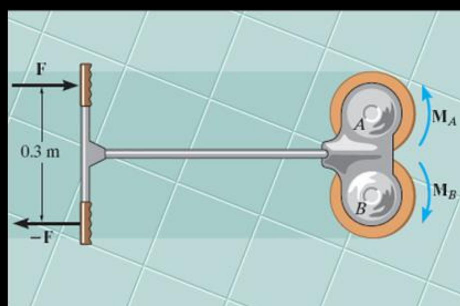
MOMENT OF A COUPLE

Today's Objectives:

Students will be able to

a) define a couple, and,

b) determine the moment of a couple.



In-Class activities:

- Reading Quiz
- Applications
- Moment of a Couple
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. In statics, a couple is defined as __________ separated by a perpendicular distance.

   A) two forces in the same direction

   B) two forces of equal magnitude

   C) two forces of equal magnitude acting in the same direction

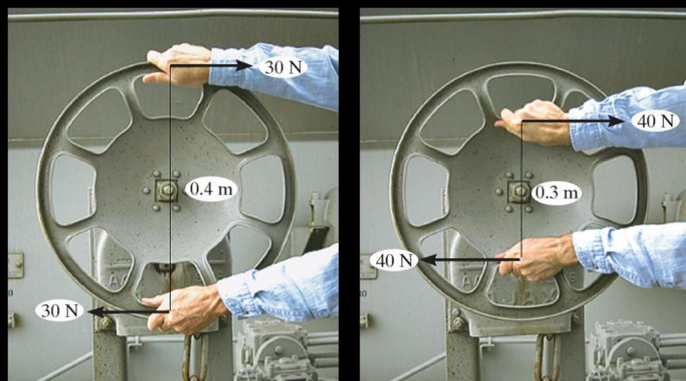
   D) two forces of equal magnitude acting in opposite directions

2. The moment of a couple is called a _________ vector.

   A) Free B) Spin

   C) Romantic D) Sliding

APPLICATIONS



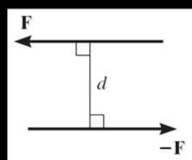
A torque or moment of 12 N · m is required to rotate the wheel. Why does one of the two grips of the wheel above require less force to rotate the wheel?

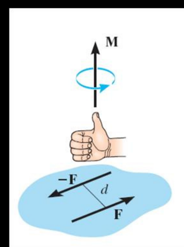
APPLICATIONS
(continued)

When you grip a vehicle's steering wheel with both hands, a couple moment is applied to the wheel.

Would older vehicles without power steering have larger or smaller steering wheels?

MOMENT OF A COUPLE





A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

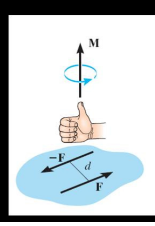
The moment of a couple is defined as

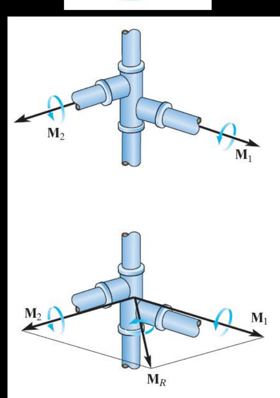
$M_O = F\ d$ (using a scalar analysis) or as

$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here ***r*** is any position vector from the line of action of ***–F*** to the line of action of ***F***.

MOMENT OF A COUPLE
(continued)



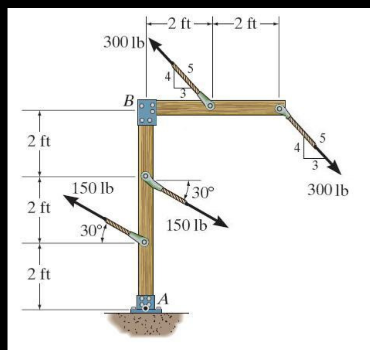


The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F *d.

Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector. It can be moved anywhere on the body and have the same external effect on the body.

Moments due to couples can be added together using the same rules as adding any vectors.

EXAMPLE - SCALAR APPROACH



Given: Two couples act on the beam with the geometry shown.

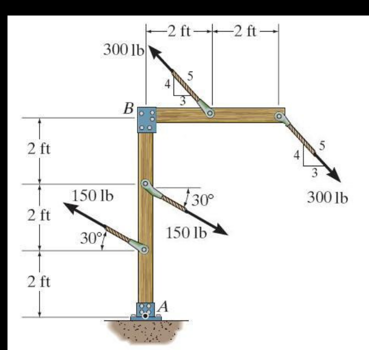
Find: The resultant couple

Plan:

1) Resolve the forces in x and y directions so they can be treated as couples.

