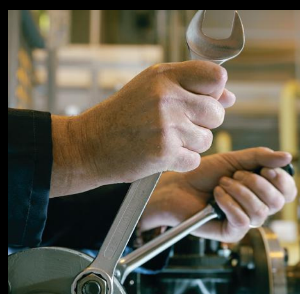
MOMENT OF A FORCE (SCALAR FORMULATION), CROSS PRODUCT, MOMENT OF A FORCE (VECTOR FORMULATION), & PRINCIPLE OF MOMENTS

Today's Objectives :

Students will be able to:

a) understand and define moment, and,

b) determine moments of a force in 2-D and 3-D cases.

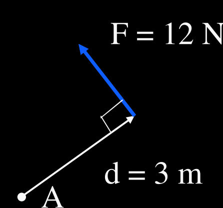


READING QUIZ

1. What is the moment of the 12 N force about point A (M_A)?

   A) 3 N·m B) 36 N·m C) 12 N·m

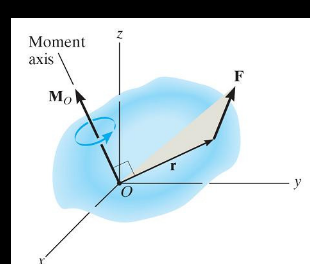
   D) (12/3) N·m E) 7 N·m



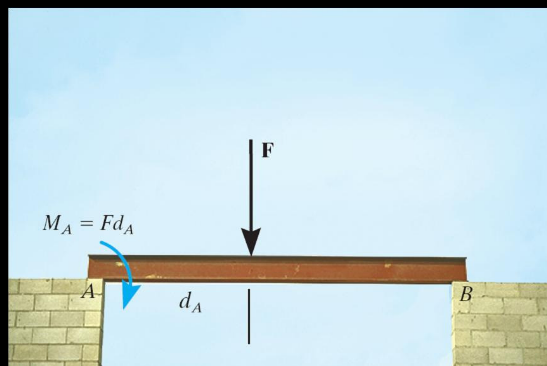
2. The moment of force ***F*** about point O is defined as $\boldsymbol{M_O}$ = ____________ .

   A) ***r* x *F*** B) ***F* x *r***

   C) ***r* • *F*** D) r * F



APPLICATIONS



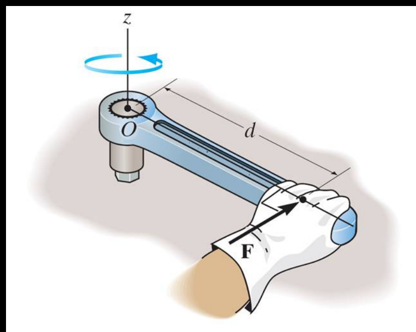
Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the beam supports.

What do you think those impacts are at points A and B?

APPLICATIONS

Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

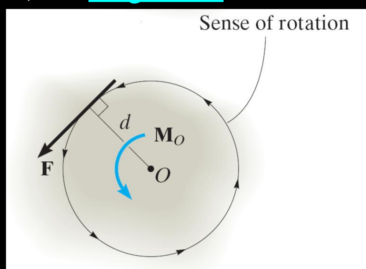
MOMENT OF A FORCE - SCALAR FORMULATION (Section 4.1)



The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

MOMENT OF A FORCE - SCALAR FORMULATION (continued)

In the 2-D case, the magnitude of the moment is $M_o = F\,d$

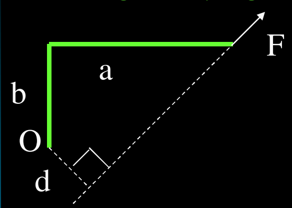


As shown, d is the perpendicular distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or counter-clockwise, depending on the tendency for rotation.

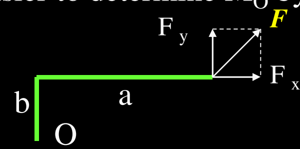
MOMENT OF A FORCE - SCALAR FORMULATION

(continued)



For example, $M_O = F\,d$ and the direction is counter-clockwise.

Often it is easier to determine M_O by using the components of ***F*** as shown.



Then $M_O = (F_Y\, a) - (F_X\, b)$. Note the different signs on the terms! The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

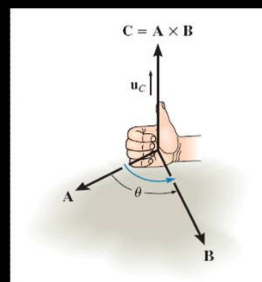
CROSS PRODUCT
(Section 4.2)

While finding the moment of a force in 2-D is straightforward when you know the perpendicular distance d, finding the perpendicular distances can be hard—especially when you are working with forces in three dimensions.

