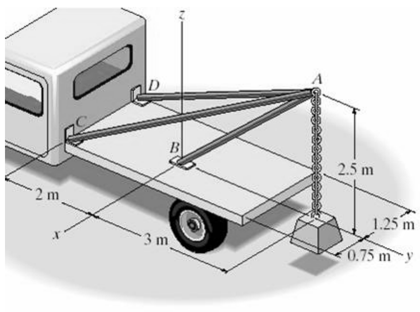
THREE-DIMENSIONAL FORCE SYSTEMS

Today's Objectives:

Students will be able to solve 3-D particle equilibrium problems by

a) Drawing a 3-D free body diagram, and,

b) Applying the three scalar equations (based on one vector equation) of equilibrium.

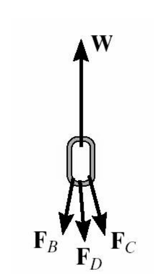


In-class Activities:

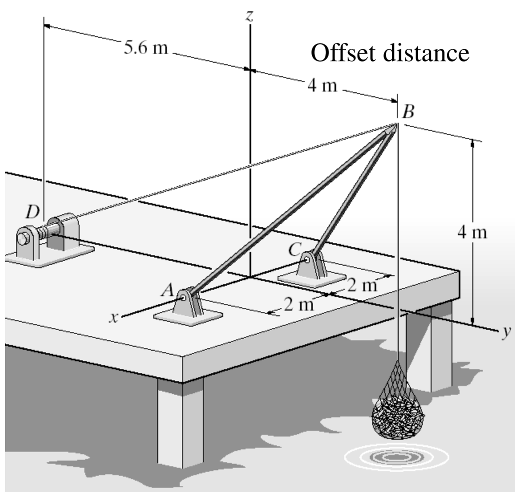
- Applications
- Equations of Equilibrium
- Concept Questions
- Group Problem Solving

APPLICATIONS

You know the weights of the electromagnet and its load. But, you need to know the forces in the chains to see if it is a safe assembly. How would you do this?



APPLICATIONS
(continued)

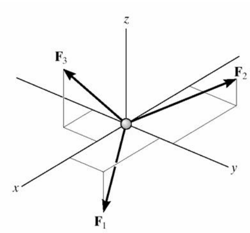


This shear leg derrick is to be designed to lift a maximum of 200 kg of fish.

How would you find the effect of different offset distances on the forces in the cable and derrick legs?

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \boldsymbol{F} = 0$).



This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$$

This vector equation will be satisfied only when

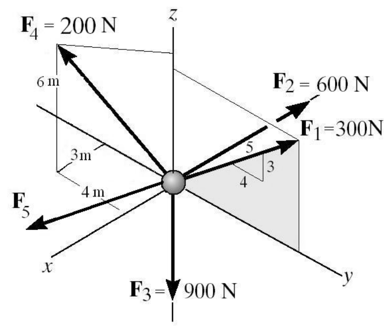
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

These equations are the three scalar equations of equilibrium. They are valid for any point in equilibrium and allow you to solve for up to three unknowns.

EXAMPLE #1

Given: The four forces and geometry shown.

Find: The force $\boldsymbol{F_5}$ required to keep particle O in equilibrium.



Plan:

1) Draw a FBD of particle O.

2) Write the unknown force as

$\boldsymbol{F_5} = \{F_x \boldsymbol{i} + F_y \boldsymbol{j} + F_z \boldsymbol{k}\}$ N

3) Write $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, $\boldsymbol{F_3}$, $\boldsymbol{F_4}$ and $\boldsymbol{F_5}$ in Cartesian vector form.

4) Apply the three equilibrium equations to solve for the three unknowns F_x, F_y, and F_z.