2) Add the two couples to find the resultant couple.

EXAMPLE - SCALAR APPROACH



The x and y components of the upper-left 300 lb force are:

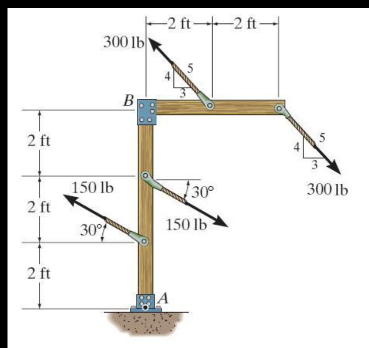
(4/5)(300 lb) = 240 lb vertically up

(3/5)(300 lb) = 180 lb to the left

Do both of these components form couples with their matching components of the other 300 force?

No! Only the 240 lb components create a couple. Why?

EXAMPLE - SCALAR APPROACH



Now resolve the lower 150 lb force:

(150 lb) (sin 30°), acting up

(150 lb) (cos 30°), acting to the left

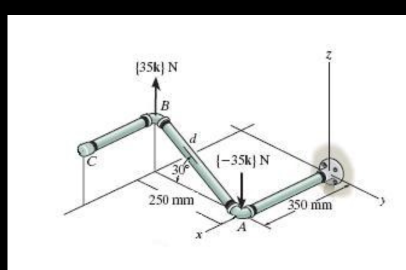
Do both of these components create a couple with components of the other 150 lb force?

The net moment is equal to:

$+\circlearrowleft \Sigma M = -(240 \text{ lb})(2 \text{ ft}) - (150 \text{ lb})(\cos 30°)(2 \text{ ft})$

$= -480 - 259.8 = -739.8 \text{ ft·lb}$

EXAMPLE – VECTOR APPROACH



Given: A 35 N force couple acting on the rod.

Find: The couple moment acting on the rod in Cartesian vector notation.

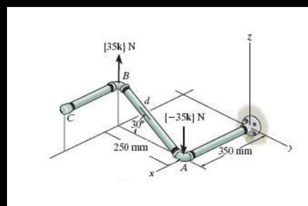
Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{35\, \boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

EXAMPLE – VECTOR APPROACH



$\boldsymbol{r}_{AB} = \{\, 0\,\boldsymbol{i} - (0.25)\,\boldsymbol{j} + (0.25 \tan 30°)\,\boldsymbol{k}\}$ m

$\boldsymbol{r}_{AB} = \{-0.25\,\boldsymbol{j} + 0.1443\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = \{0\,\boldsymbol{i} + 0\,\boldsymbol{j} + 35\,\boldsymbol{k}\}$ N

$$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & -0.25 & 0.1443 \\ 0 & 0 & 35 \end{vmatrix} \text{ N·m}$$

$$= \{(-8.75 - 0)\,\boldsymbol{i} - (0 - 0)\,\boldsymbol{j} - (0 - 0)\,\boldsymbol{k}\} \text{ N·m}$$

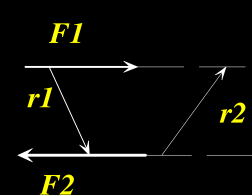
$$= \{-8.75\,\boldsymbol{i} + 0\,\boldsymbol{j} + 0\,\boldsymbol{k}\} \text{ N·m}$$

CONCEPT QUIZ

1. $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ form a couple. The moment of the couple is given by ____ .

   A) $\boldsymbol{r}_1 \times \boldsymbol{F}_1$ B) $\boldsymbol{r}_2 \times \boldsymbol{F}_1$

   C) $\boldsymbol{F}_2 \times \boldsymbol{r}_1$ D) $\boldsymbol{r}_2 \times \boldsymbol{F}_2$



2. If three couples act on a body, the overall result is that

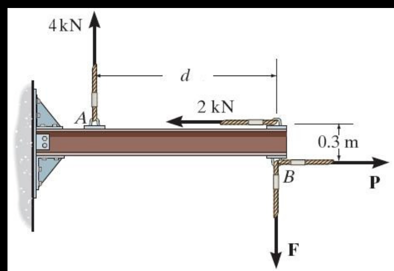
   A) The net force is not equal to 0.

