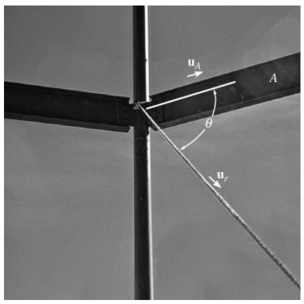
DOT PRODUCT

<u>Today's Objective</u>:

Students will be able to use the vector dot product to:

a) determine an angle between two vectors, and,

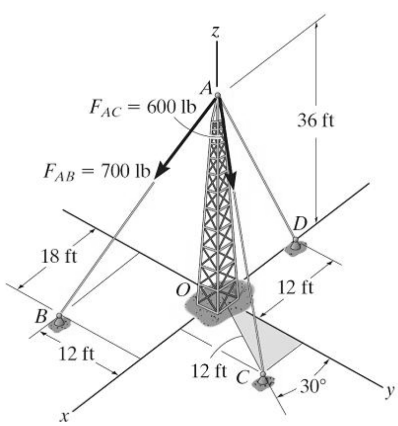
b) determine the projection of a vector along a specified line.



<u>In-Class Activities</u>:

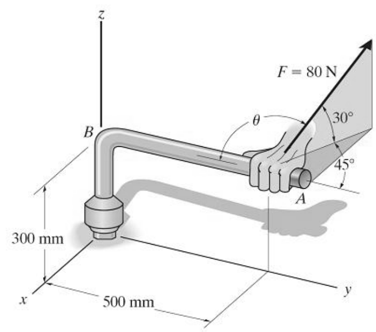
- Applications / Relevance
- Dot product - Definition
- Angle Determination
- Determining the Projection
- Group Problem Solving

APPLICATIONS



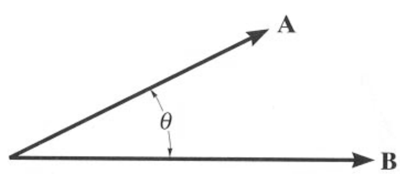
If the design for the cable placements required specific angles between the cables, how would you check this installation to make sure the angles were correct?

APPLICATIONS



For the force ***F*** being applied to the wrench at Point A, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to the pipe)?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A•B*** = A B cos θ. The angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.

<u>Dot Product Characteristics</u>:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DOT PRODUCT DEFINITON
(continued)

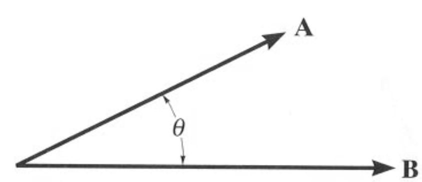
Examples: By definition, $\boldsymbol{i} \bullet \boldsymbol{j} = 0$

$\boldsymbol{i} \bullet \boldsymbol{i} = 1$

$\boldsymbol{A} \bullet \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \bullet (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$

$= A_x B_x + A_y B_y + A_z B_z$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For the given two vectors in the Cartesian form, one can find the angle by

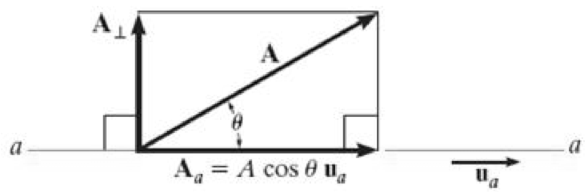
a) Finding the dot product, $\boldsymbol{A} \bullet \boldsymbol{B} = (A_x B_x + A_y B_y + A_z B_z)$,

b) Finding the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Using the definition of dot product and solving for θ, i.e.,

$\theta = \cos^{-1} [(\boldsymbol{A} \bullet \boldsymbol{B})/(A\,B)]$, where $0^\circ \le \theta \le 180^\circ$.

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\boldsymbol{u}_{aa'}$ along line aa′
2. Find the scalar projection of $\boldsymbol{A}$ along line aa′ by

$$A_{\parallel} = \boldsymbol{A} \bullet \boldsymbol{u}_{aa} = A_x U_x + A_y U_y + A_z U_z$$

DETERMINING THE PROJECTION OF A VECTOR
(continued)

3. If needed, the projection can be written as a vector, $\boldsymbol{A}_{\parallel}$, by using the unit vector $\boldsymbol{u}_{aa'}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{\parallel} = A_{\parallel}\, \boldsymbol{u}_{aa'}$$

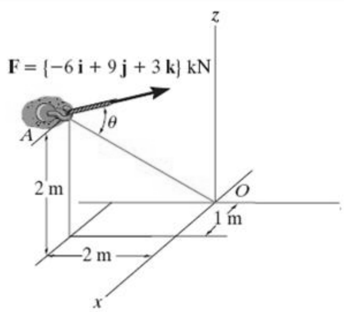
4. The scalar and vector forms of the perpendicular component can easily be obtained by

$A_\perp = (A^2 - A_{\parallel}^{2})^{1/2}$ and

$\boldsymbol{A}_\perp = \boldsymbol{A} - \boldsymbol{A}_{\parallel}$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_\perp + \boldsymbol{A}_{\parallel}$)