So a more general approach to finding the moment of a force exists. This more general approach is usually used when dealing with three dimensional forces but can also be used in the two dimensional case as well.

This more general method of finding the moment of a force uses a vector operation called the cross product of two vectors.

CROSS PRODUCT
(Section 4.2)



In general, the cross product of two vectors ***A*** and ***B*** results in another vector, ***C*** , i.e., ***C = A × B***. The magnitude and direction of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \mathrm{A\,B}\sin\theta\,\boldsymbol{u}_C$$

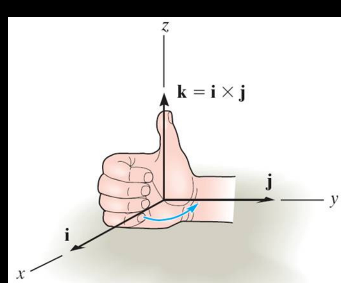
As shown, $\boldsymbol{u}_C$ is the unit vector perpendicular to both A and B vectors (or to the plane containing the A and B vectors).

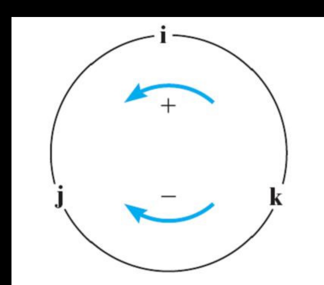
CROSS PRODUCT
(continued)

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





CROSS PRODUCT

(continued)

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

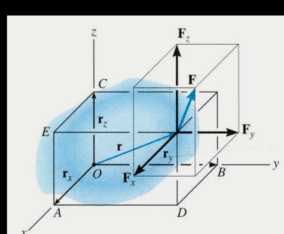
Each component can be determined using 2 × 2 determinants.

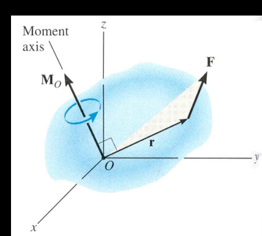
For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$ (Remember the negative sign)

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

MOMENT OF A FORCE – VECTOR FORMULATION (Section 4.3)





Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $M_O = r \times F$.

Here ***r*** is the position vector from point O to any point on the line of action of ***F***.

MOMENT OF A FORCE – VECTOR FORMULATION

(continued)

So, using the cross product, a moment can be expressed as

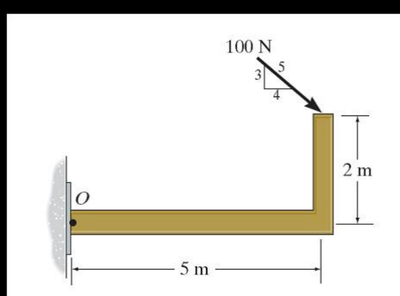
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2×2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\boldsymbol{M_O} = (r_y\ F_Z - r_Z\ F_y)\ \boldsymbol{i} - (r_x\ F_z - r_z\ F_x)\ \boldsymbol{j} + (r_x\ F_y - r_y\ F_x)\ \boldsymbol{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

EXAMPLE #1



Given: A 100 N force is applied to the frame.

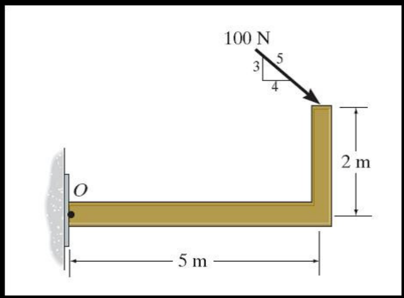
Find: The moment of the force at point O.

Plan:

1) Resolve the 100 N force along the x and y axes.

2) Determine M_O using a scalar analysis for the two force components and add those two moments together..

EXAMPLE #1 (continued)



Solution

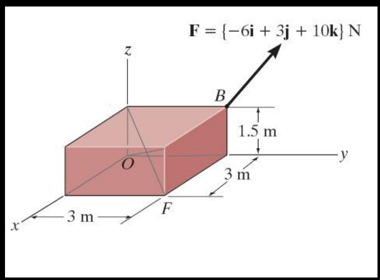
$+\uparrow F_y = -\,100\,(3/5)$ N

$+\rightarrow F_x = 100\,(4/5)$ N

$+\circlearrowleft M_O = \{-\,100\,(3/5)\text{N}\,(5\text{ m}) - (100)(4/5)\text{N}\,(2\text{ m})\}$ N·m

$= -\,460$ N·m

EXAMPLE # 2



Given: The force and geometry shown.