   B) The net force and net moment are equal to 0.

   C) The net moment equals 0 but the net force is not necessarily equal to 0.

   D) The net force equals 0 but the net moment is not necessarily equal to 0 .

GROUP PROBLEM SOLVING – SCALAR APPROACH



Given: Two couples act on the beam. The resultant couple is zero.

Find: The magnitudes of the forces P and F and the distance d.

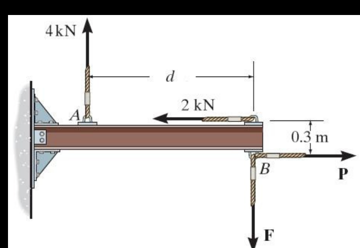
PLAN:

1) Use definition of a scalar couple to find P and F.

2) Determine the net moment (couple).

3) Equate the net moment to zero to find d.

GROUP PROBLEM SOLVING – SCALAR APPROACH



From the definition of a couple:

P = 2 kN

F = 4 kN

Determine the net moment

+↺ ΣM = (2)(0.3) – (4)(d)

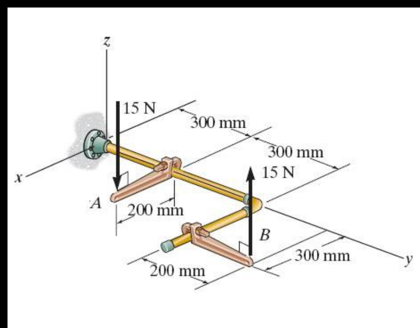
It was given that the net moment equals zero. So

+↺ ΣM = (2)(0.3) – (4)(d) = 0

Now solve this equation for d.

d = (0.6) N·m / (4) N = 0.15 m

GROUP PROBLEM SOLVING – VECTOR APPROACH



Given: $\boldsymbol{F} = \{15\,\boldsymbol{k}\}$ N and

$-\boldsymbol{F} = \{-15\,\boldsymbol{k}\}$ N

Find: The couple moment acting on the pipe assembly using Cartesian vector notation.

Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{15\,\boldsymbol{k}\}$ N .

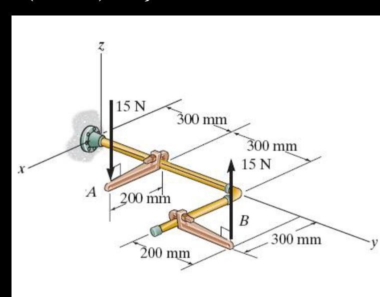
3) Calculate the cross product to find $\boldsymbol{M}$.

GROUP PROBLEM SOLVING – VECTOR APPROACH

$\boldsymbol{r}_{AB} = \{ (0.3 - 0.2)\,\boldsymbol{i} + (0.8 - 0.3)\,\boldsymbol{j} + (0 - 0)\,\boldsymbol{k} \}$ m

$= \{ 0.1\,\boldsymbol{i} + 0.5\,\boldsymbol{j} \}$ m

$\boldsymbol{F} = \{15\,\boldsymbol{k}\}$ N



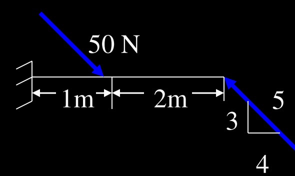
$$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 15 \end{vmatrix} \text{ N} \cdot \text{m}$$

$= \{ (7.5 - 0)\,\boldsymbol{i} - (1.5 - 0)\,\boldsymbol{j} + \boldsymbol{k}\,(0) \}$ N · m

$= \{ 7.5\,\boldsymbol{i} - 1.5\,\boldsymbol{j} \}$ N · m

ATTENTION QUIZ

1. A couple is applied to the beam as shown. Its moment equals _____ N·m.



A) 50 B) 60

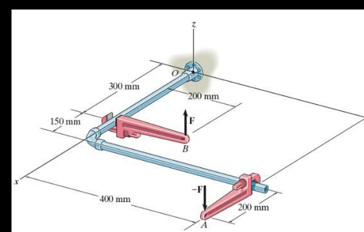
C) 80 D) 100

2. You can determine the couple moment as $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$

If $\boldsymbol{F}$ = { -20 $\boldsymbol{k}$} lb, then $\boldsymbol{r}$ is

A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AB}$

C) $\boldsymbol{r}_{CB}$ D) $\boldsymbol{r}_{BA}$



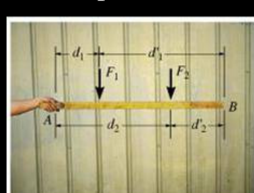
SIMPLIFICATION OF FORCE AND COUPLE SYSTEMS & FURTHER SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM

Today's Objectives:

Students will be able to:

a) Determine the effect of moving a force.

b) Find an equivalent force-couple system for a system of forces and couples.



