MAT 2379A
Midterm Examination (with Solutions)

October 20, 2010 **Professor Raluca Balan**
Time: 80 minutes

Student Number: ____________________

Family Name: ______________________ **First Name:** _________________

This is an open book examination. Only TI30 calculators are permitted. Record your answer to each question in the table below.

Question	Answer
1	B
2	A
3	C
4	E
5	E
6	D
7	A
8	C
9	C
10	B
11	A
12	D

NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.

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1. Meteorologists record the minimum temperature (in degrees Fahrnenheit) for each week of the year. The values recorded in Ottawa for the 52 weeks of the year 2009 are stored in column C1 (using Minitab). Below is the Minitab output which gives the stem-and-leaf diagram and some descriptive statistics for C1.

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Stem-and-leaf of C1  N  = 52
Leaf Unit = 1

 3   1  056
 7   2  3579
 17  3  0011244689
 21  4  2355
(7)  5  0111588
 24  6  000111233344666667778889

Descriptive Statistics: C1

Variable   N   N*    Mean   SE Mean   StDev
C1        52    0   49.58      2.35   16.95
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Find the first quartile (Q_1), the median ($\tilde{x}$), and the third quartile (Q_3).

A) $Q_1 = 35, \tilde{x} = 55.0, Q_3 = 64$
B) $Q_1 = 34, \tilde{x} = 56.5, Q_3 = 64$
C) $Q_1 = 39, \tilde{x} = 57.5, Q_3 = 69$
D) $Q_1 = 34, \tilde{x} = 58.0, Q_3 = 62$
E) $Q_1 = 36, \tilde{x} = 56.5, Q_3 = 66$.

Solution: $n = 52$ is even with $n/2 = 26$. The median is the calculated as:

$$\tilde{x} = \frac{y_{26} + y_{27}}{2} = \frac{55 + 58}{2} = 56.5$$

Since $(n+1)/4 = 53/4 = 13.25$, the first quartile is calculated as:

$$Q_1 = (0.75)y_{13} + (0.25)y_{14} = (0.75)(34) + (0.25)(34) = 34$$

(The same thing is obtained using the formula in the book: $Q_1 = (0.5)y_{13} + (0.5)y_{14} = (0.5)(34) + (0.5)(34) = 34.$)

Since $3(n+1)/4 = 159/4 = 39.75$, the third quartile is calculated as:

$$Q_3 = (0.25)y_{39} + (0.75)y_{40} = (0.25)(64) + (0.75)(64) = 64$$

(The same thing is obtained using the formula in the book: $Q_3 = (0.5)y_{39} + (0.5)y_{40} = (0.5)(64) + (0.5)(64) = 64.$)

2. Continuing with the situation in Question 1, which one of the following statements is *not* true? (There is only one statement which is not true.)

   A) the shape of the distribution is symmetric
   B) the interquartile range is 30
   C) there are no outliers
   D) the value 66 has the highest relative frequency
   E) the coefficient of variation is 34.187%.

   Solution: The first statement is not true. To see why B-E are true, we proceed as follows.

   For B),

   $$IQR = Q_3 - Q_1 = 64 - 34 = 30.$$

   For C), $1.5(IQR) = (1.5)(30) = 45$, the fences are placed at

   $$Q_1 - (1.5)IQR = 34 - 45 = -11$$
   $$Q_3 + (1.5)IQR = 64 + 45 = 109$$

   There are no values smaller than -11 or larger than 109.
   For D), 66 is the only value which appears 6 times in the diagram. For E), the coefficient of variation is:

   $$\frac{s}{\bar{x}} \cdot 100\% = \frac{16.95}{49.58} \cdot 100\% = 34.187\%$$

3. Continuing with the situation in Question 1, what percentage of the observations are within 2 standard deviations of the mean?

   A) 98.08% B)89.45% C) 96.15% D) 74.32% E)100%.

Solution: We have:

$$\begin{aligned} \bar{x} - 2s &= 49.58 - 2(16.95) = 49.58 - 33.9 = 15.68 \\ \bar{x} + 2s &= 49.58 + 2(16.95) = 49.58 + 33.9 = 83.48 \end{aligned}$$

We have to see how many values are in the interval $[15.68; 83.48]$. This would correspond to how many values are between 16 (inclusively) and 69. We see that there are only 2 values which are outside this range: 10 and 15. Hence, there are 52-2=50 observations in the range 16-69. Since 50/52=0.9615, we conclude that 50 represents 96.15% of the total number of observations.
Note: If you are *incorrectly* counting 15 as being in the interval $[15.68; 83.48]$, you get the wrong answer 51/52=0.9808, i.e. 98.08%.