Find: Moment of ***F*** about point O.

Plan:

1) Find $\boldsymbol{r_{OB}}$.

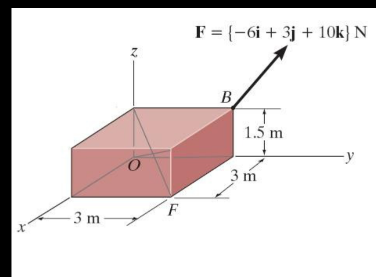
2) Determine $\boldsymbol{M_O} = \boldsymbol{r_{OB}} \times \boldsymbol{F}$.

EXAMPLE # 2 (continued)

Solution

First, find the position vector $\boldsymbol{r}_{OB}$

$$\boldsymbol{r}_{OB} = \{0\,\boldsymbol{i} + 3\,\boldsymbol{j} + 1.5\,\boldsymbol{k}\}\ \text{m}$$



Then find the moment by using the cross product.

$$M_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 3 & 1.5 \\ -6 & 3 & 10 \end{vmatrix} = [\{3(10) - 1.5(3)\}\,\boldsymbol{i} - \{0(10) - 1.5(-6)\}\,\boldsymbol{j} + \{0(3) - 3(-6)\}\,\boldsymbol{k}]\ \text{N·m}$$

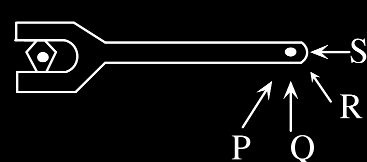
$$= \{25.5\,\boldsymbol{i} + 9\,\boldsymbol{j} + 18\,\boldsymbol{k}\}\ \text{N·m}$$

CONCEPT QUIZ

1. If a force of magnitude F can be applied in four different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

A) (Q, P) B) (R, S)

C) (P, R) D) (Q, S)

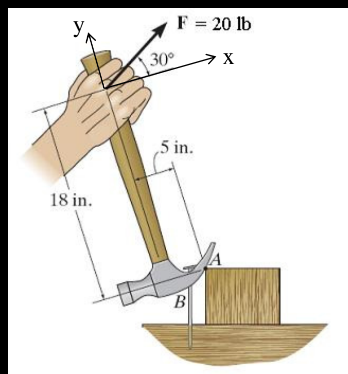


2. If $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, then what will be the value of $\boldsymbol{M} \bullet \boldsymbol{r}$?

A) 0 B) 1

C) r^2 F D) None of the above.

GROUP PROBLEM SOLVING



Given: A 20 lb force is applied to the hammer.

Find: The moment of the force at A.

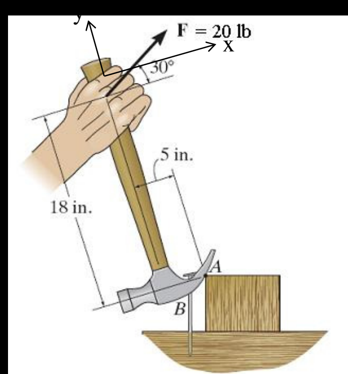
Plan:

Since this is a 2-D problem:

1) Resolve the 20 lb force along the handle's x and y axes.

2) Determine M_A using a scalar analysis.

GROUP PROBLEM SOLVING (cont.)



Solution:

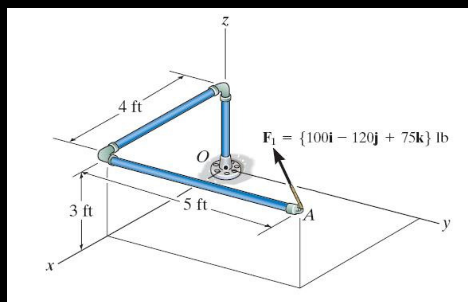
$+\uparrow F_y = 20 \sin 30° \text{ lb}$

$+\rightarrow F_x = 20 \cos 30° \text{ lb}$

$+\circlearrowleft M_A = \{-(20 \cos 30°)\text{lb } (18 \text{ in}) - (20 \sin 20°)\text{lb } (5 \text{ in})\}$

$= -351.77 \text{ lb·in} =$ 352 lb·in (clockwise)

GROUP PROBLEM SOLVING



Given: The force and geometry shown.