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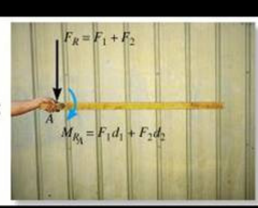


In-Class Activities:

- Reading Quiz
- Applications
- Equivalent Systems
- System Reduction
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A general system of forces and couple moments acting on a rigid body can be reduced to a ___ .

   A) single force

   B) single moment

   C) single force and two moments

   D) single force and a single moment

2. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.

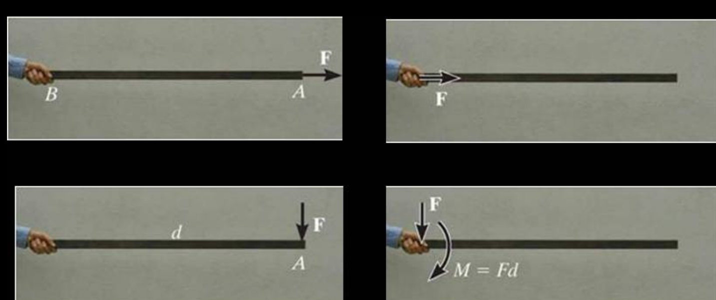
   A) internal

   B) external

   C) internal and external

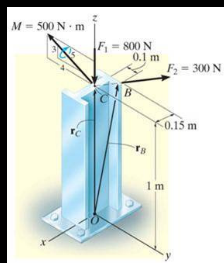
   D) microscopic

APPLICATIONS



What are the resultant effects on the person's hand when the force is applied in these four different ways?

APPLICATIONS (continued)



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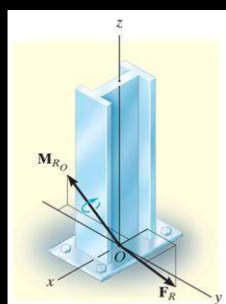
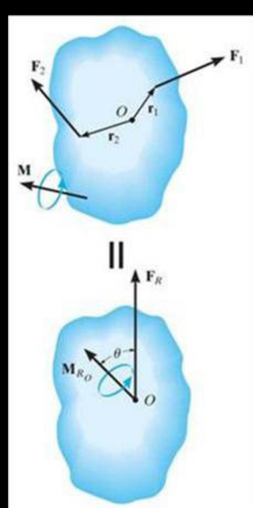


Several forces and a couple moment are acting on this vertical section of an I-beam.

For the process of designing the I-beam, it would be very helpful if you could replace the various forces and moment just one force and one couple moment at point O with the same external effect? How will you do that?

SIMPLIFICATION OF FORCE AND COUPLE SYSTEM
(Section 4.7)



When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect.

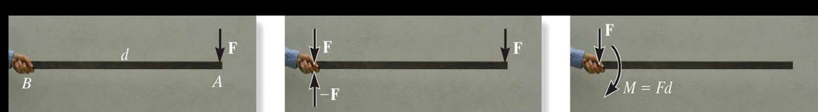
The two force and couple systems are called equivalent systems since they have the same external effect on the body.

MOVING A FORCE ON ITS LINE OF ACTION

Moving a force from A to B, when both points are on the vector's line of action, does not change the external effect.

Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

MOVING A FORCE OFF OF ITS LINE OF ACTION

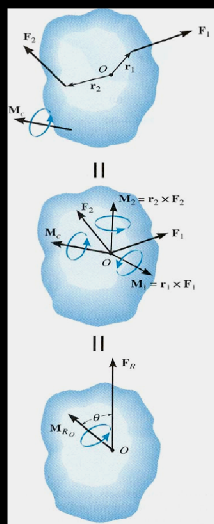


When a force is moved, but not along its line of action, there is a change in its external effect!

Essentially, moving a force from point A to B (as shown above) requires creating an additional couple moment. So moving a force means you have to "add" a new couple.

Since this new couple moment is a "free" vector, it can be applied at any point on the body.