EXAMPLE



Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

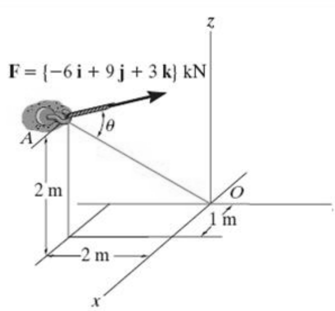
Plan:

1. Find $\boldsymbol{r_{AO}}$

2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F \ r_{AO})\}$

3. Find the projection via $F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}}$ (or $F \cos \theta$)

EXAMPLE
(continued)



$\boldsymbol{r_{AO}} = \{-1\ \boldsymbol{i} + 2\boldsymbol{j} - 2\ \boldsymbol{k}\}$ m

$r_{AO} = (1^2 + 2^2 + 2^2)^{1/2} = 3$ m

$\boldsymbol{F} = \{-6\ \boldsymbol{i} + 9\boldsymbol{j} + 3\ \boldsymbol{k}\}$ kN

$F = (6^2 + 9^2 + 3^2)^{1/2} = 11.22$ kN

$\boldsymbol{F} \bullet \boldsymbol{r_{AO}} = (-6)(-1) + (9)(2) + (3)(-2) = 18$ kN m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\ r_{AO})\}$

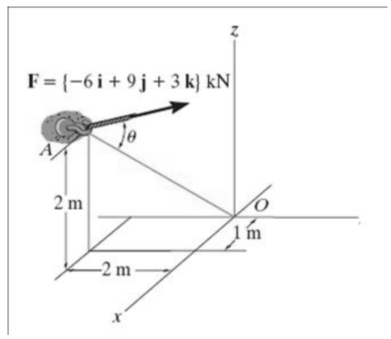
$\theta = \cos^{-1}\{18 / (11.22 * 3)\} = 57.67°$

EXAMPLE
(continued)

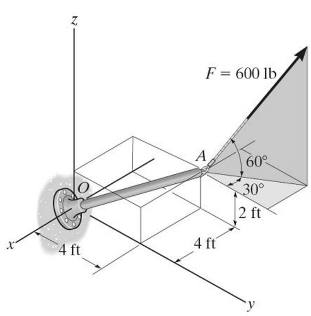
$\boldsymbol{u_{AO}} = \boldsymbol{r_{AO}}/r_{AO} = \{(-1/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} + (-2/3)\,\boldsymbol{k}\}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}} = (-6)(-1/3) + (9)(2/3) + (3)(-2/3) = 6.00$ kN

Or: $F_{AO} = F \cos\theta = 11.22 \cos(57.67°) = 6.00$ kN



GROUP PROBLEM SOLVING



Given: The force acting on the pole.

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole AO.

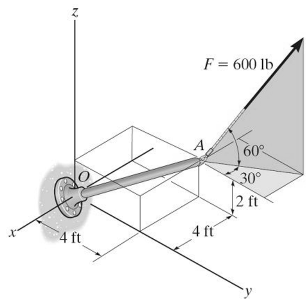
__Plan__:

1. Find $\boldsymbol{r_{AO}}$

2. Find the angle $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\ r_{AO})\}$

3. The find the projection via $F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}}$ or $F \cos\theta$

GROUP PROBLEM SOLVING (continued)



$\boldsymbol{r_{AO}} = \{4\,\boldsymbol{i} - 4\,\boldsymbol{j} - 2\,\boldsymbol{k}\}$ ft.

$r_{AO} = (4^2 + 4^2 + 2^2)^{1/2} = 6$ ft.

$F_z = 600 \sin 60° = 519.6$ lb

$F' = 600 \cos 60° = 300$ lb

$\boldsymbol{F} = \{-300 \sin 30° \boldsymbol{i} + 300 \cos 30° \boldsymbol{j} + 519.6\, \boldsymbol{k}\}$ lb

$\boldsymbol{F} = \{-150\, \boldsymbol{i} + 259.8\, \boldsymbol{j} + 519.6\, \boldsymbol{k}\}$lb

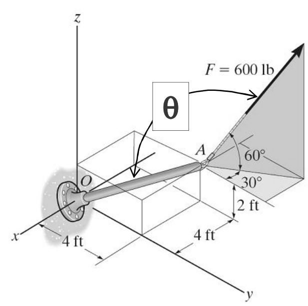
$F = (150^2 + 259.8^2 + 519.6^2)^{1/2} = 600$ lb

$\boldsymbol{F} \bullet \boldsymbol{r_{AO}} = (-150)(4)+(259.8)(-4)+(519.6)(-2) = -2678$ lb·ft

GROUP PROBLEM SOLVING (continued)

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\, r_{AO})\}$

$\theta = \cos^{-1}\{-2678 /(600 \times 6)\} = 138.1°$



$\boldsymbol{u_{AO}} = \boldsymbol{r_{AO}}/r_{AO} = \{(4/6)\, \boldsymbol{i} - (4/6)\, \boldsymbol{j} - (2/6)\, \boldsymbol{k}\}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}} = (-150)(4/6) + (259.8)(-4/6) + (519.6)(-2/6)$
$= -446$ lb

Or: $F_{AO} = F \cos \theta = 600 \cos (138.1°) = -446$ lb