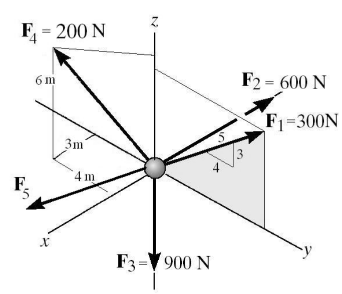
EXAMPLE #1
(continued)

$\boldsymbol{F_1} = \{300(4/5)\ \boldsymbol{j} + 300\ (3/5)\ \boldsymbol{k}\}$ N

$\boldsymbol{F_1} = \{240\ \boldsymbol{j} + 180\ \boldsymbol{k}\}$ N

$\boldsymbol{F_2} = \{-600\ \boldsymbol{i}\}$ N

$\boldsymbol{F_3} = \{-900\ \boldsymbol{k}\}$ N



$\boldsymbol{F_4} = F_4\ (\boldsymbol{r_B} / r_B)$

$= 200\text{ N } [(3\boldsymbol{i} - 4\boldsymbol{j} + 6\ \boldsymbol{k})/(3^2 + 4^2 + 6^2)^{1/2}]$

$= \{76.8\ \boldsymbol{i} - 102.4\boldsymbol{j} + 153.6\ \boldsymbol{k}\}$ N

$\boldsymbol{F_5} = \{\ F_x\ \boldsymbol{i} - F_y\ \boldsymbol{j} + F_z\ \boldsymbol{k}\}$ N

EXAMPLE #1
(continued)

Equating the respective $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ components to zero, we have

$\Sigma F_x = 76.8 - 600 + F_x = 0$; solving gives $F_x = 523.2$ N

$\Sigma F_y = 240 - 102.4 + F_y = 0$; solving gives $F_y = -137.6$ N

$\Sigma F_z = 180 - 900 + 153.6 + F_z = 0$; solving gives $F_z = 566.4$ N

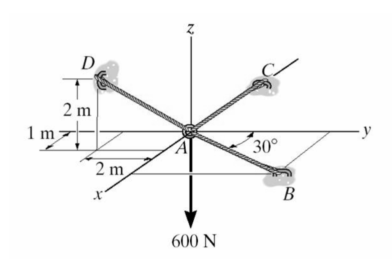
Thus, $\boldsymbol{F_5} = \{523\, \boldsymbol{i} - 138\, \boldsymbol{j} + 566\, \boldsymbol{k}\}$ N

Using this force vector, you can determine the force's magnitude and coordinate direction angles as needed.

EXAMPLE #2

Given: A 600 N load is supported by three cords with the geometry as shown.

Find: The tension in cords AB, AC and AD.



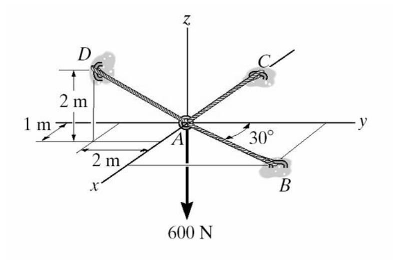
<u>Plan</u>:

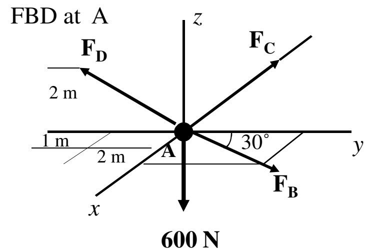
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

EXAMPLE #2 (continued)





$\boldsymbol{F_B}$ = F_B (sin 30° $\boldsymbol{i}$ + cos 30° $\boldsymbol{j}$) N

= {0.5 F_B $\boldsymbol{i}$ + 0.866 F_B $\boldsymbol{j}$} N

$\boldsymbol{F_C}$ = – F_C $\boldsymbol{i}$ N

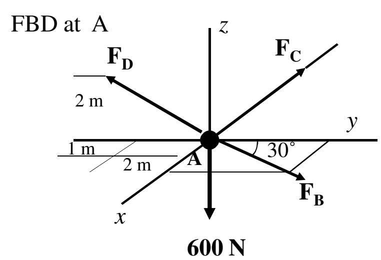
$\boldsymbol{F_D}$ = F_D ($\boldsymbol{r_{AD}}$ / r_{AD})

= F_D { (1 $\boldsymbol{i}$ – 2 $\boldsymbol{j}$ + 2 $\boldsymbol{k}$) / $(1^2 + 2^2 + 2^2)^{½}$ } N

= { 0.333 F_D $\boldsymbol{i}$ – 0.667 F_D $\boldsymbol{j}$ + 0.667 F_D $\boldsymbol{k}$ } N

EXAMPLE #2 (continued)

Now equate the respective $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ components to zero.



