EQUILIBRIUM OF A PARTICLE, THE FREE-BODY DIAGRAM & COPLANAR FORCE SYSTEMS

Today's Objectives:

Students will be able to :

a) Draw a free body diagram (FBD), and,

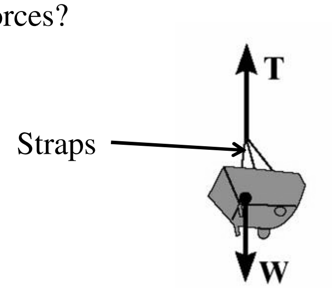
b) Apply equations of equilibrium to solve a 2-D problem.

In-Class Activities:

- Applications
- What, Why and How to draw, a FBD
- Equations of Equilibrium
- Analysis of Spring and Pulleys
- Group Problem Solving

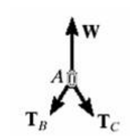
APPLICATIONS

The crane is lifting a load. To decide if the straps holding the load to the crane hook will fail, you need to know the force in the straps. How could you find the forces?

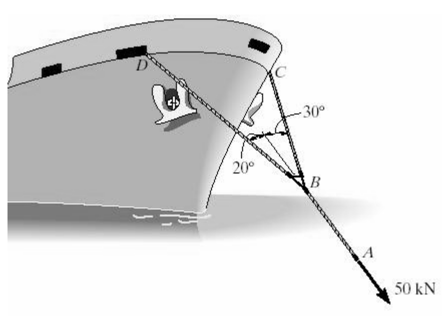


APPLICATIONS
(continued)

For a spool of given weight, how would you find the forces in cables AB and AC ? If designing a spreader bar like this one, you need to know the forces to make sure the rigging doesn't fail.



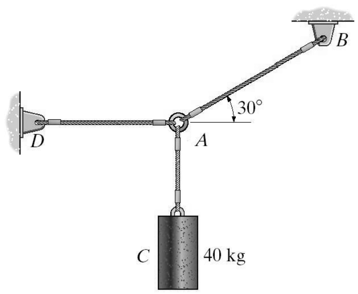
APPLICATIONS
(continued)



For a given force exerted on the boat's towing pendant, what are the forces in the bridle cables? What size of cable must you use?

COPLANAR FORCE SYSTEMS

(Section 3.3)



This is an example of a 2-D or coplanar force system.

If the whole assembly is in equilibrium, then particle A is also in equilibrium.

To determine the tensions in the cables for a given weight of the cylinder, you need to learn how to draw a free body diagram and apply equations of equilibrium.

THE WHAT, WHY AND HOW OF A FREE BODY DIAGRAM (FBD)

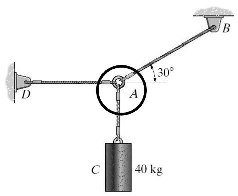
Free Body Diagrams are one of the most important things for you to know how to draw and use.

What ? - It is a drawing that shows all external forces acting on the particle.

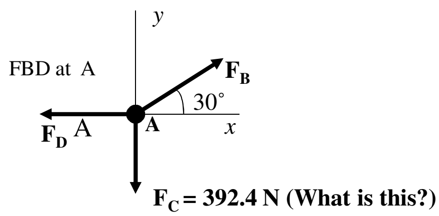
Why ? - It is key to being able to write the equations of equilibrium—which are used to solve for the unknowns (usually forces or angles).

How ?

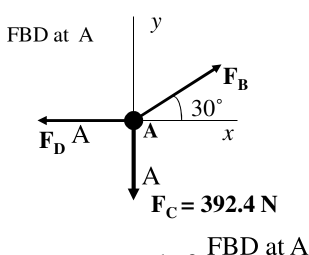
1. Imagine the particle to be isolated or cut free from its surroundings.
2. Show all the forces that act on the particle.
   Active forces: They want to move the particle.
   Reactive forces: They tend to resist the motion.
3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables .



Note : Cylinder mass = 40 Kg



EQUATIONS OF 2-D EQUILIBRIUM



Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} = 0$

or $\Sigma \boldsymbol{F} = 0$

In general, for a particle in equilibrium,

$\Sigma \boldsymbol{F} = 0$ or

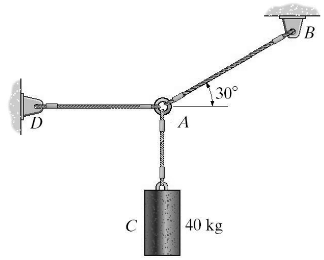
$\Sigma F_x \boldsymbol{i} + \Sigma F_y \boldsymbol{j} = 0 = 0\boldsymbol{i} + 0\boldsymbol{j}$ (a vector equation)

Or, written in a scalar form,

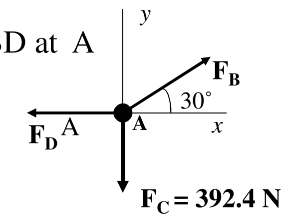
$\Sigma F_x = 0$ and $\Sigma F_y = 0$

These are two scalar equations of equilibrium (E-of-E). They can be used to solve for up to <u>two</u> unknowns.