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM



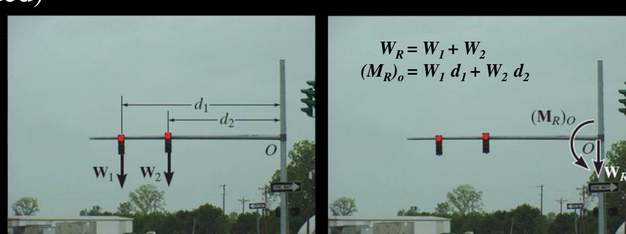
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM

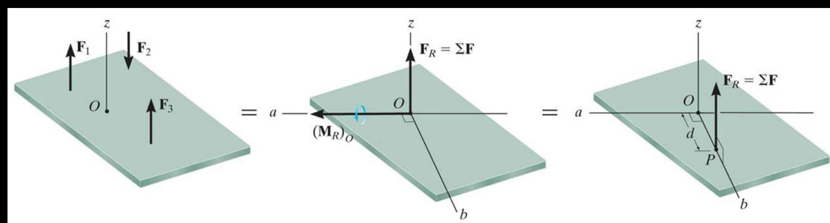
(continued)



If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

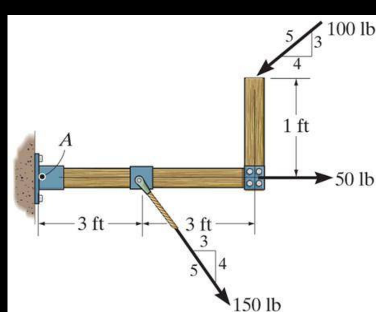
FURTHER SIMPLIFICATION OF A FORCE AND COUPLE SYSTEM (Section 4.8)



If F_R and M_{RO} are perpendicular to each other, then the system can be further reduced to a single force, F_R, by simply moving F_R from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

EXAMPLE #1



Given: A 2-D force system with geometry as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A.

Plan:

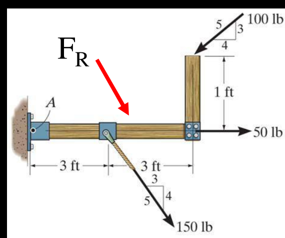
1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to A.

3) Shift F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE #1

(continued)



$+\rightarrow \Sigma F_{Rx} = 150\ (3/5) + 50 - 100\ (4/5)$

$= 60$ lb

$+\downarrow \Sigma F_{Ry} = 150\ (4/5) + 100\ (3/5)$

$= 180$ lb

$+ ↺\ M_{RA} = 100\ (4/5)\ 1 - 100\ (3/5)\ 6$

$- 150(4/5)\ 3 = -\ 640$ lb·ft

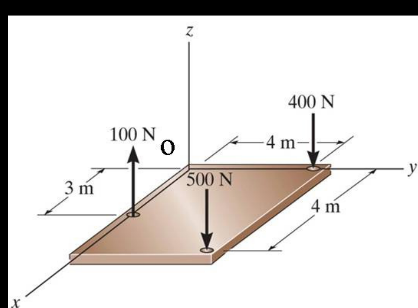
$F_R = (\ 60^2 + 180^2\ )^{1/2} = 190$ lb

↘ $\theta = \tan^{-1}\ (\ 180/60) = 71.6\ °$

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 640\ /\ 180 = 3.56$ ft.

EXAMPLE #2



Given: The slab is subjected to three parallel forces.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x, y) of the single equivalent resultant force.

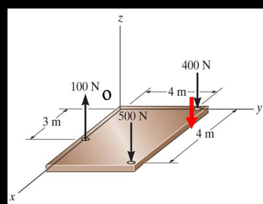
Plan:

1) Find $\boldsymbol{F_{RO}} = \Sigma \boldsymbol{F_i} = F_{Rzo}\ \boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \Sigma\ (\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\ \boldsymbol{i} + M_{RyO}\ \boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -\ M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

EXAMPLE #2
(continued)



$\boldsymbol{F_{RO}} = \{100\,\boldsymbol{k} - 500\,\boldsymbol{k} - 400\,\boldsymbol{k}\} = -\,800\,\boldsymbol{k}$ N

$\boldsymbol{M_{RO}} = (3\,\boldsymbol{i}) \times (100\,\boldsymbol{k}) + (4\,\boldsymbol{i} + 4\,\boldsymbol{j}) \times (\text{-}500\,\boldsymbol{k}) + (4\,\boldsymbol{j}) \times (\text{-}400\,\boldsymbol{k})$