$\sum F_x = 0.5\ F_B - F_C + 0.333\ F_D = 0$

$\sum F_y = 0.866\ F_B - 0.667\ F_D = 0$

$\sum F_z = 0.667\ F_D - 600 = 0$

Solving the three simultaneous equations yields

F_C = 646 N

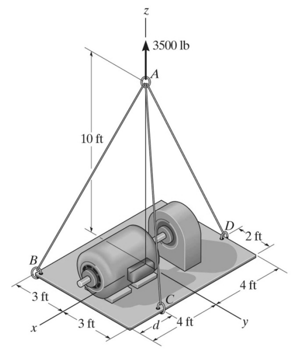
F_D = 900 N

F_B = 693 N

GROUP PROBLEM SOLVING

Given: A 3500 lb motor and plate, as shown, are in equilibrium and supported by three cables and d = 4 ft.

Find: Magnitude of the tension in each of the cables.



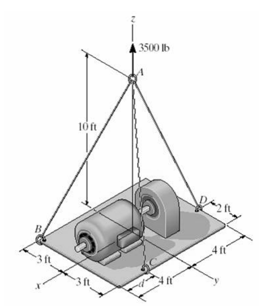
<u>Plan</u>:

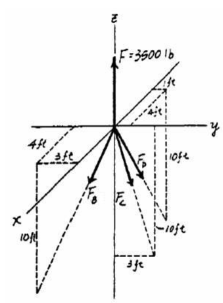
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

2) Represent each force in the Cartesian vector form.

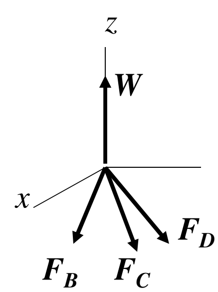
3) Apply equilibrium equations to solve for the three unknowns.

GROUP PROBLEM SOLVING (continued)





FBD of Point A



$\boldsymbol{W}$ = load or weight of unit = 3500 $\boldsymbol{k}$ lb

$\boldsymbol{F_B}$ = $F_B(\boldsymbol{r_{AB}}/r_{AB})$ = F_B {(4 $\boldsymbol{i}$ – 3 $\boldsymbol{j}$ – 10 $\boldsymbol{k}$) / (11.2)} lb

$\boldsymbol{F_C}$ = $F_C(\boldsymbol{r_{AC}}/r_{AC})$ = F_C { (3 $\boldsymbol{j}$ – 10 $\boldsymbol{k}$) / (10.4) }lb

$\boldsymbol{F_D}$ = $F_D(\boldsymbol{r_{AD}}/r_{AD})$ = F_D { (– 4 $\boldsymbol{i}$ + 1 $\boldsymbol{j}$ –10 $\boldsymbol{k}$) / (10.8) }lb

GROUP PROBLEM SOLVING (continued)

The particle A is in equilibrium, hence

$\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} + \boldsymbol{W} = 0$

Now equate the respective ***i***, ***j***, ***k*** components to zero (i.e., apply the three scalar equations of equilibrium).

$\sum F_x = (4/\ 11.2)F_B - (4/\ 10.8)F_D = 0$

$\sum F_y = (-\ 3/\ 11.2)F_B + (3/\ 10.4)F_C + (1/\ 10.8)F_D = 0$

$\sum F_z = (-\ 10/\ 11.2)F_B - (10/\ 10.4)F_C - (10/\ 10.8)F_D + 3500 = 0$

Solving the three simultaneous equations gives

$F_B = 1467$ lb

$F_C = 914$ lb

$F_D = 1420$ lb