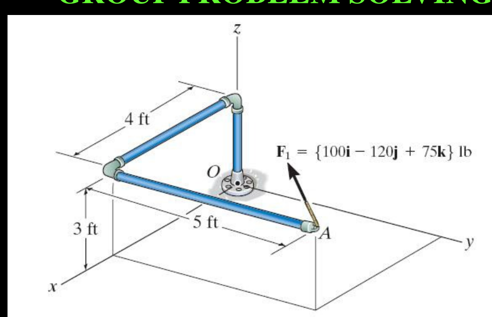
Find: Moment of F about point O

Plan:

1) Find $\boldsymbol{r}_{OA}$.

2) Determine $\boldsymbol{M}_O = \boldsymbol{r}_{OA} \times \boldsymbol{F}$

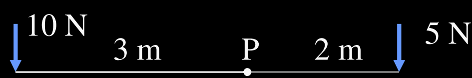
GROUP PROBLEM SOLVING (continued)



Solution: $\boldsymbol{r}_{OA} = \{ 4\,\boldsymbol{i} + 5\,\boldsymbol{j} + 3\,\boldsymbol{k} \}$ ft

$$\boldsymbol{M}_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4 & 5 & 3 \\ 100 & -120 & 75 \end{vmatrix} = \{ 735\,\boldsymbol{i} + 0\,\boldsymbol{j} - 980\,\boldsymbol{k} \} \text{ lb} \cdot \text{ft}$$

ATTENTION QUIZ



1. Using the CCW direction as positive, the net moment of the two forces about point P is

   A) 10 N ·m B) 20 N ·m C) - 20 N ·m

   D) 40 N ·m E) - 40 N ·m

2. If ***r*** = { 5 ***j*** } m and ***F*** = { 10 ***k*** } N, the moment

   r ***x*** ***F*** equals { ________ } N·m.

   A) 50 ***i*** B) 50 ***j*** C) –50 ***i***

   D) – 50 ***j*** E) 0

MOMENT ABOUT AN AXIS

Today's Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and

b) vector analysis.

READING QUIZ

1. When determining the moment of a force about a specified axis, the axis must be along _____________.

A) the x axis B) the y axis C) the z axis

D) any line in 3-D space E) any line in the x-y plane

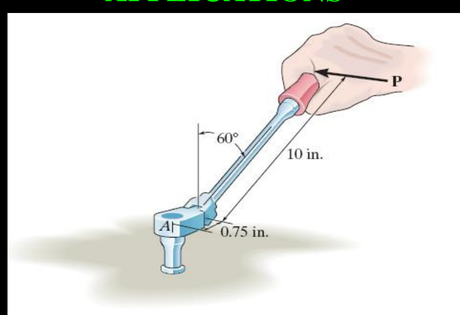
2. The triple scalar product $\boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ results in

A) a scalar quantity (+ or -). B) a vector quantity.

C) zero. D) a unit vector.

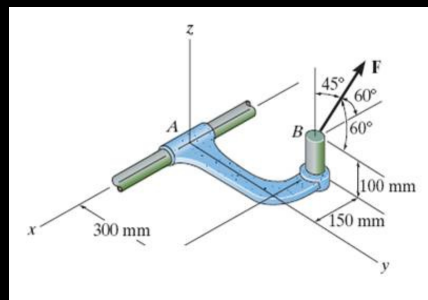
E) an imaginary number.

APPLICATIONS



With the force $\boldsymbol{P}$, a person is creating a moment $\boldsymbol{M_A}$. Does all of $\boldsymbol{M_A}$ act to turn the socket? How would you calculate an answer to this question?

APPLICATIONS



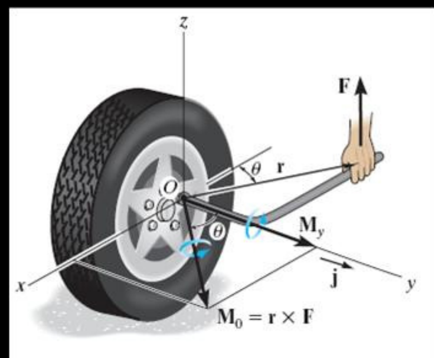
Sleeve A of this bracket can provide a maximum resisting moment of 125 N·m about the x-axis. How would you determine the maximum magnitude of F before turning about the x axis occurs?