$= \{-300\,\boldsymbol{j} + 2000\,\boldsymbol{j} - 2000\,\boldsymbol{i} - 1600\,\boldsymbol{i}\}$

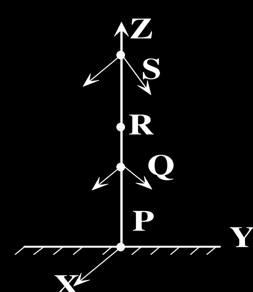
$= \{-\,3600\,\boldsymbol{i} + 1700\,\boldsymbol{j}\}$ N·m

The location of the single equivalent resultant force is given as,

$x = -\,M_{Ryo} / F_{Rzo} = (-1700) / (-800) = 2.13$ m

$y = M_{Rxo} / F_{Rzo} = (-3600) / (-800) = 4.5$ m

CONCEPT QUIZ



1. The forces on the pole can be reduced to a single force and a single moment at point ____ .

   A) P B) Q C) R

   D) S E) Any of these points.

2. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

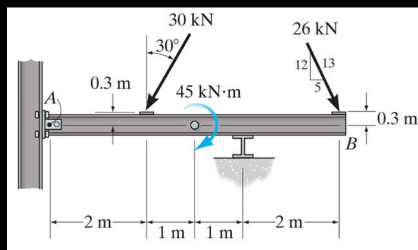
   A) One force and one couple moment.

   B) One force.

   C) One couple moment.

   D) Two couple moments.

GROUP PROBLEM SOLVING



Given: A 2-D force and couple system as shown.

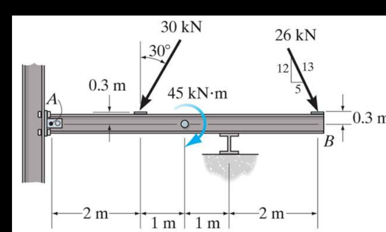
Find: The equivalent resultant force and couple moment acting at A.

Plan:

1) Sum all the x and y components of the two forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force to A and add them to the 45 kN m free moment to find the resultant M_{RA} .

GROUP PROBLEM SOLVING (continued)

Summing the force components:



$+ \rightarrow \Sigma F_x = (5/13)\ 26 - 30 \sin 30°$
$= -5$ kN

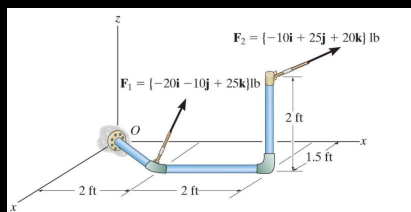
$+ \uparrow \Sigma F_y = -(12/13)\ 26 - 30 \cos 30°$
$= -49.98$ kN

Now find the magnitude and direction of the resultant.

$F_{RA} = (5^2 + 49.98^2)^{1/2} = 50.2$ kN and $\theta = \tan^{-1}(49.98/5)$
$= 84.3°$ ↙

$+\circlearrowleft M_{RA} = \{30 \sin 30° (0.3m) - 30 \cos 30° (2m) - (5/13)\ 26\ (0.3m)$
$- (12/13)\ 26\ (6m) - 45\} = -239$ kN m

GROUP PROBLEM SOLVING (continued)



Given: Forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are applied to the pipe.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i} = \boldsymbol{F_1} + \boldsymbol{F_2}$

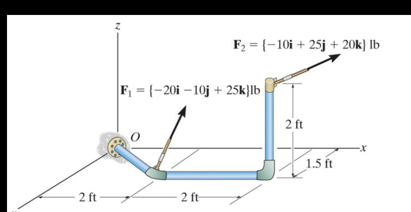
b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\, \boldsymbol{r_i} \times \boldsymbol{F_i}\,)$

where,

$\boldsymbol{M_C}$ are any free couple moments in CVN (none in this example).