EXAMPLE



FBD at A



Note : Cylinder mass = 40 Kg

Write the scalar E-of-E:

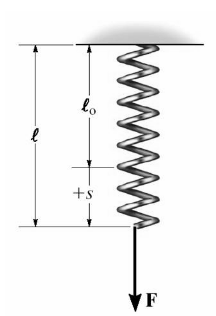
$+\rightarrow \Sigma F_x = F_B \cos 30^\circ - F_D = 0$

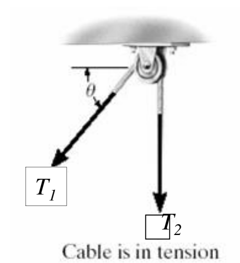
$+\uparrow \Sigma F_y = F_B \sin 30^\circ - 392.4 \text{ N} = 0$

Solving the second equation gives: $F_B = 785$ N $\rightarrow$

From the first equation, we get: $F_D = 680$ N $\leftarrow$

SPRINGS, CABLES, AND PULLEYS



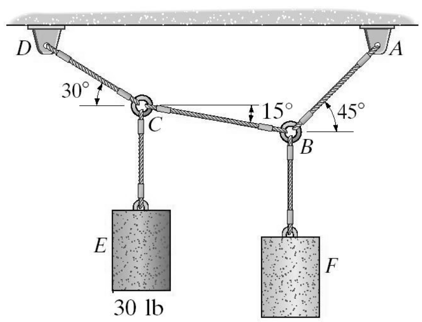


Spring Force = spring constant * deformation, or

F = k * s

With a frictionless pulley, $T_1 = T_2$.

EXAMPLE



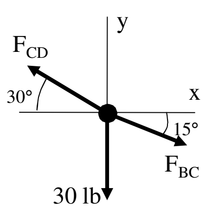
Given: Cylinder E weighs 30 lb and the geometry is as shown.

Find: Forces in the cables and weight of cylinder F.

Plan:

1. Draw a FBD for Point C.
2. Apply E-of-E at Point C to solve for the unknowns (F_{CB} & F_{CD}).
3. Knowing F_{CB} , repeat this process at point B.

EXAMPLE
(continued)



A FBD at C should look like the one at the left. Note the assumed directions for the two cable tensions.

The scalar E-of-E are:

$+\rightarrow \quad \Sigma F_x = F_{BC} \cos 15° - F_{CD} \cos 30° = 0$

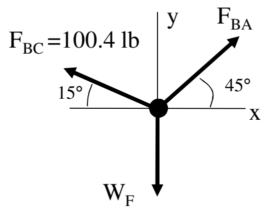
$+\uparrow \quad \Sigma F_y = F_{CD} \sin 30° - F_{BC} \sin 15° - 30 = 0$

Solving these two simultaneous equations for the two unknowns F_{BC} and F_{CD} yields:

$F_{BC} = 100.4$ lb ↘

$F_{CD} = 112.0$ lb ↖

EXAMPLE (continued)



Now move on to ring B. A FBD for B should look like the one to the left.

The scalar E-of-E are:

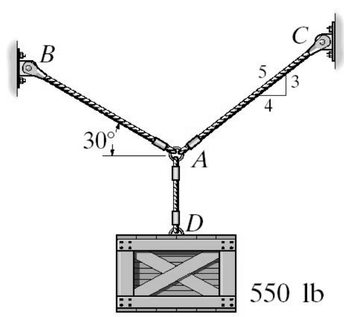
$+\rightarrow \Sigma F_x = F_{BA} \cos 45° - 100.4 \cos 15° = 0$

$+\uparrow \Sigma F_y = F_{BA} \sin 45° + 100.4 \sin 15° - W_F = 0$

Solving the first equation and then the second yields

F_{BA} = 137 lb ↖ and W_F = 123 lb ↓

GROUP PROBLEM SOLVING



Given: The box weighs 550 lb and geometry is as shown.

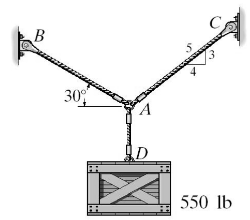
Find: The forces in the ropes AB and AC.

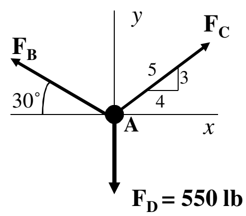
Plan:

1. Draw a FBD for point A.
2. Apply the E-of-E to solve for the forces in ropes AB and AC.

GROUP PROBLEM SOLVING

(continued)





FBD at point A

Applying the scalar E-of-E at A, we get;

$+ \rightarrow \sum F_x = F_B \cos 30° - F_C\,(4/5) = 0$

$+ \rightarrow \sum F_y = F_B \sin 30° + F_C\,(3/5) - 550 \text{ lb} = 0$

Solving the above equations, we get;

$F_B = 478$ lb ↖ and $F_C = 518$ lb ↗