SCALAR ANALYSIS

Recall that the moment of a scalar force about any point O is $M_O = F\ d_O$ where d_O is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.

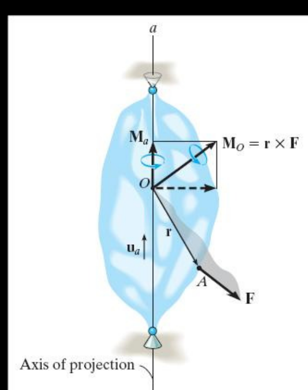
Finding the moment of a force about an axis can help answer the types of questions we just considered.

SCALAR ANALYSIS



In the figure above, the moment about the y-axis would be $M_y = F_z\,(d_x) = F\,(r \cos\theta)$. However, unless the force can easily be broken into components and the "d" found quickly, such calculations are not always trivial and vector analysis may be much easier (and less likely to produce errors).

VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the axis a'- a.

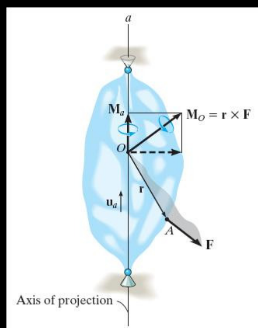
First compute the moment of ***F*** about any arbitrary point O that lies on the a'- a axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the axis a'- a using the dot product.

$$M_{a'\text{-}a} = \boldsymbol{u_a} \bullet \boldsymbol{M_O}$$

VECTOR ANALYSIS (continued)



$M_{a'-a}$ can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.

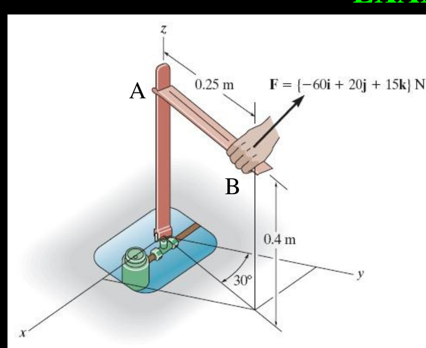
In the this equation,

u_a represents the unit vector along the axis a'-a axis,

r is the position vector from any point on the a'-a axis to any point A on the line of action of the force, and

F is the force vector.

EXAMPLE



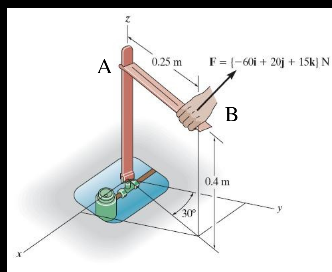
Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

Plan:

1) Use $M_z = \mathbf{u} \bullet (\mathbf{r} \times \mathbf{F})$.
2) Note that $\mathbf{u} = 1\,\mathbf{k}$.
3) The vector ***r*** is the position vector from A to B.
4) Force ***F*** is already given in Cartesian vector form.

EXAMPLE (continued)



$\boldsymbol{u} = 1\,\boldsymbol{k}$

$\boldsymbol{r_{AB}} = \{0.25 \sin 30°\, \boldsymbol{i} + 0.25 \cos 30°\, \boldsymbol{j}\}$ m

$= \{0.125\, \boldsymbol{i} + 0.2165\, \boldsymbol{j}\}$ m

$\boldsymbol{F} = \{-60\, \boldsymbol{i} + 20\, \boldsymbol{j} + 15\, \boldsymbol{k}\}$ N

Now find $M_z = \boldsymbol{u} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix}$$

$= 1\{\, 0.125\,(20) - 0.2165\,(-60)\,\}$ N·m

$= 15.5$ N·m

CONCEPT QUIZ

1. The vector operation $(\boldsymbol{P} \times \boldsymbol{Q}) \bullet \boldsymbol{R}$ equals

A) $\boldsymbol{P} \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

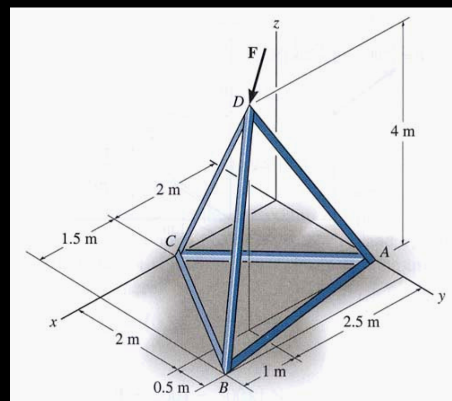
B) $\boldsymbol{R} \bullet (\boldsymbol{P} \times \boldsymbol{Q})$.