$\boldsymbol{r_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

GROUP PROBLEM SOLVING (continued)



$\boldsymbol{F_1} = \{-20\, \boldsymbol{i} - 10\, \boldsymbol{j} + 25\, \boldsymbol{k}\}$ lb

$\boldsymbol{F_2} = \{-10\, \boldsymbol{i} + 25\, \boldsymbol{j} + 20\, \boldsymbol{k}\}$ lb

$\boldsymbol{F_{RO}} = \{-30\, \boldsymbol{i} + 15\, \boldsymbol{j} + 45\, \boldsymbol{k}\}$ lb

$\boldsymbol{r_1} = \{1.5\, \boldsymbol{i} + 2\, \boldsymbol{j}\}$ ft

$\boldsymbol{r_2} = \{1.5\, \boldsymbol{i} + 4\, \boldsymbol{j} + 2\, \boldsymbol{k}\}$ ft

Then, $\boldsymbol{M_{RO}} = \Sigma\, (\, \boldsymbol{r_i} \times \boldsymbol{F_i}\,) = \boldsymbol{r_1} \times \boldsymbol{F_1} + \boldsymbol{r_2} \times \boldsymbol{F_2}$

$$\boldsymbol{M_{RO}} = \left\{ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix} \right\} \text{ lb·ft}$$

$= \{(50\, \boldsymbol{i} - 37.5\, \boldsymbol{j} + 25\, \boldsymbol{k}\,) + (30\, \boldsymbol{i} - 50\, \boldsymbol{j} + 77.5\, \boldsymbol{k}\,)\}$ lb·ft

$= \{80\, \boldsymbol{i} - 87.5\, \boldsymbol{j} + 102.5\, \boldsymbol{k}\}$ lb·ft

ATTENTION QUIZ

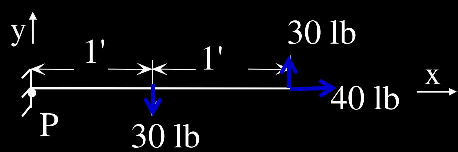
1. For this force system, the equivalent system at P is ____________ .

A) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +60 ft ·lb

B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

C) F_{RP} = 30 lb (along +y-dir.) and M_{RP} = -30 ft ·lb

D) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +30 ft ·lb



ATTENTION QUIZ

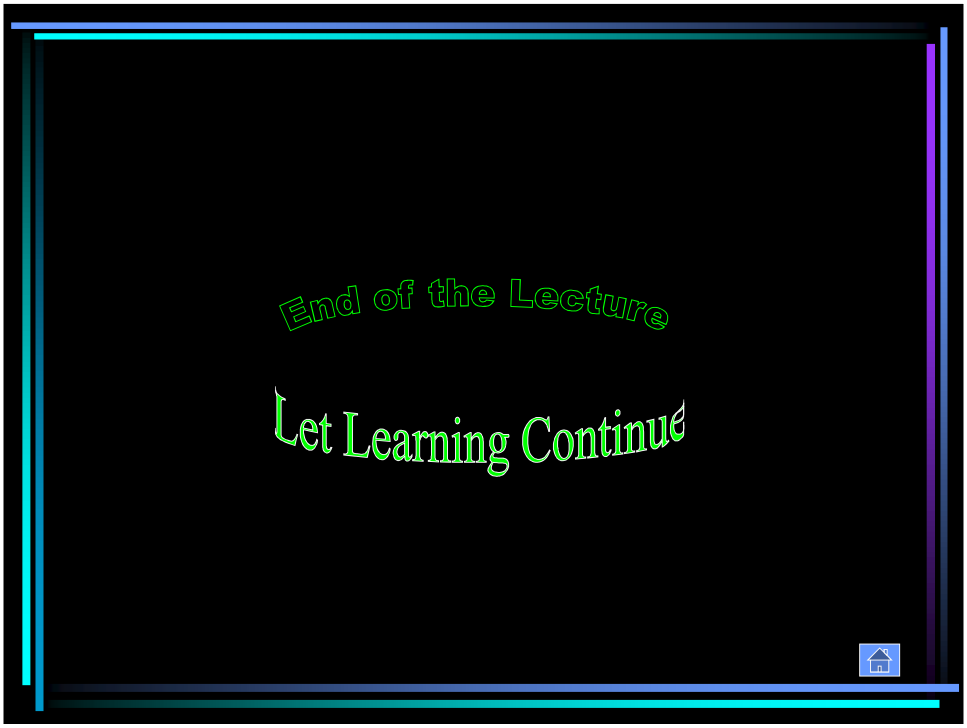
2. Consider three couples acting on a body. Equivalent systems will be _______ at different points on the body.

A) Different when located

B) The same even when located

C) Zero when located

D) None of the above.


End of the Lecture
Let Learning Continue