Introduction to Biostatistics – Mat 2379C

Solutions to Assignment 3

7.3 (10 points)

(a)

$$P(X \leq 0) = P((X-2)/0.7 \leq (0-2)/0.7) = P(Z < 2.86) = 0.0021$$

(b) Let Y be the number of seedlings which did not germinate in a tray of 25. Y has a binomial distribution with $n = 25$ trials and $p = 0.0021$ probability of success. The desired probability is $P(Y = 0) = (1 - 0.0021)^{25} = 0.9488$.

7.7 (25 points)

(a) Let X be the systolic of an individual from this population. Since

$$0.19 = P(X > 140) = 1 - P(Z < (140 - 115)/\sigma),$$

then $(140 - 115)/\sigma = 0.875$. Therefore, $\sigma = (140 - 115)/0.875 = 28.5714$.

(b) For a normal distribution the median is equal to the mean, which is equal to 115 mm Hg. Let q_1 be the first quartile, that is $P(Z < (q_1 - 115)/28.5714) = 0.25$. Thus,

$$(q_1 - 115)/28.5714 = -0.675 \Rightarrow q_1 = 115 - 28.5714(0.675) = 95.7143.$$

Let q_3 be the 3rd quartile, that is $P(Z < (q_3 - 115)/28.5714) = 0.75$. Thus,

$$(q_3 - 115)/28.5714) = 0.675 \Rightarrow q_3 = 115 + 28.5714(0.675) = 134.2857.$$

(c) We want

$$\begin{aligned} P(140 < X < 160) &= P((140-115)/28.5714 < Z < (160-115)/28.5714) \\ &= \Phi(1.58) - \Phi(0.88) = 0.9429 - 0.8106 = 0.1323. \end{aligned}$$

(d)

$$\begin{aligned} P(X > 160) &= 1 - P(Z < (160 - 115)/28.5714) \\ &= 1 - \Phi(1.58) = 1 - 0.9429 = 0.0571. \end{aligned}$$

3 (25 points)

Let X be the startle response for a person with a BAL of at least 0.015 mg/cl. Then $X \sim N(37.9, 12.4^2)$.
(a)

$$\begin{aligned} P(40 < X < 50) &= P((40 - 37.9)/12.4 < Z < (50 - 37.9)/12.4) \\ &= \Phi(0.96) - \Phi(0.17) = 0.8315 - 0.5675 = 0.264. \end{aligned}$$

(b) $P(X < 30) = P(Z < (30 - 37.9)/12.4) = \Phi(-0.64) = 0.2611.$
(c)First we find z such that $P(-z < Z < z) = 0.95$. Thus

$$0.95 = P(-z < Z < z) = 1 - P(|Z| > z) = 1 - 2P(Z < -z)$$

Therefore,

$$P(Z < -z) = 0.025 \;\Rightarrow\; -z = -1.96 \;\text{ and }\; z = 1.96.$$

In order to find the interval that contains 95% of the startle responses we need to find x_1 and x_2 such that $P((x_1 - 37.9)/12.4 < Z < (x_2 - 37.9)/12.4) = 0.95$. Thus

$$(x_1 - 37.9)/12.4 = -1.96 \;\Rightarrow\; x_1 = 37.9 - 1.96(12.4) = 13.596.$$

and

$$(x_2 - 37.9)/12.4 = 1.96 \;\Rightarrow\; x_2 = 37.9 + 1.96(12.4) = 62.204.$$

7.5 (20 points)

(a)

$$\begin{aligned} P(X > 105) &= 1 - P(Z < (105 - 100)/36) \\ &= 1 - \Phi(0.14) = 1 - 0.5557 = 0.4443. \end{aligned}$$

(b) $P(X < 98) = P(Z < (98 - 100)/36) = \Phi(-0.06) = 0.4761.$
(c)

$$\begin{aligned} P(93.5 < X < 112) &= P((93.5 - 100)/36 < Z < (112 - 100)/36) \\ &= \Phi(0.33) - \Phi(-0.18) = 0.6293 - 0.4286 = 0.2007. \end{aligned}$$

(d) $P(X < 121.5) = P(Z < (121.5 - 100)/36) = \Phi(0.60) = 0.7257.$

(e)

$$\begin{aligned} P(X > 90) &= 1 - P(Z < (90 - 100)/36) = 1 - \Phi(-0.28) = 1 - 0.3897 \\ &= 0.6103. \end{aligned}$$

(f)

$$\begin{aligned} P(82.5 < X < 117.5) &= P((82.5 - 100)/36 < Z < (117.5 - 100)/36) \\ &= \Phi(0.49) - \Phi(-0.49) = 0.6879 - 0.3121 = 0.3758. \end{aligned}$$

Below is the R output which gives the same calculations:

```
> 1-pnorm(105,100,36)
[1] 0.444769
> pnorm(98,100,36)
[1] 0.4778479
> pnorm(112,100,36)-pnorm(93.5,100,36)
[1] 0.2022004
> pnorm(121.5,100,36)
[1] 0.7248205
> 1-pnorm(90,100,36)
[1] 0.6094085
> pnorm(117.5,100,36)-pnorm(82.5,100,36)
[1] 0.3731116
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5 (20 points)

Let X be the scores made on a certain aptitude test by nursing students. Then $X \sim N(500, 10000)$.

(a)

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  > pnorm(200,500,100)
[1] 0.001349898
```

(b)

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> 1-pnorm(650,500,100)
[1] 0.0668072
```

(c)

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> pnorm(675,500,100)-pnorm(350,500,100)
[1] 0.8931336
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(d)

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> qnorm(0.2,500,100)
[1] 415.8379
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