4. Suppose 80% of college students take a statistics course, 67% take a biology course and 60% take both statistics and biology. What is the probability that a randomly selected college student takes neither statistics nor biology?

   A) 0.83 B)0.89 C) 0.90 D) 0.87 E) 0.13

   Solution: Let A be the event that the student takes statistics and B be the event that the student takes biology. We know that $P(A) = 0.8$, $P(B) = 0.67$ and $P(A \text{ and } B) = 0.93$. By the addition rule, the probability that a student takes statistics or biology is:

   $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.8 + 0.67 - 0.60 = 1.47 - 0.60 = 0.87$$

   The probability that the student takes neither statistics nor biology is:

   $$P(A^c \text{ and } B^c) = 1 - P(A \text{ or } B) = 1 - 0.87 = 0.13.$$

5. The population of bees has diminished across North America and Western Europe. This seems to be related to the pesticides used for crops. It is estimated that bees whose hives are located next to a field on which a pesticide was used have a 45% chance of surviving. Bees whose hives are located next to pesticide-free fields have an 80% chance of surviving. Suppose that 75% of the fields use pesticides. What is the probability that

a randomly selected bee will survive?

A) 0.8735 B) 0.2 C) 0.2245 D) 0.3375 E) 0.5375

Solution: Let A be the event that the bee will survive, and B be the event that its hive is located next to a pesticide field. We know that $P(A|B) = 0.45$, $P(A|B^c) = 0.8$, and $P(B) = 0.75$. By the total probability rule,

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c) = (0.45)(0.75) + (0.8)(0.25)$$
$$= 0.3375 + 0.2 = 0.5375$$

6. Continuing with the situation in Question 5, what is the probability that a randomly selected bee comes from a colony next to a field on which pesticides were used, given that the bee did not survive?

A) 0.7674 B) 0.4125 C) 0.3458 D) 0.8919 E) 0.9765

Solution: We have to calculate

$$P(B|A^c) = \frac{P(B \text{ and } A^c)}{P(A^c)}.$$

By the previous question, $P(A^c) = 1 - 0.5375 = 0.4625$. Using Venn diagrams, we see that:

$$P(B \text{ and } A^c) = P(B) - P(A \text{ and } B) = P(B) - P(A|B)P(B)$$
$$= 0.75 - (0.45)(0.75) = 0.75 - 0.3375 = 0.4125$$

Hence,

$$P(B|A^c) = \frac{0.4125}{0.4625} = 0.8919$$

7. Sleep deprivation impairs the brain's functioning ability. Attention deficits are the most important physical consequences of sleep deprivation. To measure the magnitude of these attention deficits, researchers employ a test called "the psychomotor vigilance task" (PVT) which requires the subject to press a button in response to a light stimulus at random intervals. The following table gives the scores on the PVT test for two groups of medical students: one group of 50 students slept only 5 hours the night before the test, the other group of 50 students slept 8 hours.

	5 hours of sleep	8 hours of sleep	Total
poor PVT score	29	15	44
normal PVT score	21	35	56
Total	50	50	100

(i) Calculate the probability that a randomly chosen student in this group has a poor PVT score.
(ii) Calculate the probability that a randomly chosen student in this group has a poor PVT test, given that the student slept only 5 hours.
(iii) Is the fact that a student scores poorly on the PVT test independent of the fact that the student slept only 5 hours?

A) (i) 0.44 (ii) 0.58 (iii) no;
B) (i) 0.44 (ii) 0.58 (iii) yes;
C) (i) 0.29 (ii) 0.50 (iii) yes;
D) (i) 0.44 (ii) 0.70 (iii) yes;
E) (i) 0.15 (ii) 0.35 (iii) no.

Solution: Let A be the event that the student has a poor PVT test, and B be the event that the student slept 5 hours. From the table, $P(A) = 0.44$, $P(B) = 0.5$ and $P(A \text{ and } B) = 0.29$. Hence

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.29}{0.5} = 0.58 \neq P(A) = 0.44.$$

No, A and B are not independent.

