to get full marks on this part with the incorrect inverse]

Question 2: Given the matrix and vector below, simplify the expression: $((A + I_3)^T)B + 2B$ [5 Marks]

$$A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 0 & 1 \\ 2 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A + I_3 = \begin{bmatrix} 2 & 2 & 2 \\ -1 & 1 & 1 \\ 2 & 2 & 4 \end{bmatrix}$$

$$(A + I_3)^T = \begin{bmatrix} 2 & -1 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$(A + I_3)^T B = \begin{bmatrix} 2 & -1 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 9 \end{bmatrix}$$

$$(A + I_3)^T B + 2B = \begin{bmatrix} 7 \\ 5 \\ 9 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 13 \end{bmatrix}$$

[1 Mark for knowing each step above]

Question 3: a) Solve the system. Use vector notation: $\left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 1 \\ 2 & 1 & 8 & 3 & 3 \\ -1 & 1 & 0 & 1 & 6 \end{array}\right]$ [3 Marks]

R2→R2-2R1

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 & 6 \end{array}\right]$$

R3→R3+R1

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 4 & 2 & 7 \end{array}\right]$$

R3→R3-R2

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 4 & 1 & 6 \end{array}\right]$$

R1→R1-R3

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 4 & 1 & 6 \end{array}\right]$$

R2→R2-2R1

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{3}{2} \end{array}\right]$$

We have x_4 as a free variable, this gives us $x_4 = s$.

Thus the solution set is $S = \left\{ \begin{bmatrix} -5 \\ 1 - s \\ \frac{3}{2} - \frac{1}{4}s \\ s \end{bmatrix} \middle| s \in R \right\} = \left\{ \begin{bmatrix} -5 \\ 1 \\ \frac{3}{2} \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ -1 \\ -\frac{1}{4} \\ 1 \end{bmatrix} \middle| \ s \in R \right\}$

(They may simplify this further, that is ok too!)

[2 Marks for row reducing (take off 0.5 for each error)]

[1 Mark for the solution set in vector form (take off 0.5 if not written in vector form)]

b) Determine the solution to the homogenous system given in a) above: $\left\{ s\begin{bmatrix} 0 \\ -1 \\ -\frac{1}{4} \\ 1 \end{bmatrix} \middle| \;\; s \in R \right\}$ [1 Mark]

c) Determine 2 particular solutions to the system given in a) above: [2 Marks]

One is written in our final solution: $\begin{bmatrix} -5 \\ 1 \\ \frac{3}{2} \\ 0 \end{bmatrix}$

The other can be found by subbing in any number for s. One easy example would be s=4. This will give us another solution as: $\begin{bmatrix} -5 \\ 1 \\ \frac{3}{2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -4 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ \frac{1}{2} \\ 4 \end{bmatrix}$

[1 Mark for each correct particular solution]

Question 4: If we have a matrix A that has a unique solution to Ax=b, and say we came up with a solution x=c. Give 4 ways to check if we have found the correct answer. [4 Marks]

- We can perform matrix multiplication: Ac to make sure it is = b.
- We can make sure c = $A^{-1}b$
- We can row reduce [A | b] to RREF and make sure we get [I |c]
- We can substitute c1 c2 and c3 into the linear system made by the augmented matrix of Ax=b.

Any other answers that are reasonable can also be included (verify that the linear combination of the columns = b where the coefficients are c1, c2, and c3 as another example).

[1 Mark for each correct answer]