POSITION VECTORS & FORCE VECTORS

Today's Objectives:

Students will be able to :

a) Represent a position vector in Cartesian coordinate form, from given geometry.

b) Represent a force vector directed along a line.

In-Class Activities:

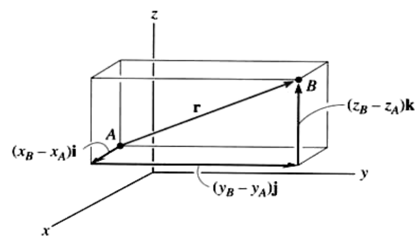
- Applications / Relevance
- Write Position Vectors
- Write a Force Vector
- Group Problem

APPLICATIONS

This awning is held up by three chains. What are the forces in the chains and how do we find their directions? Why would we want to know these things?

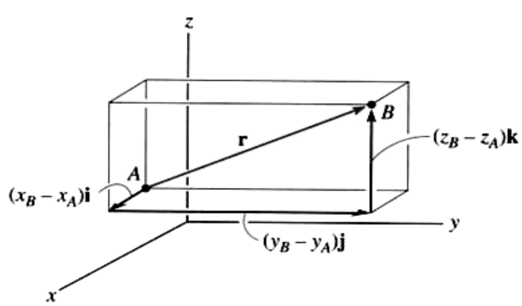
POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.



Consider two points, A and B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.

POSITION VECTOR

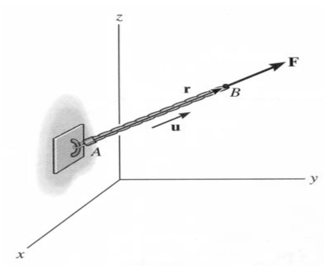


The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$$\boldsymbol{r}_{AB} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$$

Please note that B is the ending point and A is the starting point. ALWAYS subtract the “tail” coordinates from the “tip” coordinates!

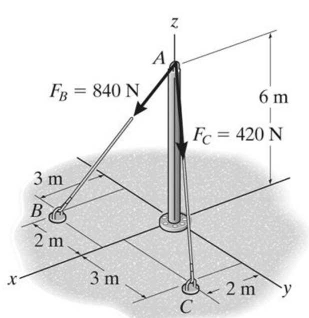
FORCE VECTOR DIRECTED ALONG A LINE (Section 2.8)



If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force's magnitude. So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.
b) Find the unit vector describing the line's direction, $\boldsymbol{u}_{AB} = (\boldsymbol{r}_{AB}/r_{AB})$.
c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = F\ \boldsymbol{u}_{AB}$.

EXAMPLE



Given: The 420 N force along the cable AC.

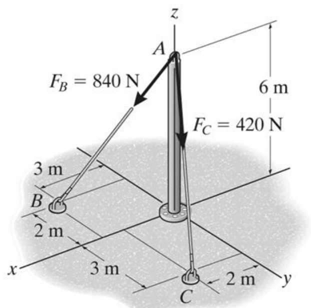
Find: The force $\boldsymbol{F}_{AC}$ in the Cartesian vector form.

Plan:

1. Find the position vector $\boldsymbol{r}_{AC}$ and the unit vector $\boldsymbol{u}_{AC}$.

2. Obtain the force vector as $\boldsymbol{F}_{AC} = 420\text{ N}\ \boldsymbol{u}_{AC}$.

EXAMPLE (continued)



As per the figure, when relating A to C, we will have to go 2 m in the x-direction, 3 m in the y-direction, and -6 m in the z-direction. Hence,

$\boldsymbol{r_{AC}} = \{2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m.

(We can also find $\boldsymbol{r_{AC}}$ by subtracting the coordinates of A from the coordinates of C.)

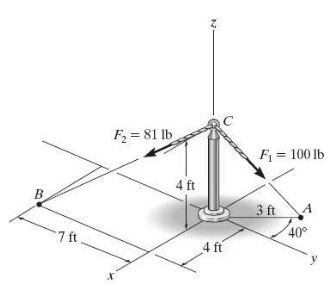
$r_{AC} = (2^2 + 3^2 + 6^2)^{1/2} = 7$ m

Now $\boldsymbol{u_{AC}} = \boldsymbol{r_{AC}}/r_{AC}$ and $\boldsymbol{F_{AC}} = 420\,\boldsymbol{u_{AC}}$ N $= 420\,(\boldsymbol{r_{AC}}/r_{AC})$

So $\boldsymbol{F_{AC}} = 420\{\,(2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 6\,\boldsymbol{k}) / 7\,\}$ N

$= \{120\,\boldsymbol{i} + 180\,\boldsymbol{j} - 360\,\boldsymbol{k}\}$ N

GROUP PROBLEM SOLVING



Given: Two forces are acting on a pipe as shown in the figure.

Find: The magnitude and the coordinate direction angles of the resultant force.

<u>Plan</u>:

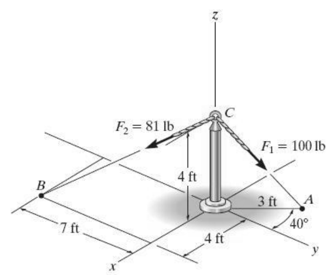
1) Find the forces along CA and CB in the Cartesian vector form.
2) Add the two forces to get the resultant force, $\boldsymbol{F_R}$.
3) Determine the magnitude and the coordinate angles of $\boldsymbol{F_R}$.

GROUP PROBLEM SOLVING
(continued)

$\boldsymbol{F_{CA}} = 100 \text{ lb } (\boldsymbol{r_{CA}}/r_{CA})$

$\boldsymbol{F_{CA}} = 100 \text{ lb } (-3 \sin 40° \, \boldsymbol{i} + 3 \cos 40° \, \boldsymbol{j} - 4 \, \boldsymbol{k})/5$

$\boldsymbol{F_{CA}} = (-38.57 \, \boldsymbol{i} + 45.96 \, \boldsymbol{j} - 80 \, \boldsymbol{k}) \text{ lb}$



$\boldsymbol{F_{CB}} = 81 \text{ lb } (\boldsymbol{r_{CB}}/r_{CB})$

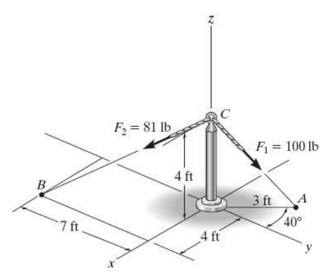
$\boldsymbol{F_{CB}} = 81 \text{ lb } (4 \, \boldsymbol{i} - 7 \, \boldsymbol{j} - 4 \, \boldsymbol{k})/9$

$\boldsymbol{F_{CB}} = \{36 \, \boldsymbol{i} - 63 \, \boldsymbol{j} - 36 \, \boldsymbol{k}\} \text{ lb}$

GROUP PROBLEM SOLVING
(continued)

$\boldsymbol{F_R} = \boldsymbol{F_{CA}} + \boldsymbol{F_{CB}}$

$\quad = \{-2.57 \, \boldsymbol{i} - 17.04 \, \boldsymbol{j} - 116 \, \boldsymbol{k}\} \text{ lb}$



$F_R = (2.57^2 + 17.04^2 + 116^2)$

$\quad = 117.3 \text{ lb} = 117 \text{ lb}$

$\alpha = \cos^{-1}(-2.57/117.3) = 91.3°$

$\beta = \cos^{-1}(-17.04/117.3) = 98.4°$

$\gamma = \cos^{-1}(-116/117.3) = 172°$