8. Some ornamental plants like Dieffenbachia and Easter lily are poisonous to family pets, after chewing on plant material. A sick cat is brought to a vet's office. With a probability of 1/2, the cat chewed on an Easter lily. It is also possible that the cat chewed on a Dieffenbachia, with a probability of 1/2. The two events are independent of each other. What is the probability that the cat chewed on one of the two plants? (i.e. the cat chewed on the Easter lily or on the Dieffenbachia, including the possibility that the cat chewed on both plants.)

A) 1 B) 1/2 C) 3/4 D) 1/3 E) 1/4

Solution: Let A be the event that the cat chewed on an Easter lily and B be the event that the cat chewed on a Dieffenbachia. We know that $P(A) = P(B) = 1/2$. Since A and B are independent, $P(A \text{ and } B) = P(A)P(B) = 1/4$. The desired probability is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 1/2 + 1/2 - 1/4 = 3/4.$$

9. When breeding, the American robins (*Turdus migratorius*) lay 3 to 5 eggs with the following probabilities:

Number of eggs	3	4	5
Probability	0.25	0.55	0.2

Let X be the number of eggs laid by a randomly chosen robin. Find the standard deviation of X.

A) 0　　B) 0.4730　　C) 0.6690　　D) 0.4475　　E) 0.2238

Solution: We first have to calculate the mean of X:

$$\mu_X = 3(0.25) + 4(0.55) + 5(0.2) = 3.95$$

The variance of X is given by:

$$\sigma_X^2 = 3^2(0.25) + 4^2(0.55) + 5^2(0.2) - (3.95)^2 = 16.05 - 15.6025 = 0.4475$$

The standard deviation of X is:

$$\sigma_X = \sqrt{0.4475} = 0.669$$

10. Assume that 13% of people are left-handed. What is the probability that among 6 randomly chosen persons, one or two are left-handed?

A) 0.3900　　B) 0.5340　　C) 0.1469　　D) 0.2670　　E) 0.1638

Solution: Let X be the number of left-handed persons in a group of 6. X has a binomial distribution with $n = 6$ trials and probability of success $p = 0.13$. We have to calculate

$$P(X = 1) + P(X = 2) = \frac{6!}{1!\ 5!}(0.13)^1(0.87)^5 + \frac{6!}{2!\ 4!}(0.13)^2(0.87)^4$$
$$= 0.3888 + 0.1542 = 0.5340$$

11. The amount of polychlorinated biphenyls (PCBs) in parts per million (ppm) in the carcasses of beluga whales from the Saint-Lawrence river follows a normal distribution with mean 564 ppm and standard deviation 80 ppm. Canadian law stipulates that anything containing more than 600 ppm of PCBs is considered toxic waste. Find the probability that a random beluga whale carcass found on the Saint-Lawrence shore is considered toxic waste.

    A) 0.3264 B) 0.5359 C) 0.6736 D) 0.5040 E) 0.4960

    Solution: Let X be the amount of PCBs found in a randomly chose beluga whale carcass. By standardization,

$$P(X > 600) = P\left(\frac{X - 564}{80} > \frac{600 - 564}{80}\right) = P(Z > 0.45)$$
$$= 1 - P(Z \leq 0.45) = 1 - 0.6736 = 0.3264$$

12. To produce ice wine, grapes that have naturally frozen outside must be picked and pressed at the right temperature. To pick up the grapes for ice wine, the temperature outside has to be between -20 and -8 degrees Celsius. Production usually begins in mid-January. Suppose that on January 15, the temperature at a particular vineyard in Ontario follows a normal distribution with mean -12 and standard deviation 5. Find the probability that the grapes for ice wine can be picked at that vineyard on January 15 for 5 years in a row.

    A) 0.7333 B) 0.3040 C) 0.0013 D) 0.2120 E) 0.1467

Solution: Let X be the temperature on January 15. By standardization,

$$P(-20 < X < -8) = P\left(\frac{-20-(-12)}{5} < \frac{X-(-12)}{5} < \frac{-8-(-12)}{5}\right)$$

$$= P(-1.6 < Z < 0.8) = P(Z < 0.8) - P(Z < -1.6)$$
$$= 0.7881 - 0.0548 = 0.7333$$

In a sequence of 5 years, let Y be the number of years that the grapes can be picked on January 15. Y has a binomial distribution with $n = 5$ trials and $p = 0.7333$ probability of success. We have:

$$P(Y = 5) = \frac{5!}{5!\ 0!}(0.7333)^5(1-0.7333)^0 = (0.7333)^5 = 0.2120$$