C) $(\boldsymbol{P} \bullet \boldsymbol{R}) \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

D) $(\boldsymbol{P} \times \boldsymbol{R}) \bullet (\boldsymbol{Q} \times \boldsymbol{R})$.

CONCEPT QUIZ

2. The force ***F*** is acting along DC. Using the triple product to determine the moment of ***F*** about the bar BA, you could use any of the following position vectors except _______.



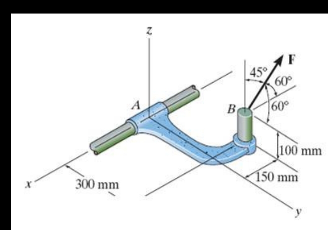
A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AD}$

C) $\boldsymbol{r}_{AC}$ D) $\boldsymbol{r}_{DB}$

E) $\boldsymbol{r}_{BD}$

GROUP PROBLEM SOLVING

Given: Sleeve A can provide a maximum resisting moment of 125 N·m about the x-axis.

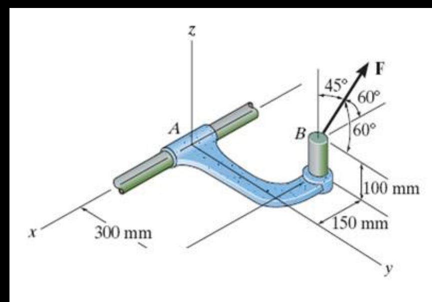


Find: The maximum magnitude of F before slipping occurs at A (the sleeve rotating around the x-axis).

<u>Plan</u>:

1) We need to use $M_X = \boldsymbol{u} \cdot (\boldsymbol{r}_{AB} \times \boldsymbol{F})$
2) Find $\boldsymbol{r}_{AB}$
3) Find ***F*** in Cartesian vector form.
4) Complete the triple scalar product & solve for F !

SOLUTION



$\boldsymbol{r}_{AB} = \{(-0.15 - 0)\,\boldsymbol{i} + (0.30 - 0)\,\boldsymbol{j} + (0.10 - 0)\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = \text{F}\,\{(-\cos 60°\,\boldsymbol{i} + \cos 60°\,\boldsymbol{j} + \cos 45°\,\boldsymbol{k})\}$ N

$= \{-0.5\,\text{F}\,\boldsymbol{i} + 0.5\,\text{F}\,\boldsymbol{j} + 0.707\,\text{F}\,\boldsymbol{k}\}$ N

SOLUTION (continued)

Now find the triple product, $M_x = \boldsymbol{u} \bullet (\boldsymbol{r}_{AB} \times \boldsymbol{F})$

$$M_X = \begin{vmatrix} 1 & 0 & 0 \\ -0.15 & 0.3 & 0.1 \\ -0.5F & 0.5F & 0.707F \end{vmatrix} \text{ N·m}$$

$M_x = 1\,\{0.3\,(0.707F) - 0.1\,(0.5F)\} + 0 + 0 = 0.162\,F$ N·m

$M_x = 125$ N·m = maximum moment along x-axis

$125 = 0.162\,F$

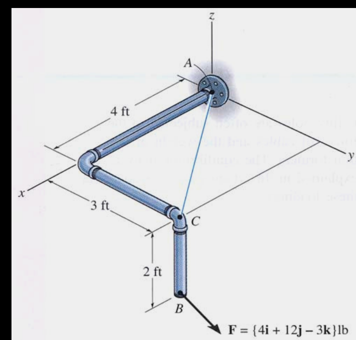
$F_{MAX} = 771$ N

ATTENTION QUIZ

1. For finding the moment of the force ***F*** about the x-axis, the position vector in the triple scalar product should be ___ .

   A) $\boldsymbol{r}_{AC}$ B) $\boldsymbol{r}_{BA}$

   C) $\boldsymbol{r}_{AB}$ D) $\boldsymbol{r}_{BC}$



2. If $\boldsymbol{r} = \{1\,\boldsymbol{i} + 2\,\boldsymbol{j}\}$ m and $\boldsymbol{F} = \{10\,\boldsymbol{i} + 20\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N, then the moment of ***F*** about the y-axis is ____ N·m.

   A) 10 B) -30

   C) -40 D) None of the above.

End of the Lecture

Let